J. Austral. Math. Soc. (*Series A*) 27 (1979), 88–90

A SERIES OF BIB DESIGNS

KISHORE SINHA

(Received 25 October; revised 22 December 1977)

Communicated by W. D. Wallis

Abstract

A series of balanced incomplete block (BIB) designs with parameters

$$v = \binom{n}{2}, \quad b = \binom{n}{s}, \quad r = \binom{n-2}{s-2} + \binom{n-2}{s},$$

$$k = \binom{s}{2} + \binom{n-s}{2}, \quad \lambda = \binom{n-3}{s-3} + \binom{n-3}{s}$$

is constructed, where $s = 2w^2 + w + 1$ and $n = 4w^2 + 4w + 3$ or $4w^2 + 2$, w any integer.

Subject classification (*Amer. Math. Soc.* (*MOS*) *1970*): primary 05 B 05; secondary 62 K 10.

Saha (1973) produced some series of partially balanced incomplete block (PBIB) designs and remarked that he had not investigated these to see whether any series yielded BIB designs under certain restrictions. Theorem 3 in his paper does give such a series. The terminology followed here is that of Raghavarao (1971).

THEOREM. *There exists a series of BIB designs with parameters*

$$v = \binom{n}{2}, \quad b = \binom{n}{s}, \quad r = \binom{n-2}{s-2} + \binom{n-2}{s}$$
$$k = \binom{s}{2} + \binom{n-s}{2}, \quad \lambda = \binom{n-3}{s-3} + \binom{n-3}{s}, \tag{1}$$

where $s = 2w^2 + w + 1$ *and* $n = 4w^2 + 4w + 3$ *or* $4w^2 + 2$, w *any (non-zero) integer.*

https://doi.org/10.1017/S1446788700016657 Published online by Cambridge University Press

PROOF. Let X be a set of n positive integers $1, 2, \ldots, n$. We identify 2-sets of X as treatments, and s-sets of X as blocks, with s and n as defined above. A block corresponding to an s-set consists of two parts A and B. Part A consists of all 2-sets formed with the s-set and part B consists of all 2-sets formed with the complement of the s-set in X. Any two sets having an integer common are first associates and with no integer common are second associates. Now the condition for balance $\lambda_1 = \lambda_2$ implies

$$\binom{n-3}{s-3}+\binom{n-3}{s}=\binom{n-4}{s-4}+\binom{n-4}{s}+2\binom{n-4}{s-2},$$

which gives the conditions stated for n and s. Thus we get a series of BIB designs with parameters (1) which can be easily verified.

When $w = 1$, $s = 4$, $n = 6$ in (1) we get a BIB design with parameters $v = 15 = b$, $r = 7 = k$, $\lambda = 3$, which is shown below:

	Block contents			Block contents	
4-sets	Part A	Part B	4-sets	Part A	Part B
1234	12, 13, 14, 23, 24, 34	56	2356	23, 25, 26, 35, 36, 56	14
1235	12, 13, 15, 23, 25, 35	46	2456	24, 25, 26, 45, 46, 56	15
1236	12, 13, 16, 23, 26, 36	45	3456	34, 35, 36, 45, 46, 56	12
1245	12, 14, 15, 24, 25, 45	36	1345	13, 14, 15, 34, 35, 45	26
1246	12, 14, 16, 24, 26, 46	35	1346	13, 14, 16, 34, 36, 46	25
1256	12, 15, 16, 25, 26, 56	34	1356	13, 15, 16, 35, 36, 56	24
2345	23, 24, 25, 34, 35, 45	16	1456	14, 15, 16, 45, 46, 56	23
2346	23, 24, 26, 34, 36, 46	15			

A BIB design with the above parameters is known, see Raghavarao (1971), p. 92. When $w = 1$, $s = 4$, $n = 11$, we get a BIB design with parameters

$$v = 55, \quad b = 330, \quad r = 162, \quad k = 27, \quad \lambda = 78.$$

The parameters of this design are a multiple by 6 of a BIB design with parameters $v = 55 = b$, $r = 27 = k$, $\lambda = 13$ which is known; see Di Paola, Wallis and Wallis (1973).

Acknowledgement

The author wishes to thank the referee for suggestions which have improved the readability of this note. Financial assistance provided by C.S.I.R., New Delhi is also gratefully acknowledged.

References

Jane W. Di Paola, J. S. Wallis and W. D. Wallis (1973), "A list of (v, b, r, k, λ) designs for $r \leqslant 30$", *Proc. 4th S.-E. Conf. Comb., G. Th. and Computing* (Florida Atlantic University, Boca Raton), 249–258.

D. Raghavarao (1971), *Constructions and Combinatorial Problems in Design of Experiments* (John Wiley and Sons Inc., New York).

G. M. Saha (1973), "On construction of T_m-type PBIB designs", *Ann. Inst. Statist. Math.* **25**(3), 605–616.

Indian Agricultural Statistics Research Institute
New Delhi 110012
India