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BOOK REVIEWS

CHATTERS, A. W. and HAJARNAVIS, C. R., *Rings with chain conditions* (Pitman, 1980), £8·00.

In the last decade quite a number of books dealing with non-commutative ring theory have been published. However, much of the material in this book has not previously been published in book form and therefore it is a welcome addition to the literature. It is concerned with certain aspects of the theory of Noetherian rings (i.e. rings which satisfy the ascending chain condition on right ideals and left ideals) though as the title suggests more general rings are also considered.

What aspects of ring theory are considered? A recurring theme is the existence of classical quotient rings Q for rings R and the interplay between R and Q. As well as presenting the early theorems of Goldie and Small an account is given of the recent work of Stafford on Noetherian rings R with $R=Q$. Among other topics discussed are Noetherian rings of Krull dimension 1 (although Krull dimension is not defined in general), Jacobson's conjecture whether the intersection of the finite powers of the Jacobson radical of a Noetherian ring is zero, serial rings, fully bounded rings, hereditary rings and more generally rings with every principal right or left ideal projective, rings with finite global dimension, the Artin Rees property and simple rings. Always the rings satisfy some chain condition on one side or on both the right and left. Many examples (often of 2×2 matrices) are given to illustrate the results and show they can't be generalised and there are a number of open questions raised by the discussion which take the form of remarks at the end of each chapter.

As one might expect the proofs use the techniques developed by Goldie and others in studying quotient rings and in particular uniform and essential modules are prominent. But a feature of the book is the repeated use of reduced rank and the Artinian radical. If R is a semiprime right Noetherian ring, with classical right quotient ring Q (which is semiprime Artinian) and M is a finitely generated right R-module, then the reduced rank is the composition length of the right Q-module $M\otimes_R Q$ and this definition can be extended to the case when R is not semiprime. Reduced rank is used in various ways perhaps the most notable being in the proof of the following generalisation of the Principal Ideal Theorem of commutative algebra: if P is a prime ideal of a right Noetherian ring R minimal over an invertible ideal then rank $P\leqq 1$. The Artinian radical of a Noetherian ring R is the sum of all Artinian right ideals of R and so is an ideal and is Artinian as a right R-module. The other striking feature of the proofs is the remarkable facility with element manipulations displayed throughout. Indeed the book is not only a tribute to the "Leeds school" of ring theory but also an expression of the "Leeds style" combining clarity, elegance and ingenuity.

This book is called a research note and as such is aimed in the first instance at experts in the field. What about other readers? If the book has a weakness it is that the style is a little terse in parts. For example, the short slick proofs of Goldie's theorem don't give as much insight as the much longer original one. Also in at least two points it is assumed that the reader knows that a non-zero ideal in a prime ring with polynomial identity contains a non-zero central element. But these are minor quibbles and potential readers should not be unduly alarmed! Anyone interested in non-commutative Noetherian rings will find this very readable and attractive book worth consulting.

P. F. SMITH

STROMBERG, KARL R., *An Introduction to Classical Real Analysis* (Wadsworth International Group, 1981), 576 pp., cloth, U.S. $29.95.

This book includes not only the material usually presented in a first rigorous course in Analysis but also much that will interest the mature mathematician.

For students taking a rigorous course the book discusses series in the setting of complex numbers and develops the exponential and trigonometric functions as sums of complex power series. Use of certain ideas (e.g. the number π) is scrupulously avoided until they have been formally presented in the text. The theory of metric spaces is treated in considerable detail in Chapter 3 with discussions of uniform convergence and the Stone-Weierstrass and Ascoli theorems. A novel idea is the introduction in Chapter 6 of the Lebesgue integral as a first integral on the real line.

For the mathematically mature the book contains a host of interesting results, historical references and exercises. The author says in his preface that he spent at least three times as much effort in preparing the exercises as he did on the main text. As a result there are hundreds of eye-catching exercises, many with subdivisions and hints. Some are routine, some lead up to a main result (e.g. that the number e is transcendental), some bear twentieth century surnames, some offer alternative methods, some introduce and illustrate concepts not dealt with in the main text (e.g. Fourier transforms, Lagrange multipliers, Bernoulli numbers, Lambert series). The exercises of the last three chapters on Integration, Infinite series and products and Trigonometric series are particularly impressive.

The production, printing and layout of the book are all pleasant and misprints seem to be few. Spare a thought though for F. Mertens (J. Reine Angew. Math. **79** (1875)), who is in danger of losing his s: in referring to his result on multiplication of series this book is common with at least three others sets the apostrophe after the *n* and before the *s*.

Students of Analysis will find the book inspiring but may also find it somewhat inhospitable in places: a case in point is the definition of continuity, another is the treatment of radius of convergence and the examples on it. Nevertheless the overall impression is of a fine achievement which will have appeal for analysts everywhere.

IAN S. MURPHY

ATIYAH, M. F. et al, *Representation Theory of Lie Groups* (L.M.S. Lecture Note Series No. 34, C.U.P., 1980). £10.95.

A research symposium was held in Oxford in 1977 and this book consists of the notes of eleven of the lectures given there. There are two parts. The first contains general and introductory lectures and the second more specialised lectures.

Although there are now many excellent texts on Lie groups, the general lectures in the present book are very valuable. They give brief accounts of several aspects of the theory. Someone who wants to find out about a particular aspect of the theory may well prefer to look here first rather than in one of the voluminous texts on the subject.

The lectures in the second part are somewhat more advanced but nevertheless they are quite accessible to non-specialists. Indeed someone who wants to see how Lie groups are used in a particular application may well find what he wants in one of these 'more specialised' lectures.

Undoubtedly there are many mathematicans who are aware that Lie groups are somehow relevant to the kinds of mathematics that interest them. A quick look at this book may well enable them to find out what the connection is and serve as a useful introduction to the literature. The book is not the specialised proceedings of a research symposium that one might expect before opening it.

ELMER REES

SCHWARZENBERGER, R. L. E. *N-dimensional crystallography* (Pitman, Research Notes in Mathematics No. 41, 1980), £6.50.

These notes are largely based on undergraduate lectures given by the author at the University of Warwick. However it contains a lot of material that could be used in courses at a higher level; in particular it could serve as a basis for some interdisciplinary seminars between mathematicians, physicists and chemists. There are many books on this topic written by chemists and physicists but there seem to be only a few written by mathematicians. This book should help mathemati-