ROGER CHAPMAN THORNE

(30 April 1929—19 May 1959)

It is with a deep sense of loss that we record the death in the first flush of his mathematical flowering of one of our youngest members, one whose career promised a contribution of high significance to Australian Mathematics.

Roger Chapman Thorne, after a distinguished undergraduate career in Sydney, was elected in 1950 to a Travelling Scholarship which enabled him to proceed to his father's college, Trinity College, Cambridge. There he took Parts II and III of the Mathematical Tripos, was awarded a Rayleigh Prize, and in 1955 graduated as Ph. D. He spent a post-doctoral year in the California Institute of Technology, working with Professor A. Erdélyi.

In 1957 Roger was appointed to a Lectureship in Applied Mathematics in the University of Sydney, a position which his father, the late Harold Henry Thorne, had occupied with distinction for thirty-three years until his death in 1953. In the short two and a half years that followed his appointment, Roger made his mark in Sydney as a clear and sympathetic lecturer, and as a wise counsellor of students outside the lecture-room. He was sincerely devoted to the work of the Church of England; among his many activities connected with the Church he was a member of the Synod of the Diocese of Sydney, and played a leading part in the Inter-Varsity Fellowship of the Evangelical Union.

Like many of the best mathematicians, Thorne worked in both Pure and Applied Mathematics, Analysis clarifying his image of the physical problem, and the physical problem suggesting to him new fields of investigation in Analysis.

His first researches * were on the diffraction of surface waves by submerged three-dimension obstacles, with special attention to the submerged sphere. In the course of his work he obtained integral expressions for all submerged multiple singularities, and their expansions near the multipole and at infinity, both in two and three dimensions, [1]. This is a very useful piece of work and is often quoted in the literature. Afterwards when he applied this to the sphere problem, he found how little was known about the behaviour of Legendre functions, and this started him on his work on asymptotic expansions.

He was mainly concerned with the problem of determining asymptotic solutions of linear second-order differential equations of the form

$$\frac{d^2w}{dz^2} = \{u^2p(z) + q(z)\}w$$

* This account of Thorne's work was written by Dr. F. Ursell, who was his Supervisor as candidate for the Ph. D., and by Dr. F. W. J. Olvers of the National Physical Laboratory with whom Thorne was in close communication after leaving Cambridge.

in which u is a large parameter and z lies in a given complex domain D, bounded or otherwise. Three cases are of especial importance: (i) D contains no singularities of $p(z)$ or $q(z)$ and no zeros of $p(z)$; (ii) D contains a simple zero of $p(z)$, a so-called turning point of the differential equation; (iii) D contains a point which is a simple pole of $p(z)$ and a regular point or double pole of $q(z)$. In each of these cases asymptotic expansions exist which are uniformly valid with respect to z and are expressed in terms of functions of a single variable — elementary functions, Airy functions and Bessel functions of fixed order, respectively.

In [2] Thorne showed that by making ingenious changes of variables the region of validity in the second case could be extended to include not only the turning point and a regular singularity at infinity (as was already known) but also a point at which $p(z)$ has a double pole. This pole is a branch point of the solution w, and the corresponding cut may be taken through the turning point creating, in effect, two turning points. The expansions then hold uniformly at the pole, the point at infinity and one of the turning points on either side of the cut.

In order to achieve expansions in these circumstances which hold at the turning points on both sides of the cut as well as at the pole and infinity a new existence theorem is required. In [3] Thorne proved that asymptotic expansions can be found in terms of Bessel functions of variable order.

In [4] the results of [2] and [3] and those of other writers are applied to the differential equation for the associated Legendre functions. New and powerful expansions are obtained for $P_n^{-m}(z)$ and $Q_n^{-m}(z)$ when n is large and positive and $0 < m < n$, $m/(n+\frac{1}{2})$ being kept fixed.

Thorne's last work led to important extensions of the theory for cases (i), (ii), and (iii). At the time of his death he was completing [5], in which very general existence theorems are established for equations of the form

$$\frac{d^2 w}{dz^2} = u^{2p} f(u, z) w.,$$

where p is a positive integer and $f(u, z)$ has an asymptotic expansion for large u of the form

$$f(u, z) \sim f_0(z) + \frac{1}{u} f_1(z) + \frac{1}{u^2} f_2(z) + \dots$$

Papers by R. C. Thorne

[1] Multipole expansions in the theory of surface waves. Proc. Cambridge Phil. Soc., 49, 1953, 707—716.

[2] The asymptotic solution of differential equations with a turning point and singularities. Proc. Camb. Phil. Soc., 53, 382—398 (1957).

[3] The asymptotic solution of linear second order differential equations in a domain containing a turning point and a regular singularity. Phil. Trans. Roy. Soc. A, 249, 585—596 (1957).

[4] The asymptotic expansions of Legendre functions of large degree and order. Phil. Trans. Roy. Soc. A, 249, 597—621 (1957).

[5] Asymptotic formulae for solutions of linear second-order differential equations with a large parameter. (In press).