

Abstracts of Australasian Ph.D. theses

Laws in torsion-free nilpotent varieties with particular reference to the laws of free nilpotent groups

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In this thesis I am concerned largely with the varieties of groups generated by the free-nilpotent-of-class- c groups of rank $n < c$, that is $\text{Var}\left(F_n(\underline{\mathbb{N}}_c)\right)$.

I show that, if $n > \max\left\{\frac{c-1}{2}, 8\right\}$,

$$(1) \quad \text{Var}\left(F_n(\underline{\mathbb{N}}_c)\right) \wedge \underline{\mathbb{N}}_{c-1} = \text{Var}F_{n+1}(\underline{\mathbb{N}}_{c-1})$$

but that (1) is not true if $n = \frac{c}{2} - 2$.

In the process of proving this I obtain, for each $w < c-1$, a set of commutator words which are homogeneous of weight $c - w$, include all the laws of weight $c - w$ in $F_{n+w}(\underline{\mathbb{N}}_{c-w})$ and are all laws in $F_n(\underline{\mathbb{N}}_c)$.

This proves that

$$\left(\left(\text{Var}F_n(\underline{\mathbb{N}}_c)\right) \wedge \underline{\mathbb{N}}_{c-w}\right) \vee \underline{\mathbb{N}}_{c-w-1} \subseteq \text{Var}\left(F_{n+w}(\underline{\mathbb{N}}_{c-w})\right) \vee \underline{\mathbb{N}}_{c-w-1}.$$

These results rely on a relationship between certain nilpotent varieties and representations of general linear groups. The relationship was first pointed out by Higman [1] for varieties of prime exponent and has subsequently been extended by Kovács and Newman to varieties generated by torsion free groups. Since Kovács and Newman's work is as yet unpublished

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the thesis contains an exposition of this extension.

Reference

- [1] Graham Higman, "Representations of the general linear groups and varieties of p -groups", *Proc. Internat. Conf. Theory of groups, Austral. Nat. Univ., Canberra*, (1965), 167-173 (Gordon and Breach, New York, London, Paris, 1967).