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New precise method for accurate modeling of thermal recoil forces

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Abstract. The exact modeling of external and internal perturbations acting on spacecraft becomes increasingly important as the scientific requirements become more demanding. Disturbance models included in orbit determination and propagation tools need to be improved to account for the needed accuracy. At ZARM (Center of Applied Space Technology and Microgravity) algorithms for the simulation and analysis of thermal perturbations have been developed. The applied methods are based on the inclusion of the actual spacecraft geometry by means of Finite Element (FE) models in the calculation of the disturbance forces. Thus the modeling accuracy is increased considerably and also housekeeping and sensor data can be included in the calculations. Preliminary results for a test case geometry of the Pioneer 10/11 mission are presented and discussed with respect to the Pioneer anomaly. It is shown that thermal effects cannot be neglected for the magnitude scale of the observed anomalous effect.

Keywords. radiation mechanisms: thermal, methods: numerical

1. Introduction

For modern spacecraft missions the requirements on perturbation knowledge and modeling accuracy become increasingly demanding. One of the sources for disturbance accelerations is the recoil force resulting from anisotropic heat radiation. For the assessment of this effect at ZARM algorithms have been developed which compute thermal perturbations based on the configuration of the spacecraft with a high level of geometric complexity and numeric accuracy. For this a finite element (FE) model of the craft is developed and a full thermal analysis is processed to calculate the resulting surface temperature distribution including material parameters and thermal boundaries.

2. Basic equations

The resulting recoil from radiation flux $dP_{d\Omega}$ directed to a specific direction characterized by $d\Omega = f(\phi, \beta)$ is

$$F_{d\Omega} = \frac{dP_{d\Omega}}{c}. \tag{2.1}$$

Assuming flat surface and Lambertian radiation the intensity is

$$I(\beta) = I_n \cos(\beta) \tag{2.2}$$

The emission hemisphere can be divided into solid angle elements $d\Omega$ bordered by the angles ϕ and β where $0 \leqslant \phi \leqslant 2\pi$, $0 \leqslant \beta \leqslant \pi/2$ and $d\Omega = \sin\beta \, d\beta \, d\phi$. The radiation flux received by the specific solid angle elements is

$$dP_{d\Omega} = L \cos\beta \sin\beta \, d\beta \, d\phi \, dA, \tag{2.3}$$

where the spectral density L at a given emissivity ϵ is defined as

$$L = \epsilon \frac{\sigma}{\pi} T^4 \quad with \quad \sigma = \frac{2\,\pi^4\,k^4}{15\,h^3 c^3}. \tag{2.4}$$

For the resulting force only flux components perpendicular to the emitting surfaces can contribute. Thus the effective power normal to the emitting surface is

$$P_\perp = L \int_0^A \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \cos\beta^2 \sin\beta \, d\beta \, d\phi \, dA = \frac{2}{3} P_{\mathrm{tot}} = \frac{2}{3} \epsilon \sigma A T^4, \tag{2.5}$$

which results in a recoil force in normal direction of the emitting plate $\mathbf{e}_\mathrm{n}$ of

$$\mathbf{F}_{\mathrm{Recoil}} = -\frac{2}{3} \frac{P_{\mathrm{tot}}}{c} \mathbf{e}_\mathrm{n}. \tag{2.6}$$

3. Computation method

The equations given above are valid for rectangular two-dimensional emitting surfaces. The computation of recoil forces for a detailed spacecraft geometry is much more complicated. At ZARM an elaborated method for the precise computation of thermal recoil forces has been developed. In a first step a complete FE model of the craft is created and thermal FE analysis are conducted to compute equilibrium surface temperatures based on spacecraft configuration, materials, thermal boundaries and loads. In a second step the results and a mathematical model of the craft are exported into a ray tracing algorithm for the computation of the resulting perturbation force including emission, absorption, reflection and shadowing models. The total force $\mathbf{F}_{\mathrm{res}}$ is composed of a) an emission component, b) an absorption component and c) a reflection component where

$$\mathbf{F}_{\mathrm{res}} = \mathbf{F}_{\mathrm{emis}} - \mathbf{F}_{\mathrm{abs}} + \mathbf{F}_{\mathrm{ref}}. \tag{3.1}$$

The emission component can be acquired with the unit normal vector of each surface and material/temperature data using equation 2.6. For complex model geometries radiation may be absorbed by other model surfaces thus reducing the effective recoil. For this emission is treated by angularly spaced rays emitted into the solid angle elements of the hemisphere, where the number of angular divisions specifies the total number of rays per element. The rays are then traced to detect intersections with other surfaces. For each hit the effective flux is computed by means of view factors. All computations in this respect are performed including shadowing effects. The processing of the reflection component is similar to the absorption computation. The rays hitting a target surface can either be reflected specularly or diffusely thus initialising new sets of reflected rays. The number of reflections or a minimum energy threshold for the processing of rays can be specified.

4. Test case: Pioneer 10/11 mission

For the Pioneer 10/11 spacecrafts a small, constant decrease of velocity has been observed. The residual perturbation acceleration has been computed to $a_{\mathrm{Pio}} = 8.74 \cdot 10^{-10} \mathrm{m/s}^2$ (in Anderson *et al.*(1998)). A conclusive explanation for this effect has not been presented until now. There are many speculations concerning new physics and unmodeled relativistic effects in the codes used for data analysis but also conventional effects may provide an explanation. In Anderson *et al.*(2002) it is stated that an anisotropy of 60 W directed against flight direction may explain a deceleration in the order of magnitude of the PA. This satisfies a closer examination of the effect of anisotropic heat radiation for the Pioneer 10/11 spacecrafts. For this an FE model including the main antenna dish and the two RTG-assemblies (composed of two RTGs each) has been developed. For a first order-of-magnitude assessment only the RTGs are considered as heat sources while the antenna is assumed to be thermally neutral (zero temperature) and only used for absorption/reflection effects. Figure 1 shows the geometry of the test case. The surface temperatures of the RTGs are acquired from a FE thermal analysis based on material, heat load and geometry data. The FE mesh is generated using quadrilateral and hexahedral thermal finite elements to include heat conduction as well as heat radiation.

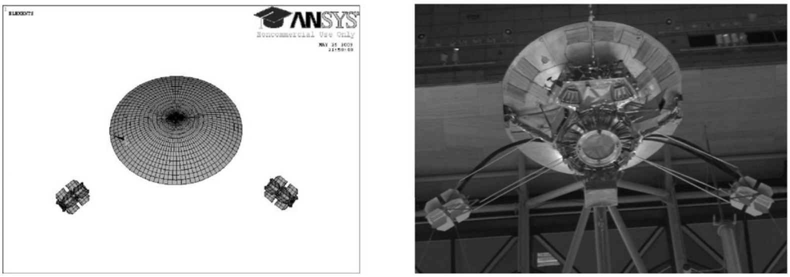

Figure 1. Test case and real Pioneer 10/11 geometry (picture courtesy Craig Markwardt).

The results of the thermal analysis (nodal solutions of the outer quadrilateral surfaces) is then exported into the ray tracing algorithm for computation of the recoil force. The analysis is conducted with varying numbers of emitted rays per surface until the solution converges. The surface model is composed of 1366 individual FE, the computation is performed for BOL power of 2500 W, white surfaces ($\epsilon = 0.9$) and a dry mass of 233 kg.

5. Preliminary results and conclusion

The solution converges for a number of approximately 90000 rays at a model size of approximately 2000 FE to a resulting acceleration component of 35 percent PA oriented against flight direction. Due to the incomplete model geometry (no equipment section, no outer payloads, RTGs only heat source) this result must not be mistaken for an exact value of the total contribution of thermal recoil to the anomalous deceleration. For this a complete model has to be processed also taking into account that the available power decreases over time due to radioactive decay. But the result points out that more detailed analyses are necessary and that thermal effects can not be neglected with respect to PA investigations. Therefore future analysis will include complete geometry, all heat sources as well as dynamic aspects. The presented method for the calculation of thermal recoil forces is of course not limited to the Pioneer case but can be used for any spacecraft mission with high requirements on perturbation knowledge. In particular fundamental physics missions like LISA, LISA pathfinder and Microscope can benefit from the improvement in modeling accuracy.

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