CORRESPONDENCE.

ON THE VALUE OF AN ASSURANCE PAYABLE AT THE INSTANT OF DEATH.

To the Editor of the Assurance Magazine.

SIR,—In his excellent work on Life Contingencies, Baily has shown that the present value of £1, payable the moment a person now aged m shall die, is

$$=\frac{i}{\log_e(1+i)}\left\{\frac{1-ia_m}{1+i}\right\}.$$

He effects the solution by dividing each year into n equal parts, estimating the value of the risk for each of these periods, adding the whole together, and diminishing n indefinitely in the final result. As a different method of demonstrating the above formula may be found interesting, I give the following:—

Let t be any fractional part of a year, then the present value of £1 payable at the end of the time t will be $\frac{1}{(1+i)^t}$; and the probability of death happening in the first year, at the instant between t and $t+dt$, would be dt if it were *certain* that the individual would die at some period of that year; but inasmuch as the chance of the latter contingency is only $q_{m(1)}$, the probability of his dying at the instant specified is $q_{m(1)}.dt$; hence the value of the risk for the first year

$$=q_{m(1)}\int_0^1\frac{dt}{(1+i)^t}.$$

The same mode of reasoning will give the value for the second, third, &c. years; and the whole risk is

$$=q_{m(1)}\int_0^1\frac{dt}{(1+i)^t}+q_{m(2)}\int_1^2\frac{dt}{(1+i)^t}+q_{m(3)}\int_2^3\frac{dt}{(1+i)^t}+, \&c.$$

$$\text{Now}\int\frac{dt}{(1+i)^t}=C-\frac{1}{(1+i)^t.\log_e(1+i)};$$

this taken from $t=0$ to $t=1$ and $\times q_{m(1)}$ gives $\frac{i}{\log_e(1+i)}.\frac{q_{m(1)}}{1+i}$

" " $t=1$ to $t=2$ and $\times q_{m(2)}$ gives $\frac{i}{\log_e(1+i)}.\frac{q_{m(2)}}{(1+i)^2}$

" " $t=2$ to $t=3$ and $\times q_{m(3)}$ gives $\frac{i}{\log_e(1+i)}.\frac{q_{m(3)}}{(1+i)^3}$

&c. &c.

The sum of these$=\frac{i}{\log_e(1+i)}.A_m$, which is Baily's result.

I am, Sir,
Yours truly,
SAMUEL YOUNGER.

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