GRAVITATIONALLY LENSED QUASARS AS COSMOLOGICAL PROBES: THE UNIQUENESS PROBLEM *

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ABSTRACT. The generalisation from circular to elliptical mass distributions acting as gravitational lenses produces a number of interesting effects. These effects include an extra indeterminacy in predictions of the time delay between the two brightest images, if only the relative brightness and separation of these images is known. This indeterminacy is removed if further information about the system (such as a measurement of the astigmatism of the lens) is available.

1. THE PROBLEM

Gravitationally-lensed quasars producing multiple images have been suggested as cosmological probes (Refsdal 1964; Young et al. 1981): the time delays between variations of the image brightnesses measure the distance of the quasar, so that the system can be used to determine the Hubble constant. This method of measuring H_0 is feasible only if the time delay as a function of distance can be predicted from the observables of the image system, the relative magnifications and separations of the images. The variables in the problem are the structure and location of the lensing mass -- since dark matter may contribute to the lens, the use of the apparent (luminous) lens structure is precluded.

For a circularly-symmetrical lens, the image positions and brightnesses are determined entirely by the masses enclosed at the image radii and the surface densities at the images. An uncertainty in the predicted time delay arises to the extent that the distribution of mass between the images is unknown (Gorenstein et al. 1985), and that the symmetry transformation of Falco et al. 1985 can be applied.

2. THE MODEL

The properties of elliptical lenses have been explored to investigae the time delays that arise from a lens model without circular symmetry. Only one simple case will be discussed here -- that of a uniform elliptical disc mass, with axial ratio b:a = 2:1. It will be assumed that the observed image structure consists only of the two brightest images with relative brightness μ_{12} and separation β_{12}. The consistency of this assumption is assured by discussing solutions only where the third

* Discussion on p.553

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image is fainter than either of the other images by at least a factor 10. The problem of the uniqueness of the time delay τ_{12} is then reduced to that of determining whether observations of (β_{12},μ_{12}) uniquely define τ_{12}, once the angular size of the lens and its surface mass density have been estimated (by other observations or by physical arguments).

The deflection angles produced by an elliptical disc correspond to an astigmatic focusing lens for images projected within the disc (a circular disc is not astigmatic). Outside the disc, the elliptical lens shows regions where the deflection angles do not tend to zero monotonically as the images move further from the disc centre, so that there are regions where magnification features unlike those present in the circular lens may appear.

3. THE RESULTS

The observed (β_{12},μ_{12}) for a given circular lens specify a unique τ_{12} at all but one (β_{12},μ_{12}). The time delays at this equivalent image configuration are of the same sense (the brighter image to fainter image delay is negative). If the third image can be detected, its location resolves the uncertainty in the lens/source configuration produced.

The elliptical lens, on the other hand, produces a single (β_{12},μ_{12}) at two different source locations over much of the (β_{12},μ_{12}) plane. For the cases of most interest, the brighter two images are projected outside the disc, and a detection of the third image may not resolve the indeterminacy in the configuration. As an example of the uncertainty in τ_{12} that can be produced, for a particular mass of the elliptical lens model, an equivalent image configuration arises at β_{12} = 2.51a, and μ_{12} = 1.57. In this case, the time delay between variations in the brightest images can be either -0.56 or +0.52τ_0, where τ_0 is a scale time depending on the distances of lens and quasar. In this example, the geometry of the two configurations is so similar that observations of all three image positions cannot resolve the ambiguity, and the difference in the relative brightness of the third image is only 0.4 mag.

The conclusion from this work is that an elliptical gravitational lens produces an extra indeterminacy in the time delay over that produced by a circular lens. Further information than simply the brightness ratios and separations of the images and the characteristics of the lens are required to remove this indeterminacy, and to predict a unique time delay that can be used to measure H_0. It is of interest to note that the best models of the double quasar of Gorenstein et al. (1985) require the use of an astigmatic lens mass resembling the elliptical model considered here, and that the removal of the indeterminacy in this case is effected by the measurement of all components of the relative magnification matrix of the brightest two images.

4. REFERENCES

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