# 1 Introduction

Quantum mechanics is often presented as an essentially probabilistic theory, where random collapses of the wavepacket with probabilities governed by the rule conjectured by Max Born (1926) play a central role. Yet, unitary evolution dictated by the Schrödinger equation is deterministic. This clash of quantum determinism of the unitary evolutions with the quantum randomness of its everyday experimental practice is at the heart of the interpretational conundrums.

We will begin with the re-examination of the textbook quantum postulates. We shall conclude that some of them need not be postulated—that they follow from the other postulates. This simplification of quantum foundations will provide us with a convenient and solid starting point. The emergence of the classical from the quantum substrate is based on this foundation of *core quantum postulates*—on our *quantum credo*.

Discussion of the postulates will be accompanied by a brief summary of their implications for the interpretation of quantum theory. We shall touch on the questions of interpretation throughout the book, and return to them in more detail only near the end. We do not need to appeal to any interpretation to arrive at our conclusions. Our results depend solely on the quantum credo, on the core quantum postulates that are uncontroversial. We assume universal validity of quantum theory. Our motivations are, however, often best explained in the context of interpretations.

## 1.1 Core Quantum Postulates: "Quantum Credo"

The difficulty of reconciling quantum determinism with quantum randomness is reflected in the inconsistency of the postulates that provide textbook summary of quantum mechanics (see, e.g., Dirac, 1958). We list them starting with the four uncontroversial core postulates. They constitute our quantum credo and will serve as four cornerstones of the *quantum theory of the classical*.

The first two postulates are familiar:

(i) *The state of a quantum system $\mathcal{S}$ is represented by a vector in its Hilbert space $\mathcal{H}_{\mathcal{S}}$.*
(ii) *Evolutions are unitary (i.e., generated by the Schrödinger equation).*

They imply, respectively, the *quantum superposition principle* and the *unitarity of evolutions*, and we shall often refer to them by citing their physical consequences. Thus, in accord with (i), the superposition principle implies that when $|r\rangle$ and $|s\rangle$ are legal quantum states, $|v\rangle = \alpha|r\rangle + \beta|s\rangle$ (where $\alpha$ and $\beta$ are complex numbers) is also an equally legal quantum state. Hilbert spaces are Euclidean: Pythagoras' theorem holds for state vectors. Therefore, when $|r\rangle$ and $|s\rangle$ are orthogonal ($\langle r|s\rangle = 0$) and normalized, then $\langle v|v\rangle = |\alpha|^2 + |\beta|^2$.

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The Schrödinger equation induces evolution of quantum states: Given a Hamiltonian $\mathbf{H}$, unitary quantum evolution is generated by the operator $U_t = e^{-i\mathbf{H}t/\hbar}$. Unitarity implies linearity—superposition evolves into a superposition of evolved ingredients. For example,

$$U_t|v_0\rangle = U_t(\alpha|r_0\rangle + \beta|s_0\rangle) = \alpha\, U_t|r_0\rangle + \beta\, U_t|s_0\rangle = \alpha|r_t\rangle + \beta|s_t\rangle = |v_t\rangle.$$

Unitary evolutions can be thought of as rotations of state vectors that preserve scalar products in the Hilbert space, $\langle r_t|s_t\rangle = \langle r_0|s_0\rangle$ (that is, angles between state vectors do not change). Unitarity implies determinism, hence it implies preservation of information.

Postulates (i) and (ii)—quantum principle of superposition and unitarity—provide an almost complete summary of the formal structure of the theory. One more postulate is often added to complete the mathematical foundation of quantum mechanics:

(o) *The quantum state of a composite object is a vector in a tensor product of the Hilbert spaces of its subsystems.*

Entanglement—the most flagrantly quantum feature—depends on the tensor structure of the quantum states of composite systems. Postulate (o) is nevertheless often omitted from textbooks, possibly because (in the light of postulates (i) and (ii)) it seems so natural.[1]

Entanglement is central to the Schrödinger cat paradox, and, more generally, for discussion of quantum measurements. Interaction between systems that results in an information flow can be regarded as a measurement. The quantum measurement problem arises because a quantum state of a collection of systems can evolve from a Cartesian product (where a definite state of the composite whole implies definite states of each constituent subsystem) into an entangled state represented with a tensor product. This entangled state of the whole is still definite and pure, but individual subsystems are no longer entitled to their own pure states: The cat is neither dead nor alive, or even $\alpha|\text{dead}\rangle + \beta|\text{alive}\rangle$. It simply does not have a pure state of its own. The same goes for the apparatus.[2]

Thus, even though the entangled state of the whole may be completely known, it is no longer possible to know equally well the states of the subsystems. By contrast, in the classical setting, completely known (pure) composite states are always given by Cartesian products of similarly known (pure) states, so that when the state of the whole is known, the state of each subsystem is also known. This difference between the nature of the states of the composite quantum systems is responsible for many of the interpretational difficulties, including the quantum measurement problem.

The existence of subsystems is then really the essence, the only part of postulate (o) that is not suggested by the superposition principle of (i) and the unitarity of evolutions of (ii). In the absence of subsystems, the Schrödinger equation leads to deterministic evolution of the state vector in an indivisible Universe, and the measurement problem disappears (Zurek, 1993, 2003a). In other words, without at least a measured system and a measuring apparatus, questions about the outcomes cannot even be posed. Interaction between a system and an apparatus leads, via unitary evolution, to entanglement, which is at the heart of the interpretational questions. Eventually we shall need at least one more ingredient—environment—to address them.

### 1.1.1 Relating Quantum Mathematics to Quantum Physics

Postulates (o)–(ii) provide a complete summary of the *mathematics* of quantum theory. They contain no premonition of either collapse or probabilities. Using them and the obvious additional ingredients (initial states and Hamiltonians) one can carry out calculations. However, such calculations would be just a mathematical exercise.

1 See Carcassi, Maccone, and Aidala (2021), for recent related discussion.

2 Throughout this book we distinguish between entangled states that have tensor product structure (see, e.g., Eq. (1.1)) and Cartesian states that allow each subsystem to have a pure state of its own. As a tensor product state is a weighted sum of Cartesian states, a Cartesian state is also a (very special) tensor product state with just one such term.

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To relate the mathematics of quantum theory to quantum physics one needs to establish a correspondence between the state vectors in the Hilbert space and experiments—hence, measurements. This task starts with the *repeatability postulate*:

(iii) *Immediate repetition of a measurement yields the same outcome.*

Postulate (iii) is idealized—it is hard to perform non-demolition measurements—but in principle this can be done. Moreover, as a fundamental postulate, repeatability of (iii) (or some equivalent thereof) is indispensable. Repeatability is the simplest case of predictability, so the very concept of a "state" embodies predictability that calls for postulate (iii): The role of states is to enable predictions, and the most basic prediction is that a state is what it is known to be. The repeatability postulate asserts that the confirmation of this prediction is in principle possible.

Postulate (iii) is uncontroversial, as repeatability is taken for granted in the classical world, where it follows from a much stronger assumption than the quantum repeatability postulated above in (iii): It stems from the objective existence of classical states. They are "ontic", a property of the system alone, and exist independently of what an observer knows.

This independence detaches their ontic role—their existence—from their epistemic role, from what is known about classical systems. In the classical world that knowledge is subjective—it represents the "state of mind" of the observer. Moreover—and in contrast to quantum states—unknown classical states can be simply "revealed" and remain unperturbed, unchanged by measurements.

The problems with the emergence of the classical from within a quantum Universe arise because—by contrast—quantum states are malleable: They are re-prepared by the attempts to find out what they are. This malleability suggests that they are epistemic—nothing more than a summary of the observer's knowledge. (Knowledge is similarly malleable—it is updated as the observer gathers additional information.) Quantum theory of the classical must therefore explain how the malleable quantum states can give rise to our perception of solid classical reality that is, in our everyday experience, independent of the information gathering by the observers.

Quantum repeatability, postulate (iii), signals a significant weakening of the role of states. Repeatability guarantees only that the presence of a *known* quantum state can be verified, but it no longer follows from its objective existence. Unlike classical states, an unknown quantum state cannot be simply found out. When observers measure observables that do not commute, each will redefine the state of the system and make the data obtained in the preceding measurements obsolete. Therefore, records of the past measurements will typically no longer determine the current state of the system.

This quantum intertwining of epistemic and ontic function of state is the central quantum feature and a key source of interpretational problems. One of our goals is to recover existence—states that are effectively objective, that survive discovery by an initially ignorant observer, so that others can confirm their continued presence. Quantum Darwinism discussed in the last part of this book shows how observers who gather information indirectly, by intercepting (as we do) fragments of the environment that decohered the system, can reach consensus about its state.

We will show that the essence of the other traditional textbook axioms that concern measurements ("amendments" we are about to discuss) can be deduced from the above quantum core, from our quantum credo that includes the three mathematical postulates (o)–(ii) and the repeatability postulate (iii) that begins to deal with information transfers (e.g., in measurements).

We note that the wording of the repeatability postulate (iii) appeals to "measurements". We retained that textbook formulation even though the use of "measurements" would signal a possible inconsistency: We aim to deduce emergence of the classical reality from *purely* quantum postulates. Thus, relying on "measurements" should be avoided. Fortunately, as we shall see in the next chapter, the demand for repeatability can be stated purely in terms of Hilbert spaces and unitary evolutions.[3]

[3] Above we use "measurement" as a shorthand for the evolution that results in a transfer of information between systems. Thus, a post-measurement state of one of them (apparatus) should reflect the state of the other (the measured system).

## 1.2 The Measurement Amendments

So far we have outlined a consistent set of core quantum postulates, (o)–(iii). They will serve as a basis for the derivation of the emergence of classical behavior in a quantum Universe. Here we consider textbook postulates: Collapse axiom (iv) and Born's rule (v), the traditional "measurement amendments" that are at odds with the quantum core.

The whole (o)–(v) list is, of course, given by textbooks. The inconsistency is usually "resolved" through some version of Bohr's strategy. That is, textbooks assume (sometimes explicitly, but often tacitly) that quantum theory can be applied only to a part of the Universe. The rest of the Universe—including observers and measuring devices—must be classical, or at the very least out of the quantum jurisdiction.

Our aim will be to show that the classical domain need not be postulated, and that the consequences of the measurement process (the focus of the amendments) follow from the quantum core, postulates (o)–(iii).

In contrast to classical physics (where an unknown pre-existing state can be found out by an initially ignorant observer) the very next item on the textbook list explicitly limits predictive attributes of quantum states:

(iv) *Measurement outcome is an eigenvalue and the corresponding eigenstate of the Hermitian operator representing the measured observable.*

Thus, in general, measurement will return something other than the pre-existing state of the system. This—compared to the classical setting—is a very significant redefinition of what a measurement does. It implies that the measurement is a multiple choice test, so that the only possible answers are the eigenvalues and the eigenstates of the measured observable.

The impossibility of determining the pre-existing state of the measured system is an immediate consequence of (iv). Repeatability postulate (iii) is in a sense an exception to this quantum undermining of the predictive role of states—it describes the situation when the measurement outcome can be predicted with certainty.

Amendment (iv) can be subdivided:

(iva) *Measurement outcomes are the eigenstates and the associated eigenvalues of an observable—a Hermitian operator.*

(ivb) *One outcome is seen in each run.*

This splitting may seem pedantic, but it is useful. Textbooks often separate our (iv) into two such axioms.

We emphasize that already (iva) limits predictive attributes of quantum states: When the measured observable does not have, as one of its eigenstates, a pre-existing state of the system, the post-measurement state of the system cannot be predicted with certainty even when the pre-existing state is perfectly known (pure). Thus, in contrast to the classical setting, perfect information about a state does not imply certainty about the measurement outcome—(iva) tells us that the state of a quantum system *per se* is not accessible.

Repeatability of postulate (iii) is an exception—it specifies when complete information guarantees certainty, at least in principle. Repeatability anticipates the essence of the collapse: The pre-existing pure state will give an unpredictable result that can, however, be confirmed and reconfirmed. What was detected (in the first measurement) will be detected again and again (thanks to repeatability).

An unknown quantum state will yield an (unpredictable) outcome that can be repeatedly reconfirmed, creating an (unwarranted) impression that also a measurement on a quantum system is just revealing a pre-existing state. Therefore, once amendment (iva) is accounted for via the uncontroversial core postulates (which we shall do in Chapter 2), then, in combination with the repeatability of (iii), the key symptom of the collapse—discreteness of quantum jumps implied by (ivb)—can be also recovered.

The textbook list is completed by Born's rule that assigns probabilities to different "legal" outcomes specified by axiom (iv):

(v) *Probability* $p_k$ *of finding an outcome* $|s_k\rangle$ *in a measurement of a quantum system that was previously prepared in the state* $|\psi\rangle$ *is given by* $|\langle s_k|\psi\rangle|^2$.

Born's rule is obviously crucial for establishing testable quantitative connection between quantum theory and experiments. We shall see that (v) does not need to be postulated: It will be derived from the core quantum postulates in Chapter 3.

## 1.2.1 Collapse of the Wavepacket

The *collapse axiom* is the first truly controversial axiom in the textbook list. In its literal form it is inconsistent with the first two postulates: Starting from a general state $|\psi_{\mathcal{S}}\rangle$ in the Hilbert space of the system (postulate (i)), an initial state $|A_0\rangle$ of the apparatus $\mathcal{A}$, and assuming (postulate (ii)) a unitary measurement-like evolution by an apparatus tested on the eigenstates of the measured observable:

$$|\sigma_k\rangle|A_0\rangle \;\Rightarrow\; |\sigma_k\rangle|A_k\rangle,$$

one is led to a superposition of outcomes:

$$|\psi_{\mathcal{S}}\rangle|A_0\rangle = \left(\sum_k a_k|\sigma_k\rangle\right)|A_0\rangle \;\Rightarrow\; \sum_k a_k|\sigma_k\rangle|A_k\rangle. \tag{1.1}$$

This is a superposition—not a single outcome. Hence, Eq. (1.1) is in an apparent contradiction with the "one outcome" posited by (iv).

The impossibility to account—starting with the core quantum postulates (o) to (ii)—for the literal collapse to a single state posited by axiom (ivb) has been appreciated since Bohr (1928) and von Neumann (1932). It was—and often still is—regarded as an indication of the insolubility of the measurement problem. It is straightforward to extend such insolubility demonstrations to various more realistic situations, e.g., by allowing the state of the apparatus to be initially unknown (i.e., mixed). As long as superposition and unitarity postulates (i) and (ii) hold, one is forced to admit that the quantum state of $\mathcal{SA}$ after the measurement corresponds to a superposition of many alternative outcomes—all the outcomes consistent with the initial state—rather than just one of them (as the literal reading of the collapse axiom and our immediate experience would have it). In particular, when the initial states of the system and the apparatus are pure, that state of $\mathcal{SA}$, Eq. (1.1), is entangled.

Given this clash between the mathematical structure of quantum theory and the literal collapse (that captures the subjective impressions of what happens in real-world measurements), one may be tempted to accept the primacy of our immediate experience, and blame the inconsistency of (iv) with the core of quantum formalism—superposition principle and unitarity, (i) and (ii)—on the nature of the apparatus: The Copenhagen Interpretation at least in its oversimplified textbook version (but see Camilleri and Schlosshauer (2015) for a more nuanced account) regards apparatus, observer, and, generally, macroscopic objects as *ab initio* classical. They do not abide by the quantum principle of superposition, and their evolutions do not need to be unitary. Therefore, according to the textbook version of the Copenhagen Interpretation, measurements are exempt from the unitarity postulate (ii). The collapse can happen on the lawless border of quantum and classical.

## 1.2.2 Information or Existence: Epiontic Quantum States?

Uneasy coexistence of the quantum and the classical is a challenge to the unification instinct of physicists. Yet, what many regard as Bohr's *ad hoc* solution has proven to be surprisingly durable.

At the heart of many approaches to the measurement problem is the desire to reduce the relation between existence and information to what it was when the fundamental theory was Newtonian

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physics. There, classical systems had real ontic states that existed independently of what was known about them. They could be found out by a measurement. Many initially ignorant observers could measure the same system without perturbing it. Their records would agree, reflecting reality of the underlying state and confirming its objective existence.

Immunity of classical states to measurements suggested that, in a classical Universe, the information was unphysical. It was a mere immaterial shadow of the real physical state, irrelevant for physics. This dismissive view of information ran into problems already when Newtonian classical physics confronted classical thermodynamics. The clash of these two classical theories led to Maxwell's demon, and is implicated in the origins of the arrow of time. The specter of information was haunting classical physics. The seemingly unphysical shadowy record state was beginning to play, already in statistical physics, the role reserved for the "real" classical state.

Attempts to solve the measurement problem often follow the strategy (which can be traced to Bohr, although he should not be held responsible for specific implementations) where the underlying state of some quantum system somehow becomes classical. This transformation usually involves modifications of quantum theory (e.g., spontaneous collapse or hidden variables that enforce protoclassical features such as localization).

It is conceivable that, one day, we may find discrepancies between quantum theory and experiments. However, evidence to date supports the view that our Universe is really quantum to the core, and we have to learn how to reconcile quantum superposition principle, unitarity, and their consequences—illustrated, e.g., by the violation of Bell's inequality—with our perceptions. Modifications that lead to explicit collapse are testable, and the past proposals (Białynicki-Birula and Mycielski, 1976; Leggett, 1980; Ghirardi, Rimini, and Weber, 1986; Penrose, 1986; Weinberg, 1989, 2012) have been severely constrained by the experimental data (see, e.g., Gähler, Klein, and Zeilinger, 1981; Pearle and Squires, 1994) that confirm the validity of quantum theory and extend its applicability to increasingly large systems.

The non-locality of quantum states and other experimental manifestations of their quantumness that stem from the combination of the quantum principle of superposition and the unitarity of evolutions, postulates (i) and (ii), are here to stay. They clash with the intuition based on our everyday experience that invites idealization of pre-existing solid classical reality, and of localized states. However, we shall see that—when it is recognized that quantum systems we encounter in our everyday experience are not isolated (so that literal applicability of the superposition principle and the unitarity of evolutions should be at least re-examined)—classical physics emerges as an approximation.

## 1.3 Let Quantum Be Quantum: Interpreting Everett's Relative States Interpretation

The strategy we shall follow to account for the definiteness of our perceptions is to start with the core quantum postulates (o)–(iii). They have a simplicity that rivals the postulates of special relativity. This "let quantum be quantum" starting point will allow us to show how (and to what extent) both of the attributes of the familiar classical world—objective existence and information about it—emerge from the quantum substrate.

The alternative to Bohr's Copenhagen Interpretation and a new approach to the measurement problem was proposed by Hugh Everett III, student of John Archibald Wheeler, well over half a century ago (Everett, 1957a,b; Wheeler, 1957). The basic idea was to abandon the literal view of the collapse and recognize that a measurement (including the appearance of the collapse) is already implicit in the evolution represented by Eq. (1.1). Everett claims that one just needs to include an observer in the wavefunction, and consistently interpret consequences of this step.

The obvious problem raised by the conflict with the literal view of the collapse axiom (ivb) is then answered by asserting that while the right-hand side of Eq. (1.1) contains all the possible outcomes,

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the observer who recorded outcome #17 will (from then on) perceive "branch #17". In other words, when the global state of the Universe is $|\Upsilon\rangle$, and my state is $|\mathcal{I}_{17}\rangle$, for me the state of the rest of the Universe collapses to $|\Upsilon_{17}\rangle = \langle\mathcal{I}_{17}|\Upsilon\rangle$. Since this is the only state I (actually, $|\mathcal{I}_{17}\rangle$!) am aware of following the interaction that led to my recording of the outcome #17, I should renormalize the state vector $|\Upsilon_{17}\rangle = \langle\mathcal{I}_{17}|\Upsilon\rangle$ of the Universe to reflect my certainty about my branch—this is now my only Universe.

This is the *Relative States Interpretation* of the measurement: The state of the rest of the Universe is defined with respect to *my* state—to the state of *my* records.

Much confusion and a heated debate has been sparked by the question of what happens to observers $|\mathcal{I}_1\rangle \ldots |\mathcal{I}_{16}\rangle$ and $|\mathcal{I}_{18}\rangle \ldots |\mathcal{I}_{\infty}\rangle$ and their "branches" of the universal state vector. If the quantum state of the whole Universe were classical—so that we could attribute real existence to the universal state vector—there would indeed be Many Worlds, each inhabited by a different $|\mathcal{I}_n\rangle$ (see, e.g., DeWitt, 1970, 1971; Deutsch, 1985, 1997; Wallace, 2012a,b).

However, the precise status of states in quantum theory is elusive—they can be confirmed (repeatability), but an unknown state cannot be found out. This inability to find out an unknown state signals the absence of "objective existence" of quantum states, and suggests a less radical possibility closer in spirit to the Relative States Interpretation.

After all, a patch in classical phase space also represents a state. When this patch collapses into a point upon measurement, this does not mean that there are other observers who from then on live in Universes with different outcomes, and have records consistent with these other outcomes.

The key difference between these two (Relative States and Many Worlds) attitudes is the extent to which a quantum state is thought to be epistemic (i.e., a carrier of information, as is a patch in phase space, representing ignorance of the observer) or ontic (as is the classical system represented by a phase space point, which can be not only confirmed, but found out by others, even when they are ignorant of its location beforehand). Only an effectively classical—ontic—view of the universal state vector would make literal Many Worlds (with all the branches equally real) inevitable.

Quantum states are not ontic—they cannot be revealed by a measurement, as axiom (iva) recognizes—so in this sense the Many Worlds Interpretation (in contrast to the Relative States view) is just too classical: It endows quantum states with objective existence to which they (unlike their classical counterparts) are not entitled. We have no stake in this debate, but we shall comment on these matters in due course, after Quantum Darwinism is introduced and after the discussion of the quantum version of objective existence and of the *Existential Interpretation*.

The aspect of Everett's views we shall, however, wholeheartedly embrace is the general applicability of quantum theory. It is indeed the principal guide and the main tool in the search for its interpretation. This "let quantum be quantum" view of the collapse is consistent with the repeatability postulate (iii); upon immediate re-measurement of the same observable, the same state will be found. Everett's (1957a) assertion: "The discontinuous jump into an eigenstate is thus only a relative proposition, dependent on the mode of decomposition of the total wave function into the superposition, and relative to a particularly chosen apparatus-coordinate value . . ." is consistent with quantum formalism: In the superposition of Eq. (1.1) record state $|A_{17}\rangle$ can indeed imply detection of the corresponding state of the system, $|s_{17}\rangle$.

### 1.3.1 Basis Ambiguity: What Happens?

Two questions immediately arise. The first one concerns the (iva) part of the collapse: What constrains the set of the preferred states of the apparatus or the observer (or, indeed, of any object that entangles with another quantum system).

By the principle of superposition (implied by postulate (i)) the state of the system or of the apparatus after the measurement can be written in infinitely many ways, each corresponding to one of the unitarily equivalent bases in the Hilbert space of the pointer of the apparatus (or memory cell of the observer) and the corresponding (usually not orthogonal) states of the system:

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$$\sum_k a_k|\sigma_k\rangle|A_k\rangle = \sum_k a'_k|\sigma'_k\rangle|A'_k\rangle = \sum_k a''_k|\sigma''_k\rangle|A''_k\rangle = \cdots . \tag{1.2}$$

This *basis ambiguity* is not limited to the pointers of measuring devices (or cats, which in the example considered by Schrödinger (1935a,b) play a role of the apparatus). One can show that also very large systems (such as satellites of planets) could evolve into very non-classical superpositions on surprisingly short timescales if they followed the Schrödinger equation (Zurek and Paz, 1995a; Zurek, 1998a). In reality, this does not seem to happen. So, there is something that (in spite of the egalitarian superposition principle enshrined in (i)) picks out certain preferred quantum states, and makes them effectively classical.

Postulate (iva) anticipates this need for preferred states—destinations for quantum jumps. Before there is a collapse (as in (ivb)), a set of preferred states (one of which is selected by the collapse) must be somehow chosen. Indeed, the discontinuity of quantum jumps that Everett emphasized in the quote above would be impossible without some underlying discontinuity in the set of the possible choices. Yet, there is nothing in Everett's writings that would provide a criterion for such preferred outcome states, and nothing to even hint that he was aware of this question. We shall show in Chapter 2 how such discontinuities arise in the framework defined by the core quantum postulates (o)–(iii).

### 1.3.2 Probabilities: How Likely Is It to Happen?

The second question concerns probabilities: How likely it is that I will become $|\mathcal{I}_{17}\rangle$ after I, the observer, measure $\mathcal{S}$? Everett was very aware of its significance.

Pointer basis and its origin are discussed in Part II of this book devoted to decoherence. However, the theory of decoherence, as it is usually practiced, employs reduced density matrices and partial trace. Their physical significance derives from averaging (Landau, 1927; Nielsen and Chuang, 2000; Zurek, 2003b) and is thus based on probabilities—on Born's rule.

Born's rule (1926), axiom (v), completes standard textbook discussions of the foundations of quantum theory. In contrast to the collapse axiom (iv), axiom (v) is not in obvious contradiction with postulates (o)–(iii), so one can adopt the attitude that Born's rule completes the core postulates (o)–(iii) and thus justify preferred basis and symptoms of collapse via decoherence. This is the usual practice of decoherence (Zurek, 1991, 1998b, 2003a; Paz and Zurek, 2001; Joos et al., 2003; Schlosshauer, 2007, 2019).

Nevertheless, as Everett and others argued, Born's rule is inconsistent with the spirit of the "let quantum be quantum" approach. Therefore, as we seek fundamental understanding of the relation between quantum theory and everyday classical reality (or, for that matter, of what happens in the laboratory experiments involving quantum systems—real quantum measurements), we shall not be satisfied with the usual approach to decoherence based on reduced density matrices. Indeed, Everett attempted to derive Born's rule from purely quantum postulates. We shall soon follow his lead, although not his strategy, which—as is now acknowledged—was flawed (DeWitt, 1971; Kent, 1990; Squires, 1990).

## 1.4 Decoherence and Einselection: a Primer

Decoherence and the *e*nvironment-*in*duced super*selection* or *einselection* will be explored at length later on in this book. However, before we get there, we shall focus in the next two sections on issues that are more fundamental and therefore more naturally discussed early on. It is nevertheless easier to proceed when some of the big picture in which decoherence plays a central role is at least sketched.

The overarching question we consider is how does the classical world—classical states that are responsible for the objective reality of our everyday experience—emerge from within the Universe that is, as we know from compelling experimental evidence, made out of quantum stuff. The short answer to this question is that, in the process of einselection, decoherence selects—from the vast

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number of quantum superpositions that populate Hilbert space—the few states that are (in contrast to all the other alternatives) stable in spite of their immersion in the environment.

All the systems we encounter are immersed in the environments such as air or photons. Decoherence is a consequence of the interaction between a quantum system $\mathcal{S}$ and its environment $\mathcal{E}$. As a result of that interaction the initially pure states of $\mathcal{S}$ and $\mathcal{E}$ entangle:

$$|\psi_{\mathcal{S}}(0)\rangle|\varepsilon(0)\rangle \xrightarrow{\mathbf{H}_{\mathcal{SE}}} |\Psi_{\mathcal{SE}}(t)\rangle. \tag{1.3}$$

The unitary evolution from the initial product state $|\psi_{\mathcal{S}}(0)\rangle|\varepsilon(0)\rangle$ to the entangled $|\Psi_{\mathcal{SE}}(t)\rangle$ is generated by the interaction Hamiltonian $\mathbf{H}_{\mathcal{SE}}$.

The environment plays the role similar to that of the apparatus in Eq. (1.1). Consequently, one is faced with the basis ambiguity problem, analogous to Eq. (1.2), as $|\Psi_{\mathcal{SE}}(t)\rangle$ can be expressed in a variety of bases at each instant $t$.

When one is interested only in the states of the systems, one can trace out the environment to get the density matrix of $\mathcal{S}$:

$$\rho_{\mathcal{S}}(t) \;=\; \mathrm{Tr}_{\mathcal{E}}|\Psi_{\mathcal{SE}}(t)\rangle\langle\Psi_{\mathcal{SE}}(t)|. \tag{1.4}$$

The density matrix is a Hermitian operator, so one can always diagonalize it. This yields:

$$\rho_{\mathcal{S}}(t) \;=\; \sum_k p_k(t)|\varsigma_k(t)\rangle\langle\varsigma_k(t)|. \tag{1.5}$$

One can also trace out the system to obtain the density matrix of the environment:

$$\rho_{\mathcal{E}}(t) \;=\; \mathrm{Tr}_{\mathcal{S}}|\Psi_{\mathcal{SE}}(t)\rangle\langle\Psi_{\mathcal{SE}}(t)| \;=\; \sum_k p_k(t)|e_k(t)\rangle\langle e_k(t)|. \tag{1.6}$$

The eigenstates of $\rho_{\mathcal{S}}$ and $\rho_{\mathcal{E}}$ come in pairs. Each such pair corresponds to the same eigenvalue $p_k(t)$—to the same probability. The entangled state can be written as a *Schmidt decomposition*:

$$|\Psi_{\mathcal{SE}}(t)\rangle \;=\; \sum_k \alpha_k(t)|\varsigma_k(t)\rangle|e_k(t)\rangle. \tag{1.7}$$

Every pure state of two systems can be expressed as such a Schmidt decomposition with a single sum over the pairs of orthonormal *Schmidt states* $|\varsigma_k(t)\rangle$ and $|e_k(t)\rangle$: The number of terms in the sum is no larger than the dimensionality of the smaller of the two Hilbert spaces. Moreover, absolute values of the Schmidt coefficients are directly related to the probabilities—to the eigenvalues of the density matrices, $|\alpha_k(t)| = \sqrt{p_k(t)}$.

One may be tempted to regard the eigenstates of $\rho_{\mathcal{S}}(t)$ as candidates for the classical states of $\mathcal{S}$ (see, e.g., Zeh, 1990; Albrecht, 1992). There are at least two problems with this. To begin with, $\rho_{\mathcal{S}}(t)$ (hence, its eigenstates) are time-dependent, so the stability one would hope for in the classical states is in question. Moreover, Schmidt decomposition is no longer unique when the eigenvalues $p_k(t)$ associated with the pairs $|\varsigma_k(t)\rangle|e_k(t)\rangle$ are degenerate. In that case any orthogonal basis that spans the Hilbert subspace corresponding to the degenerate eigenvalue is "Schmidt", and could aspire to be regarded as "preferred", as it can appear on the diagonal of the reduced density matrix.

The lack of uniqueness when the eigenvalues $p_k(t)$ become equal implies a dangerous ambiguity incompatible with the intuitive criteria for classicality: What states are classical should not depend on how likely they are.[4] Moreover, when some of the eigenvalues are nearly equal—the state is nearly degenerate—very small changes in the initial states of $\mathcal{SE}$ can dramatically change the Schmidt

[4] This lack of uniqueness would suggest, for example, that half-way through the Schrödinger cat experiment when the probabilities of the two obvious options corresponding to the two states we can euphemistically denote as $|\uparrow\rangle$ and $|\downarrow\rangle$ are equal, the two orthogonal superpositions formed from these states $|\uparrow\rangle \pm |\downarrow\rangle$ are equally valid classical alternatives.

decomposition. This hypersensitivity of Schmidt states to the initial state of $\mathcal{SE}$ disqualifies them as candidates for classicality.

A basis that is a far better candidate for classicality is the *pointer basis* (Zurek, 1981). It is selected by the Hamiltonian of interaction between the system and its environment. Let us assume that (as is often the case) the interaction Hamiltonian depends on a certain observable $\Lambda$ of $\mathcal{S}$, and, hence, commutes with it:

$$\mathbf{H}_{\mathcal{SE}} \;=\; \mathbf{H}_{\mathcal{SE}}(\Lambda); \quad [\Lambda, \mathbf{H}_{\mathcal{SE}}] = 0. \tag{1.8}$$

Observable $\Lambda$ is Hermitian, so it has (possibly degenerate) eigenstates $|s_k\rangle$. When the system starts in one of them, it will remain unperturbed by the environment. When it starts in their superposition, it will evolve into an entangled state with $\mathcal{E}$. The same entangled state we have expressed before as a Schmidt decomposition can be now written in the pointer basis:

$$|\Psi_{\mathcal{SE}}(t)\rangle \;=\; \sum_k a_k |s_k\rangle |\varepsilon_k(t)\rangle. \tag{1.9}$$

The eigenstates $|s_k\rangle$ of $\Lambda$ are obviously orthogonal, and (unlike the Schmidt states of $\mathcal{S}$) they do not depend on time. Thus, $|s_k\rangle$ are stable in spite of decoherence. The density matrix $\rho_{\mathcal{S}}(t)$ we have encountered before can be expressed in the pointer basis of the stable states $|s_k\rangle$:

$$\rho_{\mathcal{S}}(t) \;=\; \sum_k |a_k|^2 |s_k\rangle\langle s_k| \;+\; \sum_{k,l} a_k^* a_l \langle \varepsilon_k(t)|\varepsilon_l(t)\rangle |s_l\rangle\langle s_k|. \tag{1.10}$$

This is the same $\rho_{\mathcal{S}}(t)$ as before, but it is now expressed in a different basis. Its diagonal part is time-independent (thus, the states on the diagonal are stable). However, there is also a complication—the off-diagonal terms.

Stability of $|s_k\rangle$ favors pointer states as candidates for classicality. When an agent detects the system before or during decoherence in one of these states, that record will remain valid—the system will continue to be in that same pointer state later on, interaction of $\mathcal{S}$ with the environment notwithstanding.

Such states unaffected by $\mathcal{E}$ are known as pointer states precisely because the apparatus pointer will retain information about the measurement outcome when it is stored in one of its pointer states. Perfect pointer states are "decoherence-proof"—they are perfectly stable, and do not entangle with the decohering $\mathcal{E}$. We shall also eventually discuss approximate pointer states that are only "decoherence-resistant".

So what about the Schmidt states? Are they irrelevant for classicality? After all, the eigenvalues of $\rho_{\mathcal{S}}(t)$ are probabilities of the corresponding eigenstates, and probabilities of Schmidt states are measured (as relative frequencies) in experiments. That same statement cannot be made about pointer states: When the overlap of the associated environment states $\langle \varepsilon_k(t)|\varepsilon_l(t)\rangle$ is large, measurements of the observable associated with the pointer states $|s_k\rangle$ will be influenced by interference, so the coefficients $|a_k|^2$ will not be additive. Hence, they cannot be regarded as probabilities. Nevertheless, an observer who has detected one of the pointer states can count on its presence in spite of decoherence. Stability is the more important criterion, and by now there is consensus that pointer states are by far the best candidates for the classical realm.

Fortunately, this need to choose between the Schmidt states and pointer states turns out to be a false dilemma: In most circumstances, and especially when the environment is large compared to the system and decoherence is efficient, the overlap of the states of the environment correlated with the pointer states becomes vanishingly small, $\langle \varepsilon_k(t)|\varepsilon_l(t)\rangle \to 0$, and the off-diagonal terms in the density matrix expressed in the stable pointer basis, Eq. (1.10), nearly disappear. Thus:

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$$\rho_{\mathcal{S}}(t) = \sum_k p_k(t)|\varsigma_k(t)\rangle\langle\varsigma_k(t)| \overset{\langle\varepsilon_k(t)|\varepsilon_l(t)\rangle\to 0}{\Longrightarrow \cdots \Longrightarrow} \sum_k |a_k|^2 |s_k\rangle\langle s_k| = \tilde{\rho}_{\mathcal{S}}, \tag{1.11}$$

and the two density matrices nearly coincide. In that case the eigenvalues of the approximate $\tilde{\rho}_{\mathcal{S}}(t)$ diagonal in the einselected pointer basis can be safely used as probabilities of the associated pointer states. This happens when the environment correlates with the stable pointer states of $\mathcal{S}$, as in the case of perfect correlation $\langle\varepsilon_k(t)|\varepsilon_l(t)\rangle = 0$, so the time-dependent Schmidt states come to coincide with pointer states. When this occurs, decoherence is complete.

The importance of the stability of the pointer basis is best illustrated "in action", when the apparatus $\mathcal{A}$ that has already established correlation with the measured system is subject to decoherence:

$$\left(\sum_k a_k|\sigma_k\rangle|A_k\rangle\right)|\varepsilon(0)\rangle \overset{\mathbf{H}_{\mathcal{A}\mathcal{E}}}{\Longrightarrow} \sum_k a_k|\sigma_k\rangle|A_k\rangle|\varepsilon_k(t)\rangle. \tag{1.12}$$

In measurements it is imperative that the record survives decoherence (which is inevitable in macroscopic measuring devices). Above, decoherence is induced by the interaction of the apparatus with its environment, but the pointer states $|A_k\rangle$ are time-independent, immune to decoherence. Thus, the correlation established by the interaction of the measured system with the apparatus (see Eq. (1.1)) will survive intact even as the environment suppresses manifestations of quantumness between the pointer states of $\mathcal{A}$.

Our introduction to decoherence and einselection does not exhaust the subject. Moreover, it assumes much of the textbook lore (including, in particular, Born's rule) that we would like to deduce from the quantum core postulates. However, it sets the stage for the next two sections, where amendments to the core postulates will be deduced from our quantum credo—from that core.

## 1.5 Summary

In this section we have separated the textbook postulates of quantum theory into *quantum core postulates* and "amendments"—textbook *measurement axioms*. The quantum core includes three mathematical postulates (o)–(ii) dealing, respectively, with (o) composite systems, (i) the Hilbert space nature of quantum states and the quantum principle of superposition, and (ii) the unitarity of quantum evolutions.

These three postulates are included in our quantum credo. They summarize the mathematics of quantum theory. To connect mathematics with physics we will use repeatability postulate (iii). It posits that the presence of a known quantum state can be confirmed. Repeatability is then really a rudimentary version of predictability.

Postulates (o)–(iii) comprise our "quantum credo". They are uncontroversial, simple, and natural. They are consistent with all the known experiments. As we shall see, they can be used to deduce the essence of the axiom (iva)—the textbook demand that the observables should be Hermitian—and to show the discreteness of the outcomes (suggesting quantum jumps, i.e., why a single result is perceived by the observer, as in the axiom (ivb)).

Moreover, the quantum credo will allow for a derivation of Born's rule, $p_k = |\langle s_k|\psi\rangle|^2$, axiom (v). Thus, the essence of the controversial quantum axioms can be deduced from the simple and natural cornerstones of quantum theory, postulates (o)–(iii).

We have also touched on the interpretational difficulties. They are the reason for the controversies. We shall abstain from summing them up here: This would be premature, as we shall delve into the issues of interpretation throughout this book, and discuss them in more detail only near the end of this volume.

We ended this introductory section with a decoherence and einselection primer. Decoherence and einselection will be discussed at length in the body of this book.

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## 1.6 Frequently Asked Questions

FAQ#1: *In spite of the promise not to invoke interpretations, Bohr's and Everett's interpretations played a prominent role in this chapter. Why?*

The role of various quantum postulates and the problems that arise in relating quantum mechanics to our everyday experience are easiest to elucidate by pointing out the difficulties of the existing and widely known interpretations. This is why we have invoked Bohr's "Copenhagen" and Everett's "Relative States". However, we did not take sides. Indeed, our aim in this part of the book is to show how much of the standard textbook lore that includes the "measurement amendments" one can recover from the non-controversial interpretation-independent core quantum postulates—our quantum credo.

We shall come back to interpretations once we have seen how much light can be shed on the emergence of the classical without prematurely jumping to interpretational conclusions. Our aim is to investigate information transfer that is obviously crucial for the understanding of quantum measurements and of our perception of the quantum Universe we inhabit.

FAQ#2: *Feynman has famously said "nobody understands quantum mechanics". Would a quantum derivation of the classical realm contradict Feynman?*

My goal is to understand how classical reality emerges from quantum mechanics. This is different than "understanding quantum mechanics"—we shall not present, e.g., a model of entanglement that "you can explain . . . to a barmaid", to quote Rutherford's famous criterion for a successful explanation (that he gave up on after quantum tunneling was explained to him by Gamow). So, I doubt I am at odds with Feynman.

I also think that Feynman's dictum was "tongue in cheek". By contrast, I think Bohr, who said ". . . those who are not shocked when they first come across quantum theory cannot possibly have understood it . . .", can be taken at face value.

FAQ#3: *Why not start with a different set of postulates where instead "collapse axiom" is a part of the foundations, and derive what was assumed here?*

A derivation of quantum theory from the "collapse" does not exist at present. Moreover, there are at least two reasons to be wary of starting with "collapse", or indeed, with the measurement as a foundation.

The first one: One should be able, at least in principle, to describe the measurer (be it an animated agent or an inanimate apparatus) using quantum theory, even if quantum theory emerges from the derivation that involves measurements. But quantum theory includes the superposition principle and the unitarity of evolutions. Therefore, such proposals eventually have to point to some reason that makes quantum theory inapplicable to agents or measuring devices. Bohr was keenly aware of this, as were others, including von Neumann (1932), London and Bauer (1939), Heisenberg (1955, 1989), and Wigner (1961), who all grappled with this issue early on (and invoked *ab initio* classicality, self-awareness, etc.). No compelling resolution has been found until decoherence.

The second (and more important) reason is that the overwhelming majority of the consequences of quantum theory have nothing to do with measurements. Thus, atomic spectra, condensed matter physics including superfluidity, the physical basis for chemistry, nuclear, and particle physics, and many other manifestations of quantum theory are flagrantly quantum, but their quantumness has nothing to do with measurements. There is no reason to think that such quantum phenomena (or quantum exotica such as entanglement) are a consequence of rare events involving observers, especially since the essence of what is puzzling about measurements can be understood starting with our "credo".

FAQ#4: *In spite of the aversion to "measurements", postulate (iii) asserts repeatability of measurement results. Is this an inconsistency?*

It would be an inconsistency if "measurement" was indeed a "primitive"—a fundamental ingredient that does not require an explanation. We shall see (starting in Chapter 2) how this is avoided

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by treating apparatus as a quantum system. Above, we have attempted to stay close to the textbook formulations of the "credo". Hence, we have retained the language that appeals to "measurements". Repeatability can be formulated without bringing measurements in as primitive ingredients, just by considering the effect of interactions and unitary evolutions.

FAQ#5: *Can one derive "core postulates" from something else, something more palatable than the "quantum credo"?*

There were attempts at derivations of quantum postulates from other postulates. The motivation is usually to make quantum foundations more classical (and therefore, more "natural"). However, "naturalness is in the eye of the beholder".

Such derivations fall, broadly speaking, into two categories: Some (e.g., Hardy, 2001) are "operational", start with the measurement as a "primitive" in some guise, and, hence, have to accept pre-existence of measuring devices that are exempt from the quantum laws. These strategies face the problem of the "shifty split" (Bell, 1990) between quantum and classical (or between measured quantum systems and measuring devices that are exempt from the quantum superposition principle). Therefore, the measurement problem is left unresolved, much like in the Copenhagen Interpretation.

There are also derivations (Chiribella, D'Ariano, and Perinotti, 2011; Masanes, Galley, and Müller, 2019) that in addition to taking measurement as a "primitive" include an explicitly quantum element (like entanglement in some guise) in the fundamental assumptions. It is difficult to see the advantage of using such alternative axiomatics instead of the "quantum credo" in identifying the classical realm, especially as decoherence provides a unified view of the quantum Universe with the emergent classicality. This is acknowledged by some of the authors of the attempts that take measurement as a "primitive" (see, e.g., Section 6.8.2 of Galley (2018), quoted in part in the footnote).[5]

While—in the opinion of this author—none of the approaches cited above as well as in the references below is sufficiently successful and complete to compete with the quantum postulates presented, e.g., in Dirac (1958) and distilled into the the quantum credo discussed here, such efforts shed valuable light on the nature and, above all, on the uniqueness of quantum theory.

## 1.7 Further Reading

An exhaustive summary of the relevant literature would likely be longer than this chapter. Moreover, we shall return to the subject of interpretation later in this book. Here we add only a few positions to the items already cited. A summary of the interpretational effort in the first half-century after the birth of quantum physics can be found in the book of Jammer (1974). Many of the relevant papers have been collected in Wheeler and Zurek (1983). Schlosshauer's papers (2004, 2006, 2019) and his book (2007) are mainly devoted to decoherence, but they set the stage for a more general discussion of the transition from quantum to classical, and include numerous references. The voluminous work of Auletta (2000) contains even more references, and touches on a broad range of subjects, as does

[5] ". . . A derivation of the Born rule which starts from similar assumptions to [Masanes, Galley, and Müller, 2019] is the envariance based derivation . . . [It] begins by assuming the dynamical structure of quantum theory and [assumes] that quantum theory is universal, which is to say that all the phenomena we observe can be explained in terms of quantum systems interacting. Specifically the classical worlds of devices can be modeled quantum mechanically, including the measurement process. . . . [T]his is philosophically very different to the operational approach . . . [of Galley, 2018] which takes the classical world as a primitive. By assuming the dynamical structure of quantum theory . . . Zurek shows that measurements are associated to orthonormal bases, and that outcome probabilities are given by the Born rule. . . . We observe that the purification postulate [of Chiribella, D'Ariano, and Perinotti, 2011; Galley, 2018; Masanes, Galley, and Müller, 2019] seems linked to the notion that quantum theory is universal, in the sense that any classical uncertainty can be explained as originating from some pure global quantum state. This shows an interesting link to [envariant] derivation, since . . . the concept of purification is linked to the idea that quantum theory is universal" (Galley, 2018).

the followup (Auletta, 2019). The collection of John Bell's papers (Bell, 1987) as well as the book by Asher Peres (1993) are much more focused and highly recommended.

Other useful resources include the bibliographic guide to the foundations of quantum mechanics and quantum information (Cabello, 2004) as well as the classic book by Nielsen and Chuang (2000) on quantum information and quantum computation. There are also several more philosophically oriented books by d'Espagnat (1995, 2013) and a collection of debates on the foundations of quantum theory (d'Espagnat and Zwirn, 2017). Cabello (2016) provides a brief and amusing summary of various interpretations.

Everett's interpretation has been a subject of numerous publications. We have already cited several in the text. Let us add semi-popular books by Deutsch (1997), Byrne (2010), Kurizki and Gordon (2020), as well as a monograph by Wallace (2012b) and the collection of contributions edited by Saunders et al. (2010).

Other approaches that either aim at derivation of the essence of quantum theory from the principles that rely on measurements or attempt to dispose of the measurement problem in some other way include influential work of Wootters (1981, 2016), papers of Auffeves and Grangier (2016), and the Bayesian approach that has evolved (Fuchs and Peres, 2000) into QBism (Fuchs and Schack, 2013).