

## CLIMATIC SHIFT OF THE EQUILIBRIUM LINE: KUHN'S CONCEPT APPLIED TO THE GREENLAND ICE CAP

by

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### ABSTRACT

The shift of the equilibrium line in response to climatic disturbances is calculated for the International Glaciological Greenland Expedition (EGIG) profile of the Greenland Ice Cap. Data of heat balance studies carried out during EGIG 1959 and EGIG 1967 are used according to Kuhn's algorithm. The following parameters are applied: the heat transfer coefficient of the sensible heat flux; the effective emissivity of the atmosphere for long wave downward radiation; a factor relating net radiation to cloudiness; the duration of the ablation season; the altitudinal temperature gradient; the altitudinal accumulation gradient.

The altitudinal shift of the equilibrium line results for the EGIG profile + 77m/°C and -4 m per 1/10 cloudiness with 35d of ablation. Changes in heat balance by formation of superimposed ice are taken into account.

### INTRODUCTION

Kuhn (1980a) published a general algorithm, calculating by approximation the shift of the equilibrium line in response to small climatic changes. For such a calculation well founded data of the heat balance must be available. This algorithm has already been used for comparing recent positions of the equilibrium line in the Alps and for those of the younger Dryas (Kuhn 1980b). Another paper deals with the climatically conditioned shifts of the equilibrium line in the Andes (Kuhn 1981).

The reactions of large ice sheets in relation to climatic changes are the main subject of current studies (Ambach 1980; Mercer 1978; Radok and others 1982). An altitudinal shift of the equilibrium line leads to simultaneous changes of the areas of ablation and accumulation. For example, if the equilibrium line is raised, the area of ablation becomes larger while the area of accumulation becomes smaller by the same amount.

An extension of Kuhn's algorithm to the Greenland Ice Cap is elaborated in the present paper, since comprehensive heat balance studies and other glaciological investigations were made in the area of ablation and accumulation during the International Glaciological Greenland Expedition. These studies concentrate on the profile Equip Sermia - Station Centrale - Jarl Joset - Cecilia Nunatak (EGIG profile).

### KUHN'S ALGORITHM

An altitudinal shift of the equilibrium line results from disturbances of the steady state, ie with disturbances in specific cumulative accumulation ( $\delta c$ ), in net radiation balance ( $\delta R$ ) and in temperature ( $\delta Ta$ ). For a steady state of the equilibrium line, Kuhn (1980b) gives the following condition:

$$\frac{\partial c}{\partial z} \Delta h + \delta c = \frac{\tau}{L} \left[ \frac{\partial R}{\partial z} \Delta h + \delta R + \alpha \left( \frac{\partial Ta}{\partial z} \right) \Delta h + \alpha \delta Ta \right] \quad (1)$$

where  $\delta c$ ,  $\delta R$ ,  $\delta Ta$  stand for the above-mentioned

disturbances,  $\frac{\partial c}{\partial z}$ ,  $\frac{\partial R}{\partial z}$ ,  $\frac{\partial Ta}{\partial z}$  being gradients which in the

present state have been taken from measurements.  $\Delta h$  is the altitudinal shift of the equilibrium line,  $\alpha$  is the heat transfer coefficient,  $\tau$  is the number of days of ablation at the equilibrium line, and  $L$  the specific heat of the ice melt. Equation 1 is based on the idea that the specific net balance equals zero at the end of the balance year in the new position of the equilibrium line, regardless of the disturbances  $\delta c$ ,  $\delta R$ ,  $\delta Ta$ .

In the algorithm the following approaches are made:

- For the sensible heat flux ( $H_s$ ) the formulation (Kuhn 1979)

$$H_s = \alpha Ta \text{ is valid for the melting surface.}$$

- The mass loss by evaporation is negligible (Ambach 1963)

- Changes of the latent heat flux  $H_L$  and of the altitudinal gradient  $\frac{\partial H_L}{\partial z}$  are zero

- The dependence of  $c$ ,  $R$  and  $Ta$  on the altitude is taken into account in the region of the equilibrium line using the altitudinal gradients  $\frac{\partial c}{\partial z}$ ,  $\frac{\partial R}{\partial z}$  and  $\frac{\partial Ta}{\partial z}$

- The duration of the ablation season is assumed to be constant, regardless of the disturbances  $\delta c$ ,  $\delta R$ ,  $\delta Ta$ .

### APPLICATION TO THE EGIG-PROFILE

The data necessary for applying Eq. (1) are taken from data measured in the EGIG profile. The characteristics of the heat balance used for modelling are published in detail elsewhere (Ambach to be published).

Explanation of  $\delta c$ ,  $\frac{\partial c}{\partial z}$

$^{18}O$  analyses on cores in the EGIG profile showed averaged water equivalents of the annual net accumulation to have been constant back to 600 A.D. (Reeh and others 1978). Hence,  $\delta c = 0$  is assumed. The

altitudinal gradient  $\frac{\partial c}{\partial z} = 0.55 \text{ kgm}^{-2}\text{m}^{-1}$  is taken from

measurements of winter accumulation in regions between 600 and 1240 m a.s.l. (Ambach 1963, p 298, Table 53).

The fact that the melt water in the region of the equilibrium line is completely transformed into superimposed ice (DeQuervain 1969) has been taken into account. In the case of superimposed ice formation, the heat necessary for melting one annual layer is higher by a factor of 5/3 than it is in the case of melting an annual layer without any formation of superimposed ice (Ambach, to be published). Hence it holds

$$\frac{Q'}{Q''} = \frac{5}{3} \text{ respectively } \frac{m'}{m''} = \frac{5}{3}$$

where  $Q'$ ,  $Q''$  ( $\text{MJm}^{-2}$ ) are the amounts of heat necessary

for melting for both cases, with and without superimposed ice, respectively. In analogy,  $m'$  and  $m''$  ( $\text{kgm}^{-2}$ ) are the melted masses. In connection with Kuhn's algorithm this means that in the case of complete formation of superimposed ice, the gradient of cumulative accumulation must be higher by a factor of 5/3 to allow for the mass actually melted. Because of the formation of superimposed ice, the rounded off value of

$$\frac{\partial c}{\partial z} = 0.55 \cdot \frac{5}{3} \cdot 1.0 \text{ kgm}^{-2} \text{m}^{-1} \text{ is substituted for}$$

$$\frac{\partial c}{\partial z} = 0.55 \text{ kgm}^{-2} \text{m}^{-1}.$$

Explanation of  $\delta R$ ,  $\frac{\partial R}{\partial z}$

According to Kuhn (1980b), only the change of the long wave downward radiation with the air temperature ( $\delta R = \delta A$ ) needs to be taken into account in the change of the net radiation balance. The short wave radiation flux, the albedo and the long wave upward radiation flux of the surface remain constant. According to the Stefan-Boltzmann law for  $\delta R = \delta A$  it results

$$\delta A = 4\epsilon^+ \sigma T^3 \delta T_a$$

with  $\epsilon^+$  being the relative emissivity at an average cloudiness of 5/10, which is calculated from the measured dependence of the long wave radiation balance on the cloudiness. Using values measured in the area of ablation (Ambach 1963, p 70, Figure 20) an average of  $\epsilon^+ = 0.92$  is obtained. With  $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-4}$  and  $T = 273.15 \text{ K}$  it holds

$$\delta A = \alpha' \delta T_a$$

$$\frac{\delta A}{\delta z} = \alpha' \frac{\partial T_a}{\partial z}$$

$$\alpha' = 0.37 \text{ MJm}^{-2} \text{d}^{-1} \text{ } ^\circ\text{C}^{-1}$$

Explanation of  $\frac{\partial T_a}{\partial z}$ ,  $\alpha$

The altitudinal gradient of the air temperature was obtained by comparing measurements at the climate station of Jakobshavn (40 m a.s.l.) on the west coast, and at Camp IV EGIG 1959 (1013 m a.s.l.) in the ablation area. For the duration of the ablation season,  $-0.73^\circ\text{C}/100 \text{ m}$  is obtained on average (Ambach 1977, p 35). For very large glaciers, Braithwaite (unpublished) obtained values for the temperature gradient  $< -0.6^\circ\text{C}/100\text{m}$ , due to advection. The 10 m snow-temperatures in the accumulation area are about  $-1^\circ\text{C}/100\text{m}$  (DeQuervain 1969). This is due to the cooling of the snow cover with increasing altitude, on account of the heat balance, so that the air temperature cannot be substituted by the 10 m snow temperature. The gradient of  $-0.73^\circ\text{C}/100 \text{ m}$  may thus be considered as a representative value.

For calculating the heat transfer coefficient  $\alpha$ , the measurements made in the ablation area are used (Ambach 1977, p 41, Table 9b). For 38 days of melting ice surface, the value for Alpine valley glaciers given by Kuhn (1979), is obtained here too:

$$\alpha = 1.72 \text{ MJm}^{-2} \text{d}^{-1} \text{ } ^\circ\text{C}^{-1} \text{ with } \bar{T}_a = +1.8^\circ\text{C} \text{ and } \bar{u} = 7.4 \text{ m/s.}$$

It must be taken into account that at the equilibrium line there exists first of all an old snow surface and because of the formation of superimposed ice, the snow surface is changed into an ice surface. Owing to the lower roughness of the snow surface, the shear velocity obtained here will be smaller than that of an ice surface by a factor of 0.627 which is confirmed by measurements in the area of accumulation (Ambach 1979, p 47, Figure 2). At the equilibrium line during

the ablation season there will be an old snow surface for 2/3 of the period ( $\alpha = 1.08 \text{ MJm}^{-2} \text{d}^{-1} \text{ } ^\circ\text{C}^{-1}$ ). Hence a weighted mean value of  $\alpha = 1.29 \text{ MJm}^{-2} \text{d}^{-1} \text{ } ^\circ\text{C}^{-1}$  is assumed.

RESULTS

Altitudinal shift of the equilibrium line by changes in air temperature

The relations

$$\frac{\delta R}{\delta z} = \alpha' \frac{\partial T_a}{\partial z}, \delta R = \alpha' \delta T_a \text{ with } \delta c = 0 \quad (2a,b)$$

are used for transforming Equation (1) into

$$\Delta h \left[ \frac{L}{\tau} \frac{\partial c}{\partial z} - (\alpha + \alpha') \frac{\partial T_a}{\partial z} \right] = (\alpha + \alpha') \delta T_a \quad (3)$$

Altitudinal shift of the equilibrium line by changes in cloudiness

A change in cloudiness in general also causes a change in the net radiation balance. For calculating the influence of the cloudiness, the expression  $\delta R$  in Eq. (2b) is expanded as follows:

$$\delta R = \alpha' \delta T_a + \beta \delta w \text{ with } \frac{\partial w}{\partial z} = 0$$

Eq. (3) then reads

$$\Delta h \left[ \frac{\partial c}{\partial z} \frac{L}{\tau} - (\alpha + \alpha') \frac{\partial T_a}{\partial z} \right] = (\alpha + \alpha') \delta T_a + \beta \delta w \quad (4)$$

where  $\delta w$  stands for a change in cloudiness in tenths, and  $\beta$  is a numerical value resulting from the dependence of net radiation balance on cloudiness.

This dependence has been studied in detail for the EGIG profile in the areas of ablation and accumulation. In the area of ablation, the net radiation balance decreases as the cloudiness increases, in the case of a melting ice surface (Ambach 1977, p 21, Figure 4, right side). In the area of accumulation with dry snow surface, however, the net radiation balance was found to increase with increasing cloudiness. The differences in the net radiation balance with clear sky and covered sky, amounted to  $+2.8 \text{ MJm}^{-2} \text{d}^{-1}$  in the area of ablation, and  $-2.0 \text{ MJm}^{-2} \text{d}^{-1}$  in the area of accumulation. A melting old snow surface lies between the two cases, so that here net radiation balance may be taken as approximation, independent of cloudiness.

At the equilibrium line there is a melting old snow surface during 2/3 of the ablation season, with  $\beta = 0$ , whereas for 1/3 of the ablation season there exists a melting ice surface with  $\beta = -0.28 \text{ MJm}^{-2} \text{d}^{-1}$  per one tenth of cloudiness assumed in the following calculations.

Numerical results of  $\frac{\Delta h}{\delta T_a}$  and  $\frac{\Delta h}{\delta w}$

In Eq. (4) the altitudinal shift of  $\Delta h$  is expressed by two terms: the shift for a temperature disturbance at constant cloudiness  $\Delta h(\delta T_a)$ , and the shift for a cloudiness disturbance at constant temperature  $\Delta h(\delta w)$ , reading

$$\Delta h(\delta T_a) = \frac{\alpha' + \alpha}{\frac{\partial c}{\partial z} \frac{L}{\tau} - (\alpha + \alpha') \frac{\partial T_a}{\partial z}} \delta T_a \text{ with } \delta w = 0 \quad (4a)$$

$$\Delta h(\delta w) = \frac{\beta}{\frac{\partial c}{\partial z} \frac{L}{\tau} - (\alpha + \alpha') \frac{\partial T_a}{\partial z}} \delta w \text{ with } \delta T_a = 0 \quad (4b)$$

Using the numerical values of  $\alpha = 1.29 \text{ MJm}^{-2} \text{d}^{-1} \text{ } ^\circ\text{C}^{-1}$ ,

$$\alpha' = 0.37 \text{ MJm}^{-2} \text{d}^{-1} \text{ } ^\circ\text{C}^{-1}, \frac{\partial T_a}{\partial z} = -0.0073 \text{ } ^\circ\text{C m}^{-1}, L =$$

$0.3336 \text{ MJkg}^{-1}$  and  $\beta = -0.093 \text{ MJm}^{-2} \text{d}^{-1}$  per one tenth of cloudiness, the following expressions for Equations (4a,b) are obtained:

$$\frac{\Delta h}{\delta Ta} = \frac{1.66}{\frac{\partial c}{\partial z} \frac{0.3336}{\tau} + 0.0121} \left[ \frac{m}{^{\circ}C} \right] \text{ with } \delta w = 0 \quad (4a')$$

$$\frac{\Delta h}{\delta w} = \frac{-0.093}{\frac{\partial c}{\partial z} \frac{0.3336}{\tau} + 0.0121} \left[ \frac{m}{1/10} \right] \text{ with } \delta Ta = 0 \quad (4b')$$

Figure 1 shows Eqs. (4a', 4b'),  $\frac{\partial c}{\partial z}$  (kg/m<sup>2</sup>m)

being a parameter. With the values of  $\frac{\partial c}{\partial z} = 1.0 \text{ kgm}^{-2}\text{m}^{-1}$  and  $\tau = 35 \text{ d}$ , representative for the EGIG profile, the altitudinal shifts of the equilibrium line  $\frac{\Delta h}{\delta Ta} = + 77 \text{ m}^{\circ}\text{C}$

and  $\frac{\Delta h}{\delta w} = -4 \text{ m}$  per tenth of cloudiness are obtained.

These figures hold under the condition of constant value of  $\tau$ , unaffected by climatic disturbances. Hence the influence of cloudiness on the shift of the equilibrium line may be neglected under realistic assumptions. The implications of the gradient of the cumulative accumulation for mass balance modelling will appear in detail elsewhere (Ambach and Kuhn, to be published).

CONCLUSIONS

In this paper, the equilibrium line in the EGIG profile is assumed at the altitude of 1240 m a.s.l. with a horizontal distance of 37 km from the ice margin. The slope at the equilibrium line is about 1 m to 100 m (Seckel 1977, Anlage 2). For an altitudinal shift of the equilibrium line by 100 m, a 27% change of the ablation area will be obtained. Similarly, the specific net ablation averaged over the ablation area will change by 27%, if the altitudinal gradient of specific net ablation remains unaffected by changes in climate. Kuhn (to be published) however, demonstrates that the altitudinal gradient of specific net ablation most probably is affected by the climate.

Assuming that the melt water is totally converted

to the loss in mass and neglecting possible refreezing, the loss in mass by melting is obtained as follows:

$$S_a(1+0.27)\bar{a}(1+0.27) \sim S_a\bar{a}(1+0.6) \quad (5)$$

where  $S_a$  is the ablation area in a sector of the EGIG profile,  $\bar{a}$  being the mean specific net ablation in this sector. The loss in mass by melting is therefore increased by about 60% if the equilibrium line is raised by 100 m of altitude. Analogous calculations can be made for other regions of the Greenland Ice Cap.

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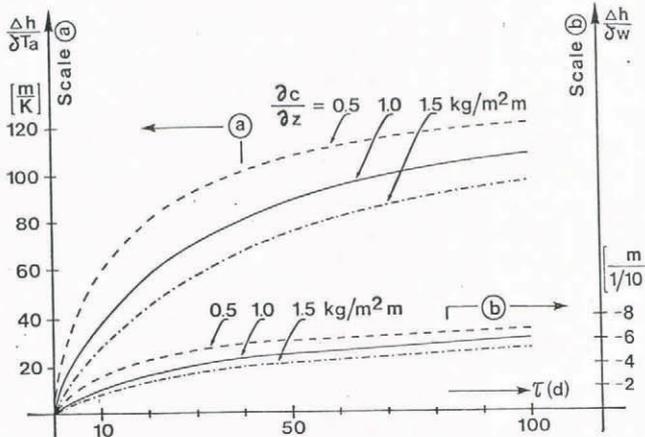


Fig.1. Altitudinal shifts of the equilibrium line  $\frac{\Delta h}{\delta Ta}$  (scale a) and  $\frac{\Delta h}{\delta w}$  (scale b) versus the duration of the ablation season  $\tau$ ,  $\frac{\partial c}{\partial z}$  (kg/m<sup>2</sup>m) being the parameter. With the values of  $\tau = 35\text{d}$  and of  $\frac{\partial c}{\partial z} = 1.0 \text{ kg/m}^2\text{m}$  the shifts read  $\frac{\Delta h}{\delta Ta} = + 77 \text{ m}^{\circ}\text{C}$  and  $\frac{\Delta h}{\delta w} = -4 \text{ m}/\frac{1}{10}$  cloudiness at the equilibrium line of the EGIG profile.