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Global MHD phenomena and their importance for stellar surfaces

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Abstract. This review is an attempt to elucidate MHD phenomena relevant for stellar magnetic fields. The full MHD treatment of a star is a problem which is numerically too demanding. Mean-field dynamo models use an approximation of the dynamo action from the small-scale motions and deliver global magnetic modes which can be cyclic, stationary, axisymmetric, and non-axisymmetric. Due to the lack of a momentum equation, MHD instabilities are not visible in this picture. However, magnetic instabilities must set in as a result of growing magnetic fields and/or buoyancy. Instabilities deliver new timescales, saturation limits and topologies to the system probably providing a key to the complex activity features observed on stars.

Keywords. MHD, turbulence, instabilities, stars: magnetic fields

1. Introduction

The imaging of the surfaces of stars other than the Sun provides us with a lot of observational facts about stellar activity. Temperature variations on the surfaces of rapidly rotating giants become visible, as shown, for example, for ζ And (Kővári *et al.* (2007) and II Peg (Carroll *et al.* 2007). Surface abundance maps of chemically peculiar stars have been published as well as maps of the surface magnetic field (e.g. Kochukhov *et al.* 2004 for 53 Cam). The magnetic fields are apparently fairly constant in time for half a century, and only a fraction of 10% or less of all A-type stars show magnetic fields. Dwarf stars are of course very interesting targets which are needed to place the Sun in the context of stellar activity. A compilation of the large-scale magnetic field geometries of, e.g., late M dwarfs is given by Morin *et al.* (2010). Since these stars are also supposed to have small-scale flares, the observations raise the question of which of the observed features represent the general topology generated by a dynamo, and which structures are local effects caused by secondary processes. The references given here are of course by no means complete. We refer the reader to other reviews for the status of the observations.

2. Dynamos from today's point of view

2.1. *Mean-field dynamos*

Motivated by convective motions of a conductive fluid, the net effect of the turbulence on the electromotive force was computed by Steenbeck *et al.* (1966). Up to a few years ago, next to all models of stellar cycles have been based on such mean-field models. The existence of this net effect, the α-effect, has been proven by numerical simulations.

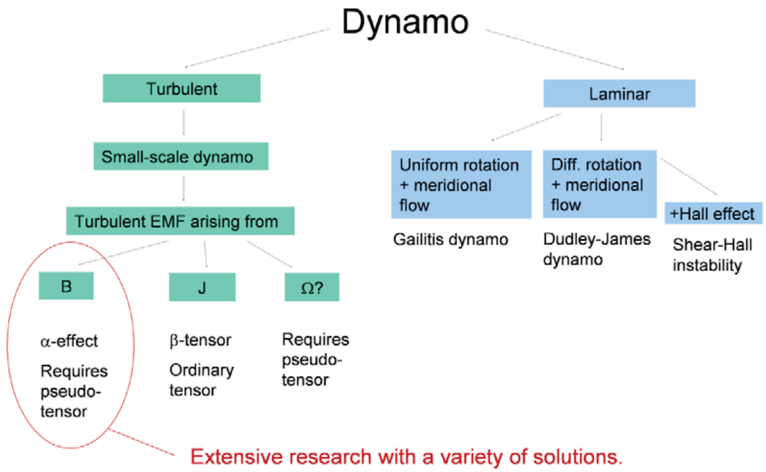

Figure 1. Possible dynamo action arising from various sources.

The presence of differential rotation leads to the generation of toroidal magnetic fields and is usually referred to as the Ω-effect. The α-effect, however, generates both components, poloidal fields from toroidal ones, and also toroidal fields from poloidal ones, just like the Ω-effect. The corresponding mean-field equation thus describes what is called an $\alpha^2\Omega$-dynamo. It is the neglect of the α-effect in the toroidal-field generation which makes the $\alpha\Omega$-dynamo which is often cited in stellar dynamos, but one should note that it is computationally inexpensive to use the full equations for any dynamo model of this kind, especially when additional subtleties play a major role in getting the cycle features right.

The other approximation is the α^2-dynamo in which the differential rotation is assumed to be zero. However, the effect of rotation is strong. Even if the differential rotation is only a 1%-effect, the nondimensional parameter for the Ω-effect in the mean-field equation is larger than the nondimensional α-parameter necessary to excite an α^2-dynamo. Differential rotation is never really negligible.

Figure 1 gives an overview of most of the possibilities to generate magnetic fields in stars continuously. The left-hand side shows the group of turbulent dynamos. Turbulence may deliver the small-scale dynamo which alone is not able to provide global magnetic fields. The generation of substantial large-scale magnetic fields through the turbulence requires the removal or cancellation of small-scale helicity. We are not going to review the issues of the problem of slow growth of the large-scale field here, but refer to Brandenburg & Subramanian (2005) for a review and assume that large-scale dynamo action can be expressed by a mean-field treatment. Numerous studies rely on the α-effect which is a pseudo-tensor acting on the axial vector $\boldsymbol{B}$. There is also the turbulent diffusivity tensor acting on gradients of $\boldsymbol{B}$ which does not only have destructive components. If rotation is present, generating terms are provided in this tensor and are called the $\boldsymbol{\Omega} \times \boldsymbol{J}$-effect. In principle, also a magnetic-field dependent pseudo-tensor acting on the angular velocity $\boldsymbol{\Omega}$ can lead to large-scale field generation (Yoshizawa 1993), but the applicability of this approach has not yet been fully explored.

While the models of kinematic dynamos based on the α-effect have become more and more sophisticated to show many of the features of the solar cycles, it is now time to

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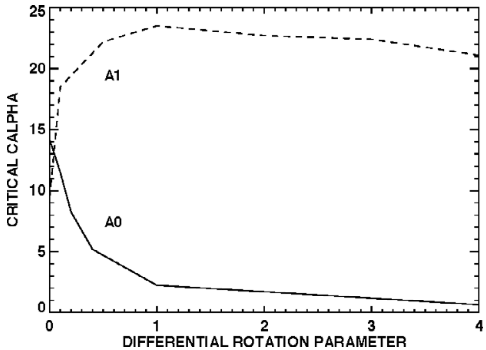

Figure 2. Critical dynamo-α in dimensionless units required for the onset of the axisymmetric dipole A0 and the oblique quadrupole A1 versus the differential rotation which mimics the solar rotation profile. In our units of the differential rotation the Sun has 6.87.

tackle the drawbacks and short-comings of the α-dynamos with a full MHD treatment that includes instabilities and the evolution of helicities in global spherical models.

2.2. *Laminar dynamos*

Only a brief mention of the group of laminar dynamos on the right-hand side of Fig. 1 is due here. They are also kinematic dynamos as there are known flow patterns – axisymmetric or non-axisymmetric – which lead to exponential growth of the magnetic field, without the addition of a turbulence source term like the α-effect. The field is always non-axisymmetric (Cowling 1934). Such dynamos would be interesting for radiative zones of Ap stars to explain their apparently strong magnetic fields, but no numerical confirmation with realistic flows from first principles has been found yet. Other problems are their slow growth and that the diversity of magnetic field topologies observed cannot be reproduced by the very regular, smooth solutions from a laminar dynamo.

2.3. *Active longitudes*

Several stars appear to exhibit persistent activity in a certain range in longitude for fractions of a stellar cycle. This long-lived non-axisymmetry raised the question of possible non-axisymmetric solutions of an $\alpha^2\Omega$-dynamo. Such solutions are in principle possible if the differential rotation of the stars is negligible. We have seen above that this is rarely the case, even if the differential rotation is only say 1%.

The observations even show changes of the active longitude by roughly 180°, mostly after a fraction of the stellar cycle length. The non-axisymmetric solutions of the $\alpha^2\Omega$-dynamo typically show, however, a stationary magnetic field pattern drifting continuously in longitude. Elstner & Korhonen (2005) found a solution in which both the axisymmetric "normal cycle" mode and the stationary non-axisymmetric mode are excited. The combination of the two resembles the flip-flop phenomenon, especially that of stars with spots at relatively high latitudes. The coexistence of the modes was basically achieved by a reduced differential rotation down to about 10% of the solar value. A small increase of the differential rotation leads to an immediate separation of the excitation thresholds of the non-axisymmetric mode and the axisymmetric one. This is demonstrated with an $\alpha^2\Omega$-dynamo with an α-effect distributed in the convection zone and a solar-like rotation profile of varying amplitude in Fig. 2.

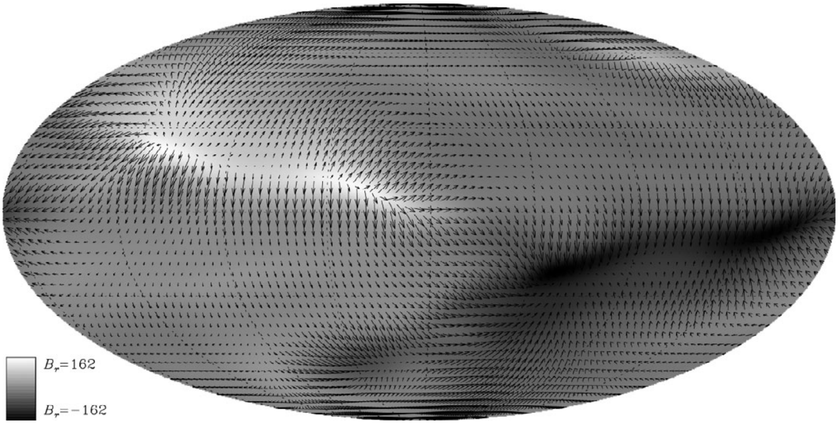

Figure 3. Surface plot of magnetic field from the Tayler instability as it appears after rising from the unstable region in the radiative envelope. The grey scale represents the radial field, the vectors the horizontal components.

It is very likely that other mechanisms than kinematic dynamo modes are at play here. Global magnetic instabilities seem to be an option, and we will elaborate on them in the following Section.

3. Instabilities

The kinematic dynamo is only a very small piece of the entire possible solutions of the MHD equations. These are the momentum equation, the induction equation, the energy equation, and mass conservation plus the constraint that $\nabla \cdot \boldsymbol{B} = 0$.

The number of known instabilities is large and the boundaries are not always clearly cut. We may group them into shear-driven instabilities, current-driven instabilities, and thermally driven instabilities. In the stellar case, shear-driven instabilities are fed by differential rotation and include the latitudinal shear instability (Watson 1981, and e.g. Dziembowski & Kosovichev 1987, Arlt *et al.* 2005) and the magneto-rotational instability (Velikhov 1959, and e.g. Arlt *et al.* 2003, Menou & Le Mer 2006). Current-driven instabilities can emerge from a magnetic field alone if it is not current-free. There is a variety of names for various types of instabilities but they all fall actually into the general class of instabilities fed by the magnetic field as studied by Vandakurov (1972) and Tayler (1973) and several authors afterwards. We will henceforth use the term Tayler instability even if modifying conditions such as differential rotation or density stratification are present. The Tayler instability very often favours non-axisymmetric modes that become unstable first.

In the first place, the Tayler instability is interesting for radiative zones like the solar tachocline – the transition from differential rotation to uniform rotation below the convection zone – and the extensive radiative envelopes of intermediate-mass stars from spectral types B to early F. Calculations by Arlt *et al.* (2007) and Kitchatinov & Rüdiger (2008) showed an upper limit for stable toroidal magnetic fields in the tachocline of less than 1000 Gauss. This contradicts the usual picture of fields up to 10^5 G which are necessary to explain a rise of flux to low latitudes forming the sunspots seen at latitudes of below about 40°. This illustrates that kinematic dynamos on the one hand (induction equation only) and thin flux-tube simulations (one-dimensional MHD) may not capture the full physics due to their approximations.

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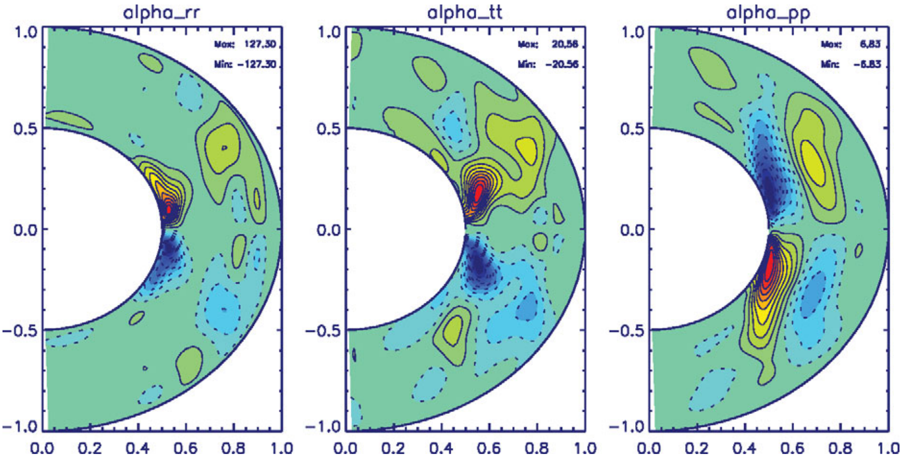

Figure 4. Diagonal elements of the α-tensor derived from the Tayler instability. Yellow areas denote positive values, blue areas represent negative values.

4. Possible dynamo action from the Tayler instability

Spruit (2002) suggested that the Tayler instability plus differential rotation may lead to dynamo action, since unstable toroidal magnetic fields will provide a source of poloidal field which in turn can be amplified by differential rotation. This idea, which is sometimes called the Tayler-Spruit dynamo, has been used by stellar evolution models. It is actually an enhanced, sustained diffusion from the continuously disrupting magnetic fields which enters these models, not the magnetic fields themselves. While simulations by Braithwaite (2006) show sustained magnetic fields when the differential rotation is maintained in the entire computational domain, other simulations, e.g. by Zahn *et al.* (2007) and Gellert *et al.* (2008), failed to obtain a dynamo. Their differential rotation was enforced only at the boundaries. Obtaining sustained dynamo action in a global, nonlinear simulation is a tricky thing because astrophysical parameters are not achievable numerically. It is thus elucidating to look for dynamo action through the presence of non-vanishing mean-field coefficients measured in nonlinear simulations.

The non-axisymmetric, unstable mode will develop other azimuthal wave numbers by nonlinear coupling and could be able to produce an average net effect of generating a large-scale axisymmetric magnetic field. If so, it may be expressed by mean-field coefficients. The following computations only employ an initial differential rotation and a poloidal magnetic field. When the generated toroidal magnetic field is supercritical, we impose a non-axisymmetric perturbation. The simulations are convectively stable and impose no rotation on the radial walls, so do not provide an energy source for sustained dynamo action. However, we can measure the mean-field coefficients arising from the Tayler instability.

We are using the MHD equations in an incompressible domain with a density constant in space and time, for the sake of simplicity. Details of the computational setup can be found in Arlt & Rüdiger (2011). The system quickly develops a toroidal magnetic field from the initial poloidal one. At the same time, Lorentz forces build up diminishing the differential rotation. The system thus evolves inevitably into a state which is Tayler unstable. When the configuration has reach the supercritical regime, we add a non-axisymmetric perturbation of azimuthal wavenumber $m = 1$ and allow the instability to grow (Fig. 3). Due to nonlinear coupling, a whole spectrum of spherical harmonics builds up including changes of the axisymmetric part of the solution.

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At the same time, we evolve a set of 27 test field equations to measure the mean-field coefficients associated with the large-scale magnetic field and first derivatives of it. The method was described by Schrinner *et al.* (2007). While the coefficients form tensors, dynamo action is typically expected from the diagonal elements of the α-tensor (the one which is associated with the large-scale field), but is in general not restricted to those. The resulting diagonal elements are plotted in Fig. 4, taken as averages from a period which is 0.0005–0.0010 diffusion times (2–3 rotations) after the non-axisymmetric perturbation was applied.

The $\alpha_{\phi\phi}$ emerging from the Tayler instability is positive in the northern hemisphere. There is a counterpart of negative α roughly along the inner tangential cylinder which is not associated with the Tayler instability. We also find that the α from the Tayler instability is closely related to a positive current helicity in this region of the computational domain, while α based on convection is related to the kinetic helicity. This is a clear sign of the magnetic nature of the Tayler instability. The α-effect measured is only about 1% of the average velocity fluctuations.

5. Concluding remarks

Instabilities provide additional time-scales and magnetic-field topologies to the dynamo. Their consequences need to be incorporated in models of solar and stellar activity. Since full MHD simulations of entire stars are still not feasible, an incorporation of the characteristics of the instabilities in large-eddy simulations or mean-field models should be a suitable choice. With compressible spherical simulations coming into reach, the necessary quantities will become accessible for stellar contexts in the near future.

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Discussion

Kosovichev: Could you see the possible Tayler-Spruit dynamo from the flux-tube like magnetic field at the bottom of the convection zone?

Arlt: The example of magnetic flux rising through a convection zone, as shown in the presentation, was only a toy model demonstrating the convective processing and it was 2D, so cannot show any dynamo effect.

Kosovichev: What is the magnetic field strength of the flux tube in the tachocline in your simulations of emerging magnetic flux?

Arlt: I chose the field strength such that it's balanced with the rotation simply that it stays in place, because otherwise a ring of magnetic field would just shrink. The field strength was somewhere with an Alfvén speed near the rotation speed, which is 10^5 Gauss or so.