Episteme (2025), 1–14
doi:10.1017/epi.2024.31

ARTICLE

Topic-sensitivity and the hyperintensionality of knowledge

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(Received 20 December 2023; revised 9 April 2024; accepted 13 June 2024)

Abstract

It is natural to assume that knowledge, like belief, creates a hyperintensional context, that is, that knowledge ascriptions do not allow for substitution of necessarily equivalent prejacents *salva veritate*. There exist a variety of different proposals for modelling the phenomenon. In the last years, the topic-sensitive approach to the hyperintensionality of knowledge has gained considerable traction. It promises to provide a natural account of why knowledge fails to be closed under necessary equivalence in terms of differences in subject matter. Here, we argue that the topic-sensitive approach, as recently put forward by Franz Berto, Peter Hawke, Aybüke Özgün, and others, faces formidable problems. The root of these problems lies in the approach's prediction that a mere grasp of subject matter may help to provide insights into necessary implications that it would seem to require more substantive epistemic work to gain.

Keywords: Knowledge; epistemic logic; hyperintensionality; topic-sensitivity; aboutness; subject matter; logical omniscience

1. Introduction

We argue that the current topic-sensitive approach to the hyperintensionality of knowledge, as recently put forward by Franz Berto, Peter Hawke, Aybüke Özgün, and others, has problematic consequences. After a brief sketch of why, and how, one might want to move beyond the intensionalist condition for knowledge given by Hintikka (1962) (§2), we go on to distinguish two main varieties of topic-sensitive accounts and briefly describe their core claims (§3), before we zoom in on the account recently advanced by Berto and Hawke (2021) and Berto (2022) (§4).

We diagnose a problem for this account and argue that analogous problems beset the other topic-sensitive accounts on the market (§5). In a nutshell, the problem is this. If φ necessarily implies ψ, so does ⌜φ ∨ ψ⌝,[1] and, whereas the topic of φ may not include the topic of ψ, the topic of ⌜φ ∨ ψ⌝ anyway does. Whenever φ and ψ are related in these ways, the account advanced by Berto and Hawke predicts that, while agents may be in no position to know ψ, relative to their total information, whenever they are in a position to know φ, relative to that same information, they will automatically be in that position

[1]For the use of corner quotes, see Quine 1981: 35–36.

relative to any total information relative to which they are in a position to know the weaker ⌜$\varphi \vee \psi$⌝. Since ψ may be necessarily implied by φ without being a logical consequence of φ, and since it may thus be unobvious that ⌜$\varphi \vee \psi$⌝ implies ψ even to agents with unlimited logical skills, information of the latter kind may not make it obvious either that ⌜$\varphi \vee \psi$⌝ implies ψ. Accordingly, the Berto–Hawke (BH) account credits topic grasping with the power to provide insights into necessary implications that it *prima facie* cannot be said to possess – not even if agents are assumed to have unlimited logical skills. Often, it would seem that substantive epistemic work is needed to gain such insights.

We review some of the strategies that have been proposed to deal with problems in this ballpark. Among these, the strategy to invoke impossible worlds, while construing necessity as truth in all possible worlds, is the *prima facie* most promising one (§6). However, as we go on to argue, a version of the problem persists (§7). Since the diagnosis generalises to other topic-sensitive accounts, we conclude that pending alternative ways to modify or prop up such accounts, their proponents must make further idealisations that go far beyond the idea that epistemic agents have unbounded logical powers (§8).

2. Hyperintensionality and epistemic logic

It is natural to suppose that true *de dicto* knowledge ascriptions more or less faithfully reflect how the content whose knowledge they ascribe is represented in the ascribee's mind. Their prejacents may be formulated in a language the agent doesn't speak; however, for those prejacents to specify the contents of the agent's knowledge, they had better be sufficiently close in cognitive significance to what is, at some level of representation, articulated in the agent's mind. This then immediately casts doubt on the adequacy of the idea, underlying many epistemic logics, that if φ and ψ are necessarily equivalent (i.e. co-intensional), then so are ⌜$K\varphi$⌝ and ⌜$K\psi$⌝ – where K is short for ⌜One knows that⌝ or ⌜The agent knows that⌝. For instance, two formulas may be true in exactly the same circumstances but differ radically in both logical complexity and range of subject matter. Yet, any epistemic agent we might be able to approximate will still be limited both in their logical skills and the range of subject matters they are in a position to entertain.

Such limitations can be illustrated thus. Classically, any formula φ is necessarily equivalent to ⌜$(\varphi \supset \xi) \supset \varphi$⌝, for arbitrary ξ. Call any such conditional a *Peircean equivalent* of φ, and ξ a *joker*. We can imagine substituting any occurrence of φ in a Peircean equivalent of φ by another Peircean equivalent of φ with a new joker. Let there be a machine that goes on repeating this operation. For any agent A like us who satisfies ⌜$K\varphi$⌝, there will be a number n such that after n operations, the result, while still co-intensional with φ, will be logically too complex for A to compute – even if A should have all the resources to mentally represent the subject matters of φ and all the jokers involved. If A cannot logically compute ψ, where ψ is the conditional that results from $m \geq n$ such operations, they cannot competently deduce ψ from φ either. Even if A satisfies ⌜$K\varphi$⌝, A will not then satisfy ⌜$K\psi$⌝. The initial Peircean equivalent of φ, i.e. ⌜$(\varphi \supset \xi) \supset \varphi$⌝, may by contrast be easy to logically compute, if ξ itself is logically simple. Yet, if ξ has a subject matter whose representation requires resources A doesn't command, then, even if A satisfies ⌜$K\varphi$⌝, A won't satisfy ⌜$K((\varphi \supset \xi) \supset \varphi)$⌝.

Epistemic logics can be seen as characterizing the structure of the total epistemic states of the agents they are concerned with. As logics, they abstract away from the kinds of contingencies afflicting real-life agents to varying degrees – such as doxastic or inferential inertia – which make the latter's epistemic states far less systematic than

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epistemic logics predict. To this extent, epistemic logics already come with substantive idealisations of epistemic agency. For instance, it won't in general be considered a good objection to a principle of epistemic logic that real-life agents frequently fail to comply with it because they cannot be bothered, or are too inattentive or time-constrained, to form certain beliefs or draw certain inferences.

It may seem but a small step to carry these idealisations further and to altogether ignore limitations of the kinds alluded to above. However, the differences between agents like us and the agents of concern to the epistemic logics in question will then threaten to no longer be a matter of degree but of principle. Any epistemic agent we might be able to approximate will still have bounded logical and bounded representational powers.

This motivates the search for logics that treat epistemic operators as creating contexts that no longer allow for substitution of co-intensional prejacents *salva veritate*. If the knowledge operator K creates such a *hyperintensional* context, it isn't closed under necessary implication either. Over the years, a number of different frameworks have been proposed to capture such hyperintensionality, including awareness logics (Fagin and Halpern 1988; Fagin et al. 1995; Grossi and Velázquez-Quesada 2015; Fernández-Fernández 2021), logics based on impossible worlds semantics (Hintikka 1975; Rantala 1982; Jago 2014; Berto and Jago 2019; Bjerring and Skipper 2019; Skipper and Bjerring 2020; Solaki 2021), and topic-sensitive logics (Hawke et al. 2020; Berto and Hawke 2021; Berto 2022).

Such logics may still, for the sake of simplicity and focus, come with *some* radical idealisations of the kind just envisaged, depending on what features of epistemic states and limitations on epistemic agency they seek to model. Constructing such a logic, one may, for example, resolve to assume, for the sake of simplicity and focus, that there are no limits on the range of subject matters an agent may entertain at any given moment but impose limits on the complexity of the logical inferences they can draw and the logical forms they can discern (Bjerring and Skipper 2019; Skipper and Bjerring 2020). Alternatively, one may resolve to assume, again for the sake of simplicity and focus, that agents are subject to no limitations on their powers of logical reasoning and discernment of logical form but impose limits on the range of subject matters they can entertain at any given moment. Topic-sensitive accounts belong in that latter camp (Hawke et al. 2020; Özgün and Berto 2021: 769; Berto and Hawke 2021: 4–5; Berto 2022).

As long as one takes the hyperintensionality of knowledge seriously enough, one might reasonably be expected to ultimately aim for a logic that respects either type of limitations (see, however, Williamson 2020 for the opposing view that we had better stick to the intensional framework lest we run the risk of overfitting).[2] In any case, though, the credentials of any such type of partly idealised approach can only properly be assessed if the idealisations are clearly set out from the start. Thus, for example, it will not do to explain away any failure to invalidate unwanted cases of closure under necessary implication by declaring such cases the outcome of some hitherto unspecified idealisation. This will be even less acceptable if, say, on topic-sensitive accounts, it turns out to be precisely the agent's grasp of topic that is responsible for validating those unwanted cases of closure.

3. Varieties of topic-sensitive accounts

Topic-sensitive epistemic logics treat agents as bounded with respect to the range of subject matters they are in a position to grasp or entertain but as unbounded in their logical skills. The key idea is that even if φ and ψ are co-intensional, or the former's

[2]See Berto (2024) for a reply to Williamson's overfitting charge.

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intension is a subset of the latter's, ⌜$K\varphi$⌝ may hold while ⌜$K\psi$⌝ does not, because the agent may grasp φ's topic without grasping ψ's topic, never mind how good they are at logical reasoning and at discerning logical forms – and the same goes, *mutatis mutandis*, for notions of knowledge relativised to bodies of information/evidence and related epistemic notions, relativised or not.

Topic-sensitive accounts of knowledge, and of cognate epistemic notions, come in two main varieties. According to accounts belonging to the first, knowledge requires that two mutually independent, individually necessary and jointly sufficient conditions be satisfied:

$$w \models^{M} \ulcorner K\varphi \urcorner \text{ iff } E \subseteq |\varphi|^{M} \text{ and } t(\varphi) \sqsubseteq \tau,$$

where the monadic operator K is to be read as ⌜The agent knows that⌝, E is the set of epistemically possible worlds left open by the agent's total information/evidence, $|\varphi|^M$ is the intension of φ according to model M, that is, the set of worlds u such that $u \models^M \varphi$, t is a function assigning topics to formulas, $\sqsubseteq$ is parthood, and τ is the fusion of all the topics grasped by the agent.

Some accounts of this variety construe E as world-dependent so that, for some f, $E = f(w)$, with f being a function from worlds to epistemically possible worlds accessible from the former (e.g. Rossi and Özgün 2023: 3–4, 14–15). Such accounts can be seen to simply add a topicality filter to the standard intensionalist account, made prominent by Hintikka (1962), according to which

$$w \models^{M} \ulcorner K\varphi \urcorner \text{ iff } f(w) \subseteq |\varphi|^{M}.$$

Other accounts of this first variety, by contrast, construe E as world-independent (for a corresponding account of evidence-based belief with this feature (see Özgün and Berto 2021: 768–71).

Topic-sensitive accounts of the second main variety focus on notions of knowledge, or of being in a position to know, that are relativised to, or conditional on, certain bodies of information/evidence. On such accounts, the truth clause for the dyadic operator in question again identifies two mutually independent, individually necessary and jointly sufficient conditions:

$$w \models^{M} \ulcorner K_i\varphi \urcorner \text{ iff } E_i \subseteq |\varphi|^{M} \textit{and } t(\varphi) \sqsubseteq \tau_i,$$

where, on some such accounts (e.g. Hawke et al. 2020: 736–37, 741), i represents a certain fragment of the agent's mind, ⌜$K_i\varphi$⌝ is to be read as ⌜The agent knows φ in i⌝, E_i is the set of epistemically possible worlds left open by the total information/evidence available in fragment i, and τ_i is the fusion of all the topics grasped in i (for a corresponding account of evidence-based belief on which, however, formulas are evaluated at pairs of worlds and intension-topic-pairs; see Berto and Özgün 2023: 948). The agent's total knowledge is then taken to be the disjunction of their knowledge in any of the fragments: $K\varphi$ iff for some i, $K_i\varphi$ (Hawke et al. 2020: 737).

On other accounts of this second variety (e.g. Berto 2022: 60–67, 85–86; Berto and Hawke 2021: 14), i itself represents information/evidence, ⌜$K_i\varphi$⌝ is to be read as ⌜Given i as her total information/evidence, the agent is in a position to know φ⌝, E_i is the set of epistemically possible worlds left open by i, and τ_i is the topic of i (or the fusion of the topic of i with τ, as on the corresponding account of evidence-based conditional belief given by Özgün and Berto 2021: 775–76).

Again, some of these accounts construe E_i as world-dependent so that, for some f, $E_i = f_i(w)$, with f being a function from pairs of worlds and information/evidence to epistemically possible worlds accessible from the former (Berto 2022; Berto and Hawke 2021). By contrast, other accounts of this variety treat E_i as world-independent

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(Hawke et al. 2020; cf. also Özgün and Berto 2021, for the case of evidence-based conditional belief).

In what follows, we will primarily focus on accounts of the second variety, more specifically on the account given by Berto and Hawke (2021) and Berto (2022) – the BH account or BH, for short. Although the primary focus is on BH, our main arguments equally apply, *mutatis mutandis*, to the other topic-sensitive accounts identified above.

4. The Berto–Hawke account

Berto and Hawke (2021) and Berto (2022) construe $\ulcorner K^{\varphi}\psi \urcorner$ as ⌜Given φ as her total information, the agent is in a position to know ψ⌝ (we here follow Berto (2022) who uses superscripts rather than subscripts). Since these authors construe the set of epistemically possible worlds left open by the agent's total information as world-dependent, a more perspicuous rendition of the truth clause for the dyadic operator is this:

(BH) $w \models^{M} \ulcorner K^{\varphi}\psi \urcorner$ iff (i) $f_{\varphi}(w) \subseteq |\psi|^{M}$ and (ii) $t(\psi) \sqsubseteq t(\varphi)$.

We call $\langle |\varphi|^{M},\ t(\varphi)\rangle$ the *thick proposition* expressed by φ (in model *M*) and, correspondingly, call $|\varphi|^{M}$ the *thin proposition* expressed by φ (in *M*) and say that $||\varphi||^{M}$ *contains* $||\psi||^{M}$ iff both $|\varphi|^{M} \subseteq |\psi|^{M}$ and $t(\psi) \sqsubseteq t(\varphi)$ (Yablo 2014: 15; Berto 2022: 25).

The notion of information at play is supposed to be non-factive (Berto 2022: 85–86). Somewhat surprisingly, so is the intended notion of being in a position to know relative to one's total information. The following simulacrum of factivity is being offered instead: $\ulcorner K^{\varphi}\psi \urcorner, \varphi \models \psi$ (Berto and Hawke 2021: 16–17; Berto 2022: 93). For, on the BH account, $|\varphi| \subseteq f_{\varphi}(w)$ holds for all φ and *w* (Berto and Hawke 2021: 14; Berto 2022: 83, 93). The authors label the simulacrum 'factivity' and call the latter principle the *Basic Constraint*.

To insist that information isn't factive is to insist that $\ulcorner K^{\varphi}\psi \urcorner \not\models \varphi$. To insist that the relevant notion of being in a position to know isn't factive is to insist that $\ulcorner K^{\varphi}\psi \urcorner \not\models \psi$. $\ulcorner K^{\varphi}\psi \urcorner \models \psi$ iff, for any *w*, $w \in f_{\varphi}(w)$. Given the simulacrum of factivity the authors accept, if $\ulcorner K^{\varphi}\psi \urcorner \models \varphi$, then $\ulcorner K^{\varphi}\psi \urcorner \models \psi$. Similarly, $\ulcorner K^{\varphi}\psi \urcorner \models \varphi$, if $\ulcorner K^{\varphi}\psi \urcorner \models \psi$ and, *in addition*, $\models \ulcorner K^{\varphi}\varphi \urcorner$, that is, $f_{\varphi}(w) \subseteq |\varphi|$. According to Berto and Hawke (2021: 27) and Berto (2022: 93), the latter fails. Note that if both $\ulcorner K^{\varphi}\psi \urcorner \models \psi$ and $\models \ulcorner K^{\varphi}\varphi \urcorner$, every formula will be provably true. So, one of them must anyway be rejected.

Berto and Hawke (2021: 28) suggest that if $\ulcorner K^{\varphi}\psi \urcorner \not\models \varphi$, then $\not\models \ulcorner K^{\varphi}\varphi \urcorner$. They write that 'if a theorist allows non-veridical information, counterexamples [to $\ulcorner K^{\varphi}\varphi \urcorner$] are obvious' since 'if an agent's total information [. . .] has a false part, then factivity assures that the agent does not know' that information (see also Berto 2022: 104). This reasoning is perplexing. On the intended interpretation of the dyadic operator, the truth of $\ulcorner K^{\varphi}\varphi \urcorner$ alone doesn't imply that (the proposition expressed by) φ is known. So, it's unclear how the factivity of *knowledge* might guarantee that if φ is false, so is $\ulcorner K^{\varphi}\varphi \urcorner$. If the authors rather mean ' . . . then the agent *is in no position to know* that information, given that information', then if, here, the principle of factivity alluded to is: $\ulcorner K^{\varphi}\psi \urcorner, \varphi \models \psi$ (as the authors' use of the term suggests), the reasoning continues to be flawed: this principle simply doesn't sanction that if φ is false, so is $\ulcorner K^{\varphi}\varphi \urcorner$. By contrast, we can make perfect sense of the quoted passage, if we take the principle of factivity in question to be $\ulcorner K^{\varphi}\psi \urcorner \models \psi$. But, as said, this is a principle the authors seem unwilling to assume. For, if they did assume it, they would have no reason to opt for the weaker principle instead.[3]

Where $w \models^{M} \ulcorner \psi \text{ CON } \xi \urcorner$ iff $||\psi||^{M}$ contains $||\xi||^{M}$, with the Basic Constraint in place, we get

[3]Berto and Hawke (2021: 28) offer another, independent reason for rejecting $\models \ulcorner K^{\varphi}\varphi \urcorner$ (i.e. $f_{\varphi}(w) \subseteq |\varphi|$), based on their diagnosis of Kripke's paradox of dogmatism.

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$$K^{\varphi}\psi \supset (\varphi \text{ CON } \psi).$$

However, given that, according to BH, $f_{\varphi}(w) \subseteq |\varphi|$ fails, this conditional cannot be strengthened to a biconditional. Note, though, that even if it were the case that, for all w and M, $f_{\varphi}(w) = |\varphi|^M$, this wouldn't imply that, for any M, ⌜$K^{\varphi}\psi$⌝ merely recorded the (world-independent) semantic fact that $||\varphi||^M$ contains $||\psi||^M$. For, epistemic facts do not reduce to semantic facts. Accordingly, even then, ⌜$f_{\varphi}(w)$⌝ would have to retain its intended interpretation in terms of the epistemic possibilities left open by φ at w.

Like other topic-sensitive accounts, BH assumes that logical constants add nothing to the topic of a formula, which topic is conceived as the fusion of the topics of the formula's atomic constituents (Berto 2022: 32–35, 64–65). This assumption is sometimes called *topic transparency* (Hawke et al. 2020: 740; Berto 2022: 32). It highlights that, according to BH, the grasp of the topic of a given formula is indifferent to the latter's logical complexity.

Consequently, conditions (i) and (ii) prove mutually independent. To see that (ii) might hold while (i) does not, note that even if $t(\psi) \sqsubseteq t(\varphi)$, and hence $t(\psi \wedge \neg\psi) \sqsubseteq t(\varphi)$, still, for non-empty $f_{\varphi}(w)$ at least, $f_{\varphi}(w) \nsubseteq |\psi \wedge \neg\psi|$. For instance, it may be that, in w, the agent's total information is that it rains, where, by topic transparency, the topic of ⌜It rains ∧ it doesn't rain⌝ is part of the topic of ⌜It rains⌝. Still, in no world accessible from w relative to that total information does it both rain and not rain. To see that (i) might hold while (ii) does not, note that even if $t(\psi) \not\sqsubseteq t(\varphi)$, and hence $t(\psi \vee \neg\psi) \not\sqsubseteq t(\varphi)$, still, for any w, $f_{\varphi}(w) \subseteq |\psi \vee \neg\psi|$. For instance, it may be that, in w, the agent's total information is that it rains, where the topic of ⌜It snows ∨ it doesn't snow⌝ isn't part of the topic of ⌜It rains⌝. Still, in all worlds accessible from w relative to that total information, it either snows or doesn't snow.

5. A problem

It is easily seen that BH still validates a principle of closure under known implication – in the sense that ⌜$K^{\varphi}\psi$⌝, ⌜$K^{\varphi}(\psi \supset \xi)$⌝ $\models$ ⌜$K^{\varphi}\xi$⌝ (Berto and Hawke 2021: 25; see Bjerring and Skipper 2024, for a criticism of this feature). Likewise, by topic transparency, BH validates the principle that, if, given one's total information, one is both in a position to know ψ and in a position to know ξ, then, given that same total information, one is in a position to know ⌜$\psi \wedge \xi$⌝ (Berto and Hawke 2021: 17–18). By contrast, BH invalidates *closure under necessary implication* (Berto and Hawke 2021: 24).

To see this, let 'Shapy' abbreviate 'the shape displayed in Figure 1', let $\Box$ be the universal necessity modal, and consider:

(1) $\Box$(Shapy is a trefoil knot ⊃ Shapy is chiral) ⊃
(K^{φ}(Shapy is a trefoil knot) ⊃ K^{φ}(Shapy is chiral)).

Since, necessarily, trefoil knots are chiral, we anyway have

(2) $\Box$(Shapy is a trefoil knot ⊃ Shapy is chiral)

The combination of (1) and (2) implies

(3) K^{φ}(Shapy is a trefoil knot) ⊃ K^{φ}(Shapy is chiral).

Given BH, (3) allows for counterexamples even when (2) holds, with the consequence that (1) proves invalid. If |⌜Shapy is a trefoil knot⌝| ⊆ |⌜Shapy is chiral⌝|, then, trivially, if $f_{\varphi}(w) \subseteq$ |⌜Shapy is a trefoil knot⌝|, $f_{\varphi}(w) \subseteq$ |⌜Shapy is chiral⌝|. Still, for suitable choices of

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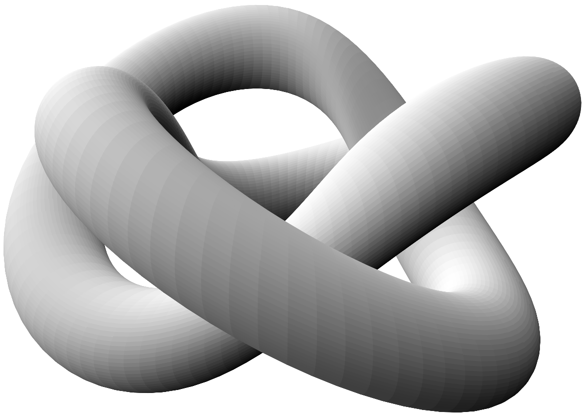

Figure 1. Shapy.

φ, the topic of φ may have the topic of ⌜Shapy is a trefoil knot⌝ as a part, without having the topic of ⌜Shapy is chiral⌝ as a part.

Importantly, however, (3) can be expected to fail for other reasons. It may fail simply because the agent is not, given that φ articulates her total information, in a position to know that trefoil knots are chiral, that is, that trefoil knots cannot be mapped onto their mirror image by rotations and translations alone. Being in a position to know the latter would seem to require expert testimony, knowledge of sophisticated math, or quite demanding exercises of mental rotation and mapping and, as the case may be, possessing the information articulated by φ as one's total information may equip one with none of these.

Proponents of BH will of course agree that, for suitable choices of φ, given that φ articulates one's total information, one may be in a position to know that Shapy is a trefoil knot, while one is not, given that same information, in a position to know that trefoil knots are chiral: the topic of ⌜Shapy is a trefoil knot⌝ may be part of $t(\varphi)$, while the topic of ⌜Trefoil knots are chiral⌝ is not.

However, the latter explanation ultimately doesn't carry far enough. For, if, in situations in which the antecedent of (3) is satisfied, its consequent might fail simply because, given one's total information, one is in no position to know trefoil knots are chiral, then (4) should be allowed to fail *for the very same reason*:

(4) K^{φ}(Shapy is a trefoil knot ∨ Shapy is chiral) ⊃ K^{φ}(Shapy is chiral).

Indeed, it's hard to see how (3) might fail because, given one's total information, one is in no position to know trefoil knots are chiral, without (4) failing, too. After all, what one is in a position to know in being in a position to know that Shapy is a trefoil knot or chiral, though richer in topic, is strictly *weaker* than what one is in a position to know in being in a position to know that Shapy is a trefoil knot. So, if the latter isn't sufficient to put one in a position to know that Shapy is chiral, how could the former nonetheless be? How could grasping the topic of ⌜Shapy is chiral⌝ alone ever make the difference, allowing one to get in the position to recognise that trefoil knots are chiral, and hence that, either way, Shapy is chiral? Being in a position to know the latter requires insights into topology or, at the very least, expert testimony, which, in this case as in the former, one's total information may fail to provide.

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However, BH validates

(5) $\Box$(Shapy is a trefoil knot $\supset$ Shapy is chiral) $\supset$
(K^{φ}(Shapy is a trefoil knot $\vee$ Shapy is chiral) $\supset$ K^{φ}(Shapy is chiral)).

Thus, given (2), BH implies (4) – irrespective of whether ⌜Shapy is chiral⌝, or any record of expert testimony to this effect, is a logical consequence of φ. For, first, if |⌜Shapy is a trefoil knot⌝| $\subseteq$ |⌜Shapy is chiral⌝|, then, equally, |⌜Shapy is a trefoil knot $\vee$ Shapy is chiral⌝| $\subseteq$ |⌜Shapy is chiral⌝|, and, second, the topic of ⌜Shapy is chiral⌝ is part of the topic of ⌜Shapy is a trefoil knot or chiral⌝. Accordingly, whatever φ is, if $f_{\varphi}(w) \subseteq$ |⌜Shapy is a trefoil knot $\vee$ Shapy is chiral⌝|, then $f_{\varphi}(w) \subseteq$ |⌜Shapy is chiral⌝|, and, if the topic of ⌜Shapy is a trefoil knot $\vee$ Shapy is chiral⌝ is part of $t(\varphi)$, then so is the topic of ⌜Shapy is chiral⌝.

For analogous reasons, BH validates

(6) $\Box$(Shapy is a trefoil knot $\supset$ Shapy is chiral) $\supset$
(K^{φ}(Shapy is a trefoil knot $\vee$ Shapy is chiral) $\supset$ K^{φ}(Shapy is a trefoil knot $\supset$ Shapy is chiral)),

– irrespective of whether ⌜Shapy is a trefoil knot $\supset$ Shapy is chiral⌝, or any record of expert testimony to this effect, is a logical consequence of φ. By contrast, BH invalidates (7) alongside (1):

(7) $\Box$(Shapy is a trefoil knot $\supset$ Shapy is chiral) $\supset$
(K^{φ}(Shapy is a trefoil knot) $\supset$ K^{φ}(Shapy is a trefoil knot $\supset$ Shapy is chiral)).

That (5) and (6) be valid, yet (1) and (7) be invalid – and, consequently, that, given (2), (4) be guaranteed to hold, while (3) might fail – is an unpalatable result. It suggests that there is a sense in which being in a position to know less implies being in a position to know more.

This result is an immediate consequence of the fact that BH licences closure under containment. Thus, on BH, we get

$$(\psi \,\mathrm{CON}\, \xi) \supset (K^{\varphi}\psi \supset K^{\varphi}\xi).$$

Since analogous principles hold on the other topic-sensitive accounts of either variety (see also Yablo 2014: 45, 117; Yablo 2017: 1059–60), the problem generalises to those accounts.

We saw that topic-sensitive accounts typically ignore the agent's *logical* limitations, idealising them away from the start. But, note that no amount of idealisation of the agent's purely logical skills will help to make the present result any more palatable. For, even expert logicians, unafflicted by doxastic or inferential inertia, are not, *eo ipso*, savants in topology.

Proponents of topic-sensitive accounts such as BH are explicit that topic-sensitivity is only one out of a whole range of hyperintensionality-inducing phenomena. For instance, Hawke et al. (2020) mention *fragmentation* and *defeasibility* as further factors. However, neither of these two factors is relevant here.

To see this, note that a similar problem afflicts the account given by Hawke et al. (2020), which implements both fragmentation and the defeasibility of knowledge by updates. As indicated, on that account, ⌜$K_i\varphi$⌝ is short for ⌜The agent knows φ in i⌝, where i is a fragment of the agent's mind, E_i is the set of epistemically possible worlds left open by the total information/evidence available in i, and τ_i is the fusion of all the topics grasped in i. Then, if the topic of ⌜Shapy is a trefoil knot $\vee$ Shapy is chiral⌝ is part of τ_i, so is the topic of ⌜Shapy is chiral⌝. Since |⌜Shapy is a trefoil knot $\vee$ Shapy is chiral⌝| $\subseteq$

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|⌜Shapy is chiral⌝|, trivially, if $E_i \subseteq$ |⌜Shapy is a trefoil knot ∨ Shapy is chiral⌝|, then $E_i \subseteq$ |⌜Shapy is chiral⌝|. Accordingly, ⌜K_i(Shapy is a trefoil knot ∨ Shapy is chiral) ⊃ K_i(Shapy is chiral)⌝ will hold *in all fragments i*, never mind how little the agent may know about topology in *i*. Since this is puzzling even before we concern ourselves with ways in which knowledge may be defeated upon update with further information, neither fragmentation nor defeasibility will help explain how this conditional might fail.[4]

6. Impossible worlds to the rescue

We argued that BH provides us with only one, rather limited explanation of why, given (2), (3) might fail – namely, t(⌜Shapy is chiral⌝) $\not\sqsubseteq t(\varphi)$ – an explanation unavailable to explain why, given (2), (4) might fail (since, if t(⌜Shapy is a trefoil knot ∨ Shapy is chiral⌝) $\sqsubseteq t(\varphi)$, then, likewise, t(⌜Shapy is chiral⌝) $\sqsubseteq t(\varphi)$). Failure to grasp the topic of ξ in the course of grasping the topic of φ is but one reason why one might fail to be in a position to know ⌜ψ ⊃ ξ⌝, given φ, in spite of being in a position to know ψ, given φ.

By contrast, whatever might ultimately help explain why, given (2), (4) might fail, will also be available as an explanation of why, given (2), (3) might fail. Just consider cases in which ⌜K^{φ}(Sharpy is a trefoil knot)⌝ holds at w, while t(⌜Shapy is chiral⌝) $\sqsubseteq t(\varphi)$ – say, because $||\varphi||^M$ contains ||⌜Shapy is chiral ∨ ¬(Shapy is chiral)⌝$||^M$.

The question accordingly is whether BH – or any of the other topic-sensitive accounts – can avail itself of resources that are sufficient to provide such an explanation and to thereby invalidate (5) and (6) alongside (1) and (7).

In the context of responding to problems in this ballpark, Hawke et al. (2020: 748) observe that, even where ψ is a necessary truth (in model M), intuitively, knowing ψ is not already part of knowing ⌜ψ ∨ ¬ψ⌝, even if ||⌜ψ ∨ ¬ψ⌝$||^M$ contains $||\psi||^M$, in the sense of 'contains' defined earlier. The same applies, *mutatis mutandis*, to knowing in fragment *i*, and to being in a position to know given that φ articulates one's total information. Yet, as we have seen, it anyway follows from BH that containment is sufficient for closure.

This is so far merely a way of stating the problem. However, Hawke et al. (2020: 748) go on to suggest that in order to heed these intuitive verdicts about knowing one thing being part of knowing another, topic-sensitive accounts might suitably be modified in such a way that containment, as defined, no longer suffices for closure. In application to BH, this would in turn require that either condition (i) or condition (ii), or both, be replaced by something more demanding, or else a third condition be added. It thus far remains unclear what these replacements or additions might consist in.

Hawke et al. (2020: 749–51) also consider problem cases in which ⌜ψ ⊃ ξ⌝ is a necessary truth (in model M) so that, accordingly, ||⌜ψ ∧ (ξ ∨ ¬ξ)⌝$||^M$ contains $||\xi||^M$, and hence, given the account they propose, ⌜K_i(ψ ∧ (ξ ∨ ¬ξ)) ⊃ K_iξ⌝ holds (in M). They go on to suggest that our reluctance to accept the latter conditional might be owing to our tendency to conflate ascriptions of knowledge of conjunctions with ascriptions of knowledge of each of their conjuncts, and that appeal to fragmentation can successfully

[4]An additional source of hyperintensionality – not investigated by Hawke et al. (2020) – are *guises* or *modes of presentations*, at least insofar as sameness of topic doesn't imply sameness of guise/mode of presentation. (The relation between topics and guises/modes of presentation is tentatively explored by Berto 2022: 37–40.) The same thick proposition may then come in different guises/modes of presentation, in such a way that the agent may fail to recognise that they are dealing with the very same thick proposition. Not even guises/modes of presentation can help in the present case, though, since we may stipulate that there is no difference in guise/mode of presentation involved when ⌜Shapy is a trefoil knot⌝ occurs on its own or as the first disjunct of ⌜Shapy is a trefoil knot ∨ Shapy is chiral⌝.

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deal with the problem of explaining why one may know each of ψ and $\ulcorner\xi \vee \neg\xi\urcorner$ without knowing ξ.

Clearly, though, whatever its merits, this strategy is of little use in the present case. If $\ulcorner\psi \supset \xi\urcorner$ is a necessary truth (in model M), then $||\ulcorner\psi \vee \xi\urcorner||^M$ likewise contains $||\xi||^M$ and so, on the account proposed by Hawke et al. (2020), $\ulcorner K_i(\psi \vee \xi) \supset K_i\xi\urcorner$ holds (in M). If we let ψ be ⌜Shapy is a trefoil knot⌝ and ξ be ⌜Shapy is chiral⌝, the present case is a case of just this sort. Yet, ⌜Shapy is a trefoil knot ∨ Shapy is chiral⌝ is not a conjunction, and, hence, our reluctance to accept ⌜K_i(Shapy is a trefoil knot ∨ Shapy is chiral) ⊃ K_i(Shapy is chiral)⌝ cannot be blamed on any such conflation. We still want to say that to know ⌜Shapy is a trefoil knot ∨ Shapy is chiral⌝ is not even in part to know ⌜Shapy is chiral⌝. The same goes for the notion of being in a position to know relative to φ as one's total information, and our reluctance to accept (4).

A *prima facie* more promising line of response is to introduce *impossible* worlds – for example, worlds in which ⌜Shapy is a trefoil knot⌝ is true but ⌜Shapy is chiral⌝ is false – and to no longer conceive of $\Box$ as the universal necessity modal (Hawke et al. 2020: 749). As long as $f_\varphi(w)$ includes such impossible worlds, while $\Box$ exclusively ranges over possible worlds, (2) may hold, while (3) and (4) both fail. For, then, $f_\varphi(w)$ may be a subset of the set of worlds in which Shapy is a trefoil knot or chiral, where this subset now includes an impossible world in which Shapy is a trefoil knot but not chiral.

However, as we shall argue in the next section, this technical fix notwithstanding, the account still is hostage to controversial assumptions that it proves hard to sustain.

7. Another bump in the carpet

Even with the introduction of impossible worlds, and the replacement of the universal necessity modal by a necessity operator exclusively ranging over possible worlds, BH remains committed to

(8) If $|\ulcorner\psi \vee \xi\urcorner|^M \subseteq |\xi|^M$, then $w \models^M \ulcorner K^\varphi(\psi \vee \xi) \supset K^\varphi\xi\urcorner$.

Indeed, BH is committed to a more general claim, namely

(9) If $|\psi|^M \subseteq |\xi|^M$ and $t(\xi) \sqsubseteq t(\varphi)$, then $w \models^M \ulcorner K^\varphi\psi \supset K^\varphi\xi\urcorner$.

That is, if all ψ-worlds are ξ-worlds, and the topic of φ has the topic of ξ as a part, the agent is in a position to know ψ, given that φ articulates her total information, only if she is likewise in a position to know ξ, given that same total information. As we shall proceed to argue, (9) has untoward consequences.

Let φ, ψ, and ξ be such that both $\varphi \not\models \xi$ and $\psi \not\models \xi$ and hence such that both $\varphi \not\models \ulcorner(\xi \vee \neg\xi) \supset \xi\urcorner$ and $\psi \not\models \ulcorner(\xi \vee \neg\xi) \supset \xi\urcorner$. Assume that $|\psi|^M \subseteq |\xi|^M$. Recall that conditions (i) and (ii) are mutually independent. Accordingly, suppose that, for a given w and M, $w \models^M \ulcorner K^\varphi\psi\urcorner$, but $t(\xi) \not\sqsubseteq t(\varphi)$. Then, $w \not\models^M \ulcorner K^\varphi\xi\urcorner$. Now, let $\varphi' = \ulcorner\varphi \wedge (\xi \vee \neg\xi)\urcorner$. Consequently, $\varphi' \not\models \xi$ and $t(\xi) \sqsubseteq t(\varphi')$.

Consider what holds in M at w when φ', rather than φ, is the agent's total information. Plausibly, the agent is *in no worse* position to know ψ relative to φ' than she is relative to φ. After all, $\ulcorner\xi \vee \neg\xi\urcorner$ doesn't serve as a defeater for knowledge of ψ; in fact, it has no bearing at all on the epistemic standing of ψ (cf. Berto and Hawke 2021: 18–22, for a discussion of such defeaters). But, if so, then, according to BH, $w \models^M \ulcorner K^{\varphi'}\psi\urcorner$.

Given $|\psi|^M \subseteq |\xi|^M$ and $w \models^M \ulcorner K^{\varphi'}\psi\urcorner$, it follows that $f_{\varphi'}(w) \subseteq |\xi|^M$. BH thus implies that, likewise, $w \models^M \ulcorner K^{\varphi'}\xi\urcorner$. This commits the proponents of BH to saying that the

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agent accordingly is *in a better* position to know ξ relative to φ' than she is relative to φ. At best, this might happen if grasping the topic of ξ puts the agent in a position to realise that ξ holds if ψ holds. For example, if ψ is ⌜Jane and Jill are sisters⌝ and ξ is ⌜Jane and Jill are siblings⌝, then grasping the topic of ξ, the agent can work her way from knowing that ψ holds to knowing that ξ holds, as 'is a sibling' is defined as 'is a brother or sister'. However, there is no guarantee that there will be such a transparent, definitional link for *all* choices of ψ and ξ. Indeed, there is no such link connecting ⌜is a trefoil knot⌝ and ⌜is chiral⌝ that would allow the agent to simply read off the definition of the latter that whatever falls under the former falls under the latter (for an illustration of this, see Dehn 1914).

Returning to our earlier example, if ψ is ⌜Shapy is a trefoil knot⌝ and ξ is ⌜Shapy is chiral⌝, then, plausibly, the agent is *in no better* position to know ξ relative to φ' than she is relative to φ. Just suppose that φ is ⌜Sam knows that Shapy is a trefoil knot⌝ (where Sam ≠ the agent). Yet, if the agent is in no better position to know ξ relative to φ' than she is relative to φ, then, according to BH, for these choices of ψ and ξ, it after all cannot be that $|\psi|^M \subseteq |\xi|^M$. Since, necessarily, trefoil knots are chiral, M must therefore include impossible worlds at which ψ holds, but ξ does not, assigning such worlds to $|\psi|^M$, so that $|\psi|^M \nsubseteq |\xi|^M$. That's the technical fix.

Someone might complain that this technical fix involves an illicit change of meaning. For, it would seem that, if |⌜Shapy is a trefoil knot⌝$|^M \nsubseteq$ |⌜Shapy is chiral⌝$|^M$, then, relative to M, ⌜Shapy is a trefoil knot⌝ and ⌜Shapy is chiral⌝ can no longer be understood to attribute the properties of being a trefoil knot and of being chiral, respectively, as nothing instantiates the former property without instantiating the latter (cf. Williamson 2020: 247–48, for a related concern). Epistemic agents may consider worlds as possible that are in fact impossible. But, if their actual meanings are any guide, neither ⌜Shapy is a trefoil knot⌝ nor ⌜Shapy is chiral⌝ concerns what epistemic agents consider possible; these formulas simply wouldn't seem to attribute any epistemic or otherwise agent-relative properties. So, what other properties might ⌜Shapy is a trefoil knot⌝ and ⌜Shapy is chiral⌝ be understood to attribute in M? Whatever these properties are, M would seem to imbue the two formulas with meanings that differ from the intended ones.

However, there is a rejoinder to this general complaint about the effects of countenancing impossible worlds. Once impossible worlds are being introduced, we should think of the intension of a given formula ψ relative to a model M as the union of two sets, the set of possible worlds at which ψ is true according to M and the set of impossible worlds at which ψ is true according to M, so that $|\psi|^M = |\psi|^{\text{poss}M} \cup |\psi|^{\text{imposs}M}$. It might accordingly be suggested in reply that when it comes to the *meaning* of ψ, only $|\psi|^{\text{poss}M}$ matters. Since, for all that has been said about M, it continues to be the case that |⌜Shapy is a trefoil knot⌝$|^{\text{poss}M} \subseteq$ |⌜Shapy is chiral⌝$|^{\text{poss}M}$ – so that, to this extent, M remains faithful to the intended meanings of the formulas involved – the fact that |⌜Shapy is a trefoil knot⌝$|^M \nsubseteq$ |⌜Shapy is chiral⌝$|^M$ need thus imply no illicit change of meaning.

But, what general guarantee is there, even once impossible worlds are added to the mix, that whenever it *does* hold that $|\psi|^M \subseteq |\xi|^M$, it is automatically transparent to the agent that ξ holds if ψ holds, if her total information enables her to grasp the topic of ξ (e.g. if her total information is articulated by φ' rather than φ)? We surely cannot constrain the assignment of intensions to K-free formulas in the light of our intuitive, pretheoretic verdicts on what the agent is, or isn't, in a position to know, or is in a position to work out by attending to the subject matter of what is *de facto* implied by what she is in a position to know. This would be an illicit case of reverse engineering, solely designed to guarantee the material adequacy of BH. But, similarly, neither can the assignment of intensions to K-free formulas be constrained by features of their topics, so as to guarantee that, for example, whenever $|\psi|^M \subseteq |\xi|^M$, the definition of what ξ is about is a generalisation of the definition of what ψ is about such that anyone familiar with both can deduce ξ from ψ, at least if suitably logically competent (in which event, after all, $\psi \models \xi$).

Once the *K*-free formulas are assigned their intensions, as well as their topics, BH determines which *K*-formulas are true, relative to some function *f* from pairs of formulas and worlds to sets of worlds. Whether the latter provides representations faithful to our intuitive verdicts about what the agent is in a position to know, relative to varying pieces of total information – that is, whether BH is materially adequate – might be adjudicated by appropriate choices of *f*. But, it cannot be a matter decided by revisiting the assignment of intensions to *K*-free formulas and making adjustments accordingly (e.g. by making it the case that $|\psi|^M \nsubseteq |\xi|^M$, solely to ensure that $f_{\varphi'}(w) \subseteq |\psi|^M$, but $f_{\varphi'}(w) \nsubseteq |\xi|^M$). This would be to put the cart before the horse.

The problem comes into starker relief, once we set out to give a natural interpretation of what $f_\varphi(w)$, $f_{\varphi'}(w)$, *etc.* stand for. Asking for such an interpretation seems legitimate, as, on the BH account, the converse of the Basic Constraint fails, and so, $f_\varphi(w) \neq |\varphi|^M$, for some *M*. While they say rather little about the way in which φ and *w* conspire to determine $f_\varphi(w)$, Berto and Hawke (2021: 14) give the following gloss on $f_\varphi(w)$: $w' \in f_\varphi(w)$ if, and only if, relative to *w*, *w*′ 'is not ruled out by knowledge that can be based on the total information' φ. Given its impredicativity, this gloss still allows for more informative interpretations. One such interpretation, suggested by the little that the authors do say about the way in which φ and *w* conspire to determine $f_\varphi(w)$, is in terms of *undefeated information*, where the agent's total undefeated information can be understood to be the combination of (a) that part of the agent's total information that constitutes her evidence, *E*, and (b) everything in her total information that *E* 'carries information about' such that no other piece of her total information, however misleadingly, defeats the claim that *E* does so (cf. Berto and Hawke 2021: 18–22). Accordingly, the present suggestion is that $f_\varphi(w)$ is the strongest thin proposition implied by the undefeated information the agent has when it is φ that articulates her total information.[5]

But, now, on a suitably externalist reading of 'evidence' and 'carrying information', it may well happen that the agent's evidence *E* carries information about something that is not itself a logical consequence of φ. More specifically, it may happen that, while the agent has unlimited logical skills and $f_\varphi(w)$ implies both $|\psi|^M$ and $|\xi|^M$, the former implication is transparent to the agent, whereas the latter implication is not. For instance, if the agent's sole evidence is that Shapy is a trefoil knot (or that Sam knows that Shapy is a trefoil knot) – so that the agent is in a position to know that Shapy is a trefoil knot, given her total information – then, even if this evidence carries information about Shapy's being chiral, where nothing implied by the agent's total information defeats this connection, the agent may nonetheless fail to be in a position to realise, or acknowledge, or be responsive to that fact. The point then is that nothing might change in this regard if the agent's total undefeated information furthermore contains that either Shapy is chiral or Shapy isn't chiral, without yet having ⌜Shapy is chiral⌝ as a logical consequence.

Accordingly, the problem – of unduly crediting facts of topic inclusions with the power to render intensional connections transparent to the agent – persists, never mind whether the worlds of the model include impossible worlds. No idealisation of the agent's *logical*

[5]We say that, for any sets of worlds, *X* and *Y*, and any topics *x* and *y*, respectively assigned to *X* and *Y*, *X* implies *Y* iff $\langle X, x\rangle$ implies $\langle Y, y\rangle$ iff $\langle X, x\rangle$ implies *Y* iff *X* implies $\langle Y, y\rangle$ iff $X \subseteq Y$. See Berto and Özgün (2023: 947) for a framework in which topics are assigned directly to sets of worlds, without the need of formulas as vehicles.

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skills can diminish the badness of this result. Nonlogical intensional connections of the kind at issue are to be found in many areas of thought, where it will continue to be implausible to presume that grasping the topics involved already suffices for such connections to suddenly become transparent. Besides attributing unlimited logical skills, we would have to assume in addition that the agent is maximally competent in whatever area of thought both ψ and ξ belong to, where such maximal competence is likely to not only require unlimited computational prowess but substantive knowledge of theory (again see Dehn 1914 for an illustration of what this might involve in the case of topology).

8. Conclusion

The topic-sensitive approach to the hyperintensionality of knowledge aims to model the epistemic states of agents that are at most logically, but not representationally unbounded. Its analyses of epistemic states combine an intensional condition – that is, truth in all epistemically possible worlds – with a topicality filter. The approach may take different forms, and we distinguished at least two main varieties. Accounts belonging to the first employ the familiar monadic knowledge operator and demand that the topic of its prejacent be included in the totality of topics grasped by the agent. Accounts belonging to the second variety employ a dyadic operator – for knowledge relative to fragments of the agent's mind or for being in a position to know relative to the agent's total information – and demand that the topic of its prejacent be included in the topic of the relevant fragment or in that of the agent's total information.

Accounts of either variety make overly strong predictions, even for logically unbounded agents: they predict that such facts of topic inclusion suffice in order for the agent to gain insights into necessary implications that it requires substantive epistemic work to gain – including insights into necessary implications that are not purely logical in nature. Unless they are suitably modified or propped up by adding further conditions, topic-sensitive accounts would therefore seem to presuppose more radical idealisations than are involved in crediting agents with unlimited logical skills. Extant attempts to modify or prop up such accounts, so as to avoid the need for further idealisations of this kind, prove ill-suited to forestall the overly strong predictions they make.[6]

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[6]The authors would like to thank Aybüke Özgün, Timothy Williamson, an anonymous reviewer for this journal, and members of the audience at the LOGOS Graduate Reading Group for their helpful comments on earlier drafts. Work on this paper received funding from the research project PID2021-122566NB-I00, financed by the Spanish Ministry of Science, Innovation and Universities, and the Marîa de Maeztu grant CEX2021-001169-M (funded by MICIU/AEI/10.13039/501100011033).

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Cite this article: Rossi N. and S. Rosenkranz (2025). "Topic-sensitivity and the hyperintensionality of knowledge." *Episteme* 1–14. https://doi.org/10.1017/epi.2024.31