ASTROMETRY BY ASTEROID OCCULTATIONS

L. KAHL KRISTENSEN
Institute of Physics and Astronomy
University of Aarhus, DK-8000 Aarhus, Denmark

Abstract. For the determination of the dynamical equinox in fundamental star catalogues observed by astrometric satellites it is proposed to observe occultations of stars by selected asteroids. Being a long term programme now is the time for planning. Quadrature observations and photometric back-up are essential.

1. Introduction

The Hipparcos satellite provides a high accuracy fundamental catalogue at epoch 1991.5. However, the equator and ecliptic, which by tradition fix the coordinate axes, require separate determinations. The equator and precession must be determined by instruments taking part in the rotation of the Earth, as for instance VLBI radio observations. The determination of the ecliptic requires observations of planets. Direct observations by Hipparcos are affected by systematic errors due to the deviation of the photocentres at the large phase angles 15-25^0 caused by the scanning mode. This displacement is also indicated in high accuracy photographic positions (Kristensen 1980). In the JPL series of ephemerides, partly based on radar and radio observations, coordinates of the Earth are available with superior accuracy. The problem is, however, to determine the rotation between the two systems. Here we shall investigate the possibility of using asteroid occultations as a direct link between the stars and the planets.

2. Accuracy of occultation observations

Asteroids have been proposed for fundamental astrometry by several authors (Dyson 1929, Numerov 1932, Brouwer 1935, Strömgren 1950) because their positions may be obtained relative to the stars with high systematic

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accuracy. The most accurate positions of asteroids relative to stars are obtained from occultations. Formally an event timed to ± 0.2 s gives, with the typical daily motion 0.25^0 , a position error of order $\pm 0.002''$; corresponding to 2-3 km on the surface. Ultimately the errors will depend on how well the shape can be modelled and on surface irregularities. A mean error $\sigma = \pm 5$ km in the relative positions on the sky-plane does not seem unrealistic. The rotation periods and poles must be determined from lightcurves and the occultations themselves will then contribute to the determination of shape. This work can be made separately and requires access to and combination of all the original observations. The results give the simple input to the astrometric programme as **four** numbers for each event: time, relative coordinates, asteroid minus star, and a star identification label. For many years the unknown shape will be of minor importance compared to the present $\pm$ 0.15 $''$/century error in proper motions. Full benefit of occultations must await a new astrometric satellite!

3. Determination of rotation to dynamical frame

A simplified model illustrates the essential aspects in the determination of the rotation. Nearly circular orbits in the (X,Y)-plane are assumed for the Earth and the asteroid. The Earth has orbital radius a and the coordinates taken from the ephemeris are rotated through the angles (p, q, r) around the X, Y and Z axes. The elements of the asteroid are supposedly determined by more than three occultations. We assume that the eccentricity, perihelion and semi-major axis a′ are well separated from the rotations (p′, q′, r′) by a uniform distribution of the observations. The perfect situation would be if these three quantities were determined in the system of the inner planets by transmitters or reflectors on the surface of the asteroids. This would give the additional condition (p′,q′,r′)=(p,q,r) and make the adjustment trivial. Let λ and λ' be the longitudes of the Earth and the asteroid, E the elongation and β the phase angle. The left hand side of the equations of condition in the Z-direction on the sky plane is:

$$a'p'\sin\lambda' - a'q'\cos\lambda' - ap\sin\lambda + aq\cos\lambda \tag{1}$$

and in the direction of motion:

$$a'r'\cos\beta + ar\cos E \tag{2}$$

These equations will have the same weights because their mean errors are equal and of order $\sigma = \pm 5$ km.

The unknowns really wanted are simply (p, q, r), the corresponding elements for the planet being dummies. We note immediately that if observations are concentrated near opposition ($\lambda' = \lambda$) it is not possible to separate a·(p, q, r) from a′·(p′, q′, r′). Classical transit or photographic observations tend to concentrate at opposition and occultations have a great advantage in this respect. To separate the variables we must have different coefficients in (1) and (2) so the difference $\lambda' - \lambda = 180^0$-E- β must be as large as possible. The phase angle β(E) is a function of the elongation E. Other thing being equal, distant asteroids are preferable by increasing the differences between the coefficients i (1) and (2). Equation (2) shows that quadrature observations separate r from r′ (cos E = 0,sin $\beta = a/a'$). The separation of the node and inclination variables p and q is also mainly determined by the distribution of the observations in elongation. We solve (1) by least squares and assume symmetry with respect to opposition and average the normal equations over λ . Define the average $< \lambda' - \lambda >_k$ of N_k observations for asteroid k by:

$$N_k \cos < \lambda' - \lambda >_k = -\sum_{j=1}^{N_k} \cos (E_{jk} + \beta_{jk}) \ . \tag{3}$$

The mean error ϵ in the determination of p and q is then estimated by

$$\epsilon^2 = 2\sigma^2/a^2 \sum_k N_k \sin^2 < \lambda' - \lambda >_k \ . \tag{4}$$

The optimal accuracy is thus obtained if $\lambda' - \lambda$ is uniformly distributed around $\pm 90^0$ or elongation E_k given by $\tan E_k = a_k/a$.

4. The two stages of an occultation programme

Lunar occultations have proved their importance in fundamental astrometry and Newcomb (1878) demonstrated their accuracy and increasing value with time. A main point is that it is a "dependency" method where re-reductions are easily made and advantage can be taken of improved star places. High accuracy photographic observations must be stated by 3-4 dependencies to keep their value but for occultations this is simplified to a single star.

Work on asteroid occultations was pioneered by G.E.Taylor and is today well organized by D.W.Dunham and others. The main difficulty till now is the uncertain predictions. Today star errors in large surveys are at best $\pm 0.3''$ (PPM North). Orbits are based on observations with residuals $\pm 1.0''$ mainly due to plate measurements. Ephemerides average hundreds of such positions and many are at present more accurate than the stars. The star errors reduce to $\sigma^* = \pm\ 0.040''$ when the Tycho Catalogue becomes available and the predictions will immediately be improved. In a second stage the orbit can be determined by three occultations and the accuracy of the ephemeris will be comparable to that of the stars. To have 68% probability (1σ limit) that an event will actually occur on the central-line of a predicted track, the diameter D must satisfy:

$$2\sigma^*\Delta \leq D \quad (5)$$

When occultation observations at this second stage become more a routine than happy chance, the necessary number of observations for each object may be secured. The number of chords may not be large though. Examples are known where only two well- placed chords give good accuracy (Kristensen 1981). Equation (5) has important consequences for the brightness of the stars. For a distant ($a' = 3.30$) object $D > 134$ km and assuming a low albedo $p_V = 0.04$ the planet magnitude is V(a,0) = 12.9 mag. The star must be at least 0.5 mag brighter in order to have a 1 mag drop and V* < 12.4 mag. The table gives the faintest V* for albedoes 0.04, 0.10 and 0.16. At quadrature phase effects may correct V* by + 0.7 mag, especially for dark (C-type) objects.

a′	D km	0.04	0.10	0.16	80°	60°	Yearly	
2.20	70	11.5	10.5	10.0	2.0	4.5	1.4	3.1
2.75	102	12.0	11.0	10.5	1.6	2.4	1.3	1.9
3.30	134	12.4	11.4	10.9	1.0	1.5	0.8	1.3

In the case of V*=10.0 a search in the Hipparcos catalogue is nearly complete but the effective density of stars is a little reduced by unusable fainter stars. In the opposite situation V*=12.4 some faint events are lost in the catalogue but all 24 stars per square degree are usable. The areas within the parallax $8.80''/\Delta$ from the opposition loops to elongations 80° and 60° are multiplied by 24 stars/sq.deg. to give the mean number of predictions per opposition. The last two columns give the number of events per year.

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