Some of the exercises (such as the proof of Schröder–Bernstein's Theorem in Chapter 10) are not immediate for the first-time reader, but I assume that they are included as a sort of complement or to be done during the lecture.

Regarding the use of this book as a manual for instructors, Kirby himself suggests in the preface different combinations of the chapters according to the total number of teaching hours as well as to the prerequisites of the targeted audience of students. Part I comprises Chapters 1–5 and introduces the basic syntactic and semantic notions of formulae and structures. Part II, comprising Chapters 6–11, focuses on theories. Whilst Kirby decides not to introduce formal deduction and thus it does not present a proof of Gödel's completeness theorem, it does present the compactness theorem, which is one of the main tools in model theory (and yet one of the results most difficult to fully grasp and internalise at the beginning). Chapter 11 contains an adaptation of Henkin's method (at a purely semantic level without formal deductions) to produce a canonical model whose universe consists of interpretations of the constants. This is crucial in order to obtain Löwenheim–Skolem Theorems (downwards and upwards) in Chapters 12 and 13, which belong to Part III (Chapters 12–16), in which more advanced notions such as elementary substructures and extensions as well as categoricity are introduced. Part IV (Chapters 17–22) presents quantifier elimination, one of the fundamental aspects of model theory concerning the study of definable sets. The notions presented in Parts III and IV reappear in Part V, in which complete types are introduced, and in particular the omitting types theorem is proved. However, Kirby does not assume a knowledge of topology for his audience, so the topological properties of the space of types are not explored in detail (in particular, the notion of an isolated type is given purely in terms of atomic or principal formulae). The last chapters of Part V concern the notions of prime models as well as saturated models. The construction of a countable saturated model is presented in Chapter 26 for countable complete small theories (or in Kirby's notation 0-stable). Part VI (Chapters 28–32) presents the theory of algebraically closed fields as an archetype for the comprehension of a tame mathematical structure (or a class thereof) from a model theoretic point of view, relating fundamental notions from algebra to their model-theoretic counterparts: Chevalley's theorem in Chapter 31 and quantifier elimination, or Hilbert's Nullstellensatz and model-completeness in Chapter 32.

In my personal opinion, this book is well suited for two different audiences: advanced researchers in mathematics who would like to get a first acquaintance with some of the notions and results in model theory without having to spend a considerable amount of time with some classical notions whose relevance may not be clear at the beginning. The second audience is students (or rather, those faculty members considering offering an introductory course in model theory) at universities which do not offer many courses in mathematical logic, but are interested in broadening their curricula with a first introductory course in model theory.

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Edwin Mares. ***The Logic of Entailment and its History***. Cambridge University Press, Cambridge, UK, 2024, xv + 264 pp.

At the heart of logic lies the conditional notion of *entailment*; certain arguments are logically good in virtue of having a conclusion which follows *from* the premises in some sense. From the very birth of the research program, relevantists have insisted upon a substantial notion of from-ness; a proposition may be true in virtue of its logical form, yet need not for that reason follow *from* any and every collection of premises. Edwin Mares' book *The History and Philosophy of Entailment* is a fresh and updated account of the philosophy of and the

https://doi.org/10.1017/bsl.2024.31 Published online by Cambridge University Press

history leading up to the relevant logic E, the very same logic, then, set forth at the very onset of Anderson and Belnap research program of relevant logics as their theory of entailment. In many ways, Mares' book can be seen as a continuation of Anderson and Belnap's account of entailment as set forth in the first volume of their relevant magnus opus *Entailment: The Logic of Relevance and Necessity*, Princeton, 1975, with the addition of two main new ideas: (1) the logical theorems of E on the form $A \to B$ are to be read as expressing *proof plans*, and (2) the formal account of entailment is a theory of *theory closure*. I will get back to these two feature, but let me also point out an important feature of Anderson and Belnap's theory of entailment which is all but missing from Mares' account, namely their theory of enthymematical consequence: One of the well-known properties of E is that B is derivable—in the "standard", or "official" as A&B called it, sense of the term—from a set of formulas Γ if and only if $A \wedge t \to B$ is a logical theorem of E, where t is a primitive or contextually definable truth-constant and A is a conjunction of a finite subset of Γ.[1] Although Anderson and Belnap derided what they called the "official" notion of the standard notion of derivability for failure to account for a substantial notion of from-ness, they did acknowledge it as one "often used in this book, and indeed one could hardly get along without" (*Entailment*, p. 259). Although Mares does mention this notion of derivability in connection with the discussion of Barcan Marcus' result of the failure of the deduction theorem for Lewis' modal logic S2, Mares' only ever uses the restricted notion of axiomatic derivability for which premises must all be logical theorems. Relevantists might be accustomed to this, but an upper level undergraduate student having read the relevant parts of, say, Priest book *An Introduction to Non-Classical Logic*, Cambridge, 2008, might find this strange. My main point, however, is simply to underscore that Mares' account of E is different than that of Anderson and Belnap.

The book is divided into three main parts, and is also outfitted with a helpful introductory chapter, and a technical appendix. The latter has the pleasing effect of substantially increasing readability and underscores the philosophical nature of the project. The result is a book which ought to be of interest to both students and researchers with an interest for non-classical accounts of logical consequence.

Part I consists of four chapters under the heading *Entailment in the Twentieth Century* and starts off by a nice presentation of C. I. Lewis' attempts at expressing logical consequence using the strict conditional, as well as short accounts of both Everett Nelson's connexive as William Parry's analytic implication, and various issues connected to possible world semantics. These accounts, according to Mares, all treat nested entailments inadequately in that they yield logical theorems $A \to B$, where, intuitively, a proof plan which assumes A in order to derive B isn't a good one. Regrettably, the notion of a good or a bad proof plan remains rather intuitive throughout the book, which is somewhat paradoxical since one of the lessons Mares draws from his account of Lewis is the importance of clear job description (cf. Ch. 2.13). The historical account then moves on to Anderson and Belnap's use-restricted Fitch calculus. Like Anderson and Belnap, Mares insists that this calculus does force a hypothesis A to be used in obtaining B in order for it to be possible to conclude $A \to B$ and that this "real use" requirement "is what makes the system a *relevant* system" (p. 75). It is this proof system which is set forth as an account of the idea of a proof plan. Regrettably, Mares does not discuss critiques of this idea such as the possibility of constructing such a use-restricted calculus for classical logic.[2] Nor is the critique laid out in Routely, Meyer,

[1]The result goes back to Meyer's *E and S4*, *Notre Dame Journal of Formal Logic*, vol. 11 (1970), pp. 181–199.

[2]For such a calculus, see Brady, *Natural Deduction Systems for Some Quantified Relevant Logics*, *Logique et Analyse*, vol. 27 (1984), pp. 355–277, in which a logic *RF* is outfitted with such a calculus. *RF* proves every logical theorem of classical logic as recognized in Brady, *Rules in Relevant Logic—II: Formula Representation*, *Studia Logica*, vol. 52 (1993), pp. 565–585.

Plumwood, and Brady's *Relevant Logics and Their Rivals*, Ridgeview, 1982, of E as the logic of entailment discussed. Without, then, a clear account of why a "bad" proof plan such as one which starts off assuming p in order to obtain $q \vee \neg q$, cannot still be a valid entailment, I at least found motivation for thinking that the Fitch calculus for E separates the true states of entailment from false ones, somewhat wanting.

The philosophical interpretation of E as a theory of entailment is given from part 2 and onwards. Mares starts by giving an illuminating account of both Tarski's and Kuratowski's axioms for a closure operator and in line with the Anderson–Belnap take on from-ness, opts in favor of Kuratowski's axiom that $\mathcal{C}(\emptyset) = \emptyset$. This, then, is a different theory than that of proof plans which does yield the set of logical theorems of E from the empty set. The two theories are, however, connected by way of Mares' theory of theory closure. The notion of a closure operator is then expanded to include conjunction so that, by definition, for any set of formulas $\Gamma \cup \{A, B\}$, $A, B \in \mathcal{C}(\Gamma)$ if and only if $A \wedge B \in \mathcal{C}(\Gamma)$. A weak logic called *Tarski Logic*, TL, is then introduced which has an entailment conditional $\rightarrow$ which reflects, in a sense introduced in Chapter 5, the rules of the a conjunctive-infused notion of a closure operator. The first proper consequence relation $\mathcal{C}_{TL}$—defined so that $B \in \mathcal{C}_{TL}(\Gamma)$ if and only if $A \rightarrow B$ is a logical theorem of TL, where A is some conjunction of the formulas in Γ—is then introduced. Although Mares' claim (p. 127) that Tarski's axioms *determine the logic TL* seems rather audacious—Tarski's axiom do not involve any connective—there is a pleasing naturalness to Mares' way of building up a theory of consequence. This is the part of the book that I found most interesting, although a discussion of similar attempts—especially that of the relevant "rival" attempt found in *Relevant Logics and Their Rivals*—would have made Mares contribution even more rewarding.

Mares main idea is that the theory of entailment is a theory of theory closure. Mares generalizes the closure operator $\mathcal{C}_{TL}$ to allow for $\mathcal{C}_a$, where a, then, is itself any theory of entailment, that is, an object which validates $A \wedge B$ if it validates both conjuncts, and which validates B if it validates A and $A \rightarrow B$ is a logical theorem of the logic in question. Intuitive rules governing such closure operators are then argued to yield the conjunction-implicative fragment of E. The semantics Mares ends up with advocating is a version of Kit Fine's semantics for relevant logic first presented in Fine's *Models for Entailment*, *Journal of Philosophical Logic*, vol. 3 (1974), pp. 347–372, and which has gained renewed interest in recent years. Part 3 of the book lays out this semantics showing how it can be used to interpret and justify the Fitch system for E, and with it, Mares' proof plan idea of entailment. I should hasten to note that Mares expands upon Fine's semantics in two important direction: negation is given a modal incompatibility reading so that $\neg A$ is to hold in a theory a just in case A fails to hold in every a-compatible theory. The semantics is then expanded to deal with quantifiers using the model theory developed by Mares himself and Robert Goldblatt.

The book ends with the last chapter of part 3 entitled *Entailment and Reasoning* dealing with various topic relating to theories which don't contain any entailment statements. Such theories end up being closed under the logical rules of FDE. Excluded middle and disjunctive syllogism need not hold, therefore, for such theories, and so if one is to model a classical theory, Mares advocates the use of "side conditions" and an extra "internal" conditional. The chapter reads much like a note for future work, which I think it, and indeed the book as a whole, will inspire to.

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SAMUELE IAQUINTO AND GIULIANO TORRENGO. ***Fragmenting Reality: An Essay on Passage, Causality and Time***. Bloomsbury Academic, London, 2022, x + 208 pp.