THE LONGITUDE DIFFERENCE MERATE - MILANO DERIVED FROM DANJON ASTROLABE OBSERVATIONS BY MEANS OF AN ONE STEP ADJUSTMENT USING AN EXTENDED MODEL

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ABSTRACT

As a part of the establishment of a unified longitude system for the European Triangulation Network the difference in longitude between the reference points Merate and Milano was measured with a Danjon Astrolabe. This paper describes the results of a one-step adjustment of these observations including additional parameters for effects like personal equations and catalogue errors.

1. INTRODUCTION

A substantial basis for the new adjustment of the European Triangulation Network is a homogeneous net of astronomical longitude reference points. As a part of the establishment of such a reference system the differences in longitude between München (Germany) = geodetic reference point, Merate (Italy) = BIH station (MIA) and Milano (Italy) = geodetic reference point and former BIH station (MII) have been determined from measurements with a Danjon Astrolabe in München, Merate, Milano and again München at the end of 1977.

This paper describes the derivation of the longitude difference between Merate (MIA) and Milano (MII) from a common adjustment of all star transits observed at these two observatories in one step, in contrast to the usual way of groupwise adjustment and subsequent computation of the stations longitudes in a second step. The measurements were done by two observers who alternated with each other and who observed within 13 nights 65 groups with altogether 1271 stars.

2. ADJUSTMENT MODEL

The fundamental relation for the evaluation of equal altitude observations is the well known cosine theorem in the astronomic triangle. In a common adjustment of all observations at both stations it has to be solved for the following parameters:

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Latitude $\varphi_i = \varphi_{io} + d\varphi_i$, longitude $\lambda_i = \lambda_{io} + d\lambda$,

difference of personal equations $d\varphi_p$ respectively $d\lambda_p$,

zenith distance $z_k = z_{ko} + dz_k$

with i [1:2] = station index, k [1:65] = group index.

In addition to these 71 fixed parameters we introduced into the model right ascension corrections $d\alpha$ which were set up iteratively for those stars whose mean residuals applying the t-test turned out to be significant on the 95% level. Thus the observation equation for a star transit s becomes

$$\begin{aligned} v_s = & - \cos a_s \, d\varphi_i - \frac{j-1}{2} \cos a_s \, d\varphi_p - \cos \varphi_{io} \sin a_s \, d\lambda_i \\ & - \frac{j-1}{2} \cos \varphi_{io} \sin a_s \, d\lambda_p - dz_k + (\cos \varphi_{io} \sin a_s \, d\alpha_r) - l_s \end{aligned} \quad (1)$$

with

a_s = north azimuth, j[1,3] = observer index,

r = index of stars for which corrections $d\alpha$ had to be estimated,

l_s = free term computed in the usual way from the observed time of transit, approximate values of the unknowns and the apparent places; l_s includes a refraction term and is corrected for polar motion using the pole coordinates and UT1-UTC published by the BIH in circular D.

Introducing the notations

$$\begin{aligned} x_f^T &= (d\varphi_1, d\varphi_2, d\varphi_p, d\lambda_1, d\lambda_2, d\lambda_p, dz_1, \ldots, dz_{65}), \\ x_v^T &= (\ldots, d\alpha_r, \ldots), \quad v^T = (v_1, \ldots, v_{1271}), \quad l^T = (l_1, \ldots, l_{1271}), \end{aligned} \quad (2)$$

B_f = coefficient matrix of the fixed parameters x_f,

B_v = coefficient matrix of the variable parameters x_v,

Q_l = variance-covariance matrix of the observations,

the system of observation equations has the least squares solution

$$\begin{pmatrix} x_f \\ x_v \end{pmatrix} = [(B_f, B_v)^T Q_l^{-1} (B_f, B_v)]^{-1} (B_f, B_v)^T Q_l^{-1} l. \quad (3)$$

The normal equation system is partitioned into a fixed and a variable part in order to have not to invert the whole system again in each iteration. The variable part is successively extended until no further star turns out to be in need of a right ascension correction $d\alpha$. With the abbreviations

$$\begin{aligned} N_{ff} &= B_f^T Q_l^{-1} B_f, \quad N_{fv} = B_f^T Q_l^{-1} B_v, \quad N_{vv} = B_v^T Q_l^{-1} B_v, \\ Q_{ff,1} &= N_{ff}^{-1}, \quad Q_{vv} = (N_{vv} - N_{fv}^T Q_{ff,1} N_{fv})^{-1}, \\ Q_{fv} &= - Q_{ff,1} N_{fv} Q_{vv}, \quad Q_{ff,2} = - Q_{ff,1} N_{fv} Q_{fv}^T, \end{aligned} \quad (4)$$

one gets

$$x_f = \underbrace{Q_{ff,1} B_f^T Q_1^{-1} l}_{\text{fixed}} + \underbrace{Q_{ff,2} B_f^T Q_1^{-1} l + Q_{fv} B_v^T Q_1^{-1} l}_{\text{variable}} \tag{5}$$

$$x_v = Q_{fv}^T B_f^T Q_1^{-1} l + Q_{vv} B_v^T Q_1^{-1} l.$$

The variance-covariance matrix Q_1 of the observations is not known a priori but one can assume that the covariances are negligible and that the variances within one group are equal because they seem first of all to depend on the weather conditions and on the observer's disposition. Under this prerequisite Q_1 is diagonal and its elements q_k can be estimated from the observations in an iterative process (Kubik 1967). Starting from any, for instance equal variances for all groups one gets new estimates q_k of the variances after each adjustment from the equations.

$$q_k = \frac{1}{n_k} v_k^T v_k - b_k, \quad b_k = -\frac{1}{n_k} \mathrm{tr}[(B_f, B_v)_k Q_x (B_f, B_v)_k^T] \tag{6}$$

with n_k = number of stars observed in group k,

v_k = subvector of v belonging to group k,

$(B_f, B_v)_k$ = submatrix of (B_f, B_v) belonging to group k.

3. RESULTS

According to the given formulae system a common adjustment of all 1271 observed star transits was made, and that in two versions:

A. An adjustment without iterative estimation of the 65 group variances. In this case of assuming equal weights for all observations the computer program made 42 iterations for deriving corrections $d\alpha$ of 22 FK4 and 19 FK4 Sup stars.

B. An adjustment including the estimation of the group variances from the observations themselves according to equations (6). In this case the program needed two iteration steps for the estimation of variances which changed no more significantly in a further iteration. Within one variance iteration 47 iterations for deriving corrections $d\alpha$ of 25 FK4 and 21 FK4 Sup Stars have been computed.

The different number of iterations in the two adjustments is due to the different variance matrices of the observations. As main results of the two adjustments the longitudes of Merate (MIA) λ_1 and of Milano (MII) λ_2, the difference of personal equations $d\lambda_p$ and the derived longitude difference $\Delta\lambda$ with their r.m.s. errors are given in table 1.

From the observations at the reference point München done before and after those at the Italian stations it was evident that the personal equations in longitude did not alter.

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Parameter	A	B
λ_1, m_{λ_1}	$37^m42^s.7856 \pm 0^s.0012$	$37^m42^s.7833 \pm 0^s.0011$
λ_2, m_{λ_2}	$36\ 45.8322 \pm 0.0012$	$36\ 45.8314 \pm 0.0009$
$d\lambda_p$, $m_{d\lambda_p}$	$-\ 0.0069 \pm 0.0014$	$-\ 0.0067 \pm 0.0012$
$\Delta\lambda$, $m_{\Delta\lambda}$	$56^s.9534 \pm 0^s.0013$	$56^s.9519 \pm 0^s.0011$

Table 1: Results of the Adjustment

The conventional longitude used in the BIH 1968 reference system was derived for Milano (MII) from observations in the period 1966.50-1967.45, whereas that of Merate (MIA) was established by the observatory and was not based on astronomical observations (Guinot 1978). We have assumed that the actual longitudes of these stations may be obtained by adding the term a' determined by the BIH for the last year of operation; the value is given for MII in the BIH Annual Report for 1969, and for MIA in 1977 by Guinot (1978). The resulting longitudes are:

Merate (MIA) $\lambda_1 = 37^m42^s.7665$ Milano (MII) $\lambda_2 = 36^m45^s.8359$

Thus the difference in longitude in the BIH system between these two stations is approximately $56^s.931$.

Summarizing the results of this paper and comparing them with those given in the BIH system one may make the following statements:

- The common adjustment of all observations using the described model proved useful. The estimation of the group variances within the adjustment yields a remarkable improvement in the inner accuracy of the derived parameters.
- The longitudes of Milano (MII) agree fairly well within 4 ms. In the case of Merate (MIA) there results a difference of about 17 ms which is however imaginable if one considers that the r.m.s. error of coefficient a'_{1977} is $\pm$ 7.9 ms (Guinot 1978).
- It should be considered whether longitude differences like the observed one with accuracies of a few ms could be used for an improvement of the BIH longitude reference system.

4. REFERENCES

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DISCUSSION

S. Debarbat: Am I correct in my deduction, from the small errors that you quote, that the Danjon astrolabe is a very suitable instrument for the determination of differences of longitude?

K. Kaniuth: Yes, in my opinion it is. But I should add that in the case of Merate and Milano, because of the small latitude difference between the stations, nearly identical star programs could have been observed.