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NOTES

Tax evasion and debt dynamics with endogenous growth

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Abstract
In this note, we study the relationship between tax evasion and economic growth in a model where public expenditure allows to improve private capital productivity, and it is financed by both taxes and public debt. Here, we define debt to be sustainable if the debt/GDP ratio resulting from agents optimization converges toward a finite equilibrium that is endogenous to the model. We show that: (i) the level of public expenditure which maximizes growth does not depend on audit parameters, (ii) evasion reduces the range of parameters for which the debt/GDP ratio is sustainable, and (iii) the debt/GDP ratio is sustainable if the total factor productivity is sufficiently high.

Keywords: dynamic tax evasion; public debt sustainability; endogenous growth

1. Introduction

The optimal tax–debt mix to finance public expenditure provision has generated a lively debate in the literature. Public debt may create fiscal illusion and ultimately undermine growth (Khalid and Guan 1999); taxes reduce fiscal illusion by making agents aware of the costs of public provision, but they may create dead-weight losses (Tresch 2002). Furthermore, an increment in taxes may also generate or worsen tax evasion, which is a wide spread phenomenon and whose magnitude is quite relevant (Cebula and Feige, 2012). Recent estimates show that intentional under-reporting of income is about 18–19% of total reported income in the US (i.e. a tax gap of about 500 billion dollars) which may increase to about one trillion dollars if we take into account tax avoidance (Davison 2021). In Europe, the level of tax evasion is about 20% of GDP, with a loss of about 1 trillion Euros (Buehn and Schneider 2012; Murphy 2014; Albarea et al. 2020).

In some countries (Greece and Italy for example) tax evasion is associated with a high public debt, but the relationship between these variables and growth is not clear, in spite of a growing literature (Argentiero and Cerqueti 2021; Halkos et al. 2020; Schilirò 2019). Public debt allows to increase public expenditure without raising the tax rate or reducing tax evasion; however, this process may undermine economic growth if debt becomes unsustainable.

In this note we study the effects of tax evasion on growth and debt sustainability by using an approach in the spirit of Blanchard et al. (1990). We define debt to be sustainable if the dynamics of the debt/GDP ratio follows a mean reverting process that converges toward a stable equilibrium (which is endogenous to the model; see Debrun et al., 2020) instead of in terms of an outside threshold for the debt/GDP ratio (Alloza et al. 2020; Fournier and Fall 2017; Caner et al. 2010).

A representative consumer optimally chooses the intertemporal consumption and the evasion. If the agent is caught evading, a fine must be paid on the evaded yield. We show that, although the level of public expenditure which maximizes growth may not depend on these parameters, the latter reduces the range of parameters for which the debt/GPD ratio is mean reverting. This implies that the level of expenditure that maximizes growth and welfare may not be compatible with debt sustainability because of the burden that interests and debt create on the economy. The adverse impact of tax evasion is even more pronounced in economies with slower growth or those heavily reliant on public expenditure (where private capital productivity is relatively low). In such case, the debt-to-GDP ratio remains mean reverting only if the total factor productivity is sufficiently strong. From a policy perspective, our model highlights a trade-off: while tolerating tax evasion can boost expected optimal economic growth, it comes at the cost of undermining government debt stability. As a result, minimizing tax evasion is particularly crucial for economies that rely heavily on public expenditure to sustain economic growth.

2. Related literature

Debt sustainability is a controversial issue: the literature which has investigated sovereign debt sustainability (see Lindgren 2021; Debrun et al., 2020; Willems and Zettelmeyer 2022 for a review) has not reached a definite conclusion.

Fournier and Fall (2017) use OECD data to show that Governments are vulnerable to changes in macroeconomic conditions and interest rates.

In a controversial paper, Reinhart and Rogoff (2010) showed that debt/GDP above 90% may undermine growth, but according to Herndon et al. (2014) this is due to errors in the coding and the dataset, while, in actual fact, a high debt/GDP ratio could even slightly improve growth, as suggested by most of the theoretical papers that do not find any endogenous "dangerous" threshold for debt/GDP ratio (see Willems and Zettelmeyer (2022) for a review.).

From an empirical point of view, the World Bank (Caner et al., 2010) argues that an alert threshold for the debt/GDP ratio is about 77% for mature economies and 64% for emerging ones, while Eberhardt and Presbitero (2015) argue that a negative relationship may exist, but a common debt threshold does not exist when observed and unobserved heterogeneity across countries is taken into account.

The relationship between tax evasion, public debt, and growth has been rather unexplored from a theoretical point of view[1] while from an empirical point of view (see Loayza, 2016) there seems to be only a negative relationship between the size of the informal sector and growth. Our model adds to the present literature by studying under which conditions the debt-to-GDP ratio follows a mean reverting path also in the presence of tax evasion without setting a specific threshold for the debt/GDP ratio.

3. The model

We model a stylized general equilibrium where Government sets the level of public expenditure that maximizes welfare. The agent is endowed with a production technology that allows to transform capital into yield through a constant return to scale production function whose arguments are capital and public expenditure as in Barro (1990); Futagami et al. (2008); Minea and Villieu (2013); Mirrlees et al. (2011).

The representative agent maximizes the expected present value of their future utility by controlling their intertemporal consumption and tax evasion. Tax evasion adds uncertainty to these lifetime decisions because of random audits: if the agent is caught evading, they have to pay a fine on top of evaded taxes. Tax revenues are a source of uncertainty also for the government which may have to issue debt if the tax revenue is lower than expected. In this environment, government has to set the level of public expenditure that maximizes the agent's value function, under the constraint that debt is sustainable; we assume that this condition is satisfied if the debt/GDP stochastic process is mean reverting, that is, the debt-to-GDP ratio converges to a stable equilibrium.

We first solve the problem for the agent in order to find their optimal consumption and tax evasion; we then compute the dynamics of the optimal debt/GDP ratio and define under which conditions it is mean reverting. Finally, we compute the public expenditure that maximizes growth and met the mean reverting conditions.

As in Barro (1990), we model an economy with a representative individual. GDP y_t is produced by using both private capital k_t and public expenditure G_t through a constant return to scale technology:

$$y_t = AG_t^{\beta} k_t^{1-\beta},$$

where A is the constant total factor productivity and β is the elasticity of the product w.r.t. public expenditure. We assume that public expenditure is set to be a constant proportion of private capital, that is,

$$G_t = gk_t,$$

and, accordingly, the production function can be written as:

$$y_t = Ag^{\beta} k_t. \tag{1}$$

Contrary to Barro (1990), our representative consumer takes g (rather than G_t) as a fixed parameter, that is, they can predict that a change in k_t will cause G_t to adjust, that is, we rule out fiscal illusion[2].

Public expenditure is financed through a proportional tax τ on income and through public debt. However, a fraction ϕ of the total tax revenue τy_t is lost in tax collection activities: the higher the ϕ, the more inefficient is Government. The agent is fully rational, that is, they can see the effect that public expenditure has on economic growth and can also observe collection cost.

B_t is the total amount of public debt which: (i) increases because of its service at the rate r, (ii) increases because of the public expenditure G_t, (iii) decreases because of taxes collected on declared yield, and (iv) decreases when evasion (e_t) is caught and the agent must pay a fine $\eta(\tau)$ on the evaded income $e_t y_t$.

As in Levaggi and Menoncin (2013); Bernasconi et al. (2015), we assume that the audit happens according to a Poisson jump process. If we call $d\Pi_t$ this process, it may have value either 1 (if an audit happens) with probability λdt or 0 (if no audit happens). The constant parameter λ is the jump intensity.[3] Accordingly, the dynamics of the public debt is:

$$dB_t = \left(B_t r + gk_t - (1-\phi)\,\tau(1-e_t)\,y_t\right) dt - (1-\phi)\,\eta(\tau)\,e_t y_t d\Pi_t, \tag{2}$$

where we assume that the same inefficiency ϕ applies also when collecting fines following an audit. We immediately see that if an audit occurs (i.e. $d\Pi_t = 1$) the debt reduces by the fee paid by the caught evader. Instead, if no audit happens, the last term of (2) is zero. On average the expected fine cashed by the government is

$$\mathbb{E}_t\left[(1-\phi)\,\eta(\tau)\,e_t y_t d\Pi_t\right] = (1-\phi)\,\eta(\tau)\,e_t y_t \lambda dt.$$

The audit regimes can be modeled through the shape of the function $\eta(\tau)$ which allows to take into account several actual forms of fines.[4]

The private capital k_t: (i) increases because of production y_t, (ii) decreases because of taxes on the non-evaded income $\tau(1-e_t)\,y_t$, (iii) decreases because of consumption c_t, (iv) increases because of the interest rate paid by Government to the bond holders $B_t r$, (v) decreases because of the loans to Government dB_t, and (vi) decreases because of the fee that must be paid on the audited income. Thus, we can write

$$dk_t = \left(y_t - \tau(1-e_t)\,y_t - c_t + B_t r\right) dt - dB_t - \eta(\tau)\,e_t y_t d\Pi_t. \tag{3}$$

After substituting dB_t into the capital dynamics we get:

$$\frac{dk_t}{k_t} = \left[(1-\phi\tau(1-e_t))\,Ag^{\beta} - g - \frac{c_t}{k_t}\right] dt - \phi\eta(\tau)\,Ag^{\beta} e_t d\Pi_t. \tag{4}$$

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If there are no collection costs (i.e. $\phi = 0$) evasion does not play any role: it is just a reallocation of fund between the agent and Government. The expected value of (4) is

$$\mathbb{E}_t\left[\frac{dk_t}{k_t}\right] = \left((1-\phi\tau)\,Ag^{\beta} - g - \frac{c_t}{k_t} + \phi(\tau - \lambda\eta(\tau))\,Ag^{\beta}e_t\right)dt,$$

and tax evasion is profitable, on average, if $\tau > \lambda\eta(\tau)$, that is, if the tax that must be paid is higher than the fee weighted by the intensity of being caught. Finally, the debt/GDP ratio has the following dynamics:

$$d\left(\frac{B_t}{y_t}\right) = \left(\frac{g^{1-\beta}}{A} - (1-\phi)\,\tau(1-e_t) - \frac{B_t}{y_t}\left[(1-\phi\tau(1-e_t))\,Ag^{\beta} - g - \frac{c_t}{k_t} - r\right]\right)dt$$
$$-\left(\frac{1-\phi}{\phi} - \frac{B_t}{y_t}Ag^{\beta}\right)\frac{\phi\eta(\tau)\,e_t}{1-\phi\eta(\tau)\,e_tAg^{\beta}}d\Pi_t. \tag{5}$$

3.1 The agent's optimization problem

Over an infinite time horizon, a representative consumer maximizes his/her intertemporal utility which depends on the consumption of a private good (c_t). We assume that agent's utility has a constant relative risk Aversion (δ) and agent's (constant) subjective discount is ρ. Thus, the optimization problem is

$$\max_{\{c_t,e_t\}_{t\in[t_0,\infty[}} \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty}\frac{c_t^{1-\delta}}{1-\delta}e^{-\rho(t-t_0)}dt\right], \tag{6}$$

with k_t following the dynamics in (4).

Proposition 1. *Given the capital dynamics (4), the optimal consumption and evasion that solve Problem (6) are*

$$\frac{c_t^*}{k_t} = \frac{\rho}{\delta} + \frac{\delta-1}{\delta}\left((1-\phi\tau)\,g^{\beta}A - g\right)$$
$$+\frac{\tau}{\eta(\tau)}\left(1 - \left(\frac{\eta(\tau)\,\lambda}{\tau}\right)^{\frac{1}{\delta}} - \frac{1}{\delta}\left(1 - \frac{\eta(\tau)\,\lambda}{\tau}\right)\right), \tag{7}$$

$$e_t^* = \frac{1}{\phi\eta(\tau)\,g^{\beta}A}\left(1 - \left(\frac{\eta(\tau)\,\lambda}{\tau}\right)^{\frac{1}{\delta}}\right). \tag{8}$$

Proof. *See Appendix A.* □

Corollary 2. *Given the capital dynamics (4), if the optimal evasion is zero (i.e. $\tau = \eta(\tau)\,\lambda$) the optimal consumption that solves Problem (6) is*

$$\left.\frac{c_t^*}{k_t}\right|_{\lambda=\frac{\tau}{\eta(\tau)}} = \frac{\rho}{\delta} + \frac{\delta-1}{\delta}\left((1-\phi\tau)\,g^{\beta}A - g\right). \tag{9}$$

Proof. *It is sufficient to substitute $\lambda = \frac{\tau}{\eta(\tau)}$ in the result of Proposition 1.* □

From (7) and (8) we note that both the optimal consumption and the optimal evasion are constant fractions of income (we recall that, in this model, income is a linear transformation of capital as in (1)). This result is perfectly in line with the so-called "consumption smoothing" behavior that is often observed in the market.

It is interesting to note that if evasion is expedient (i.e. $e^* > 0$) and $\delta > 1$, the optimal consumption with evasion is always higher than that without evasion. To show this, let us consider the term

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in (7) that depends on the fiscal parameters η and λ, and set $m := \frac{\tau}{\eta(\tau)\lambda}$. Consumption is higher with tax evasion if:

$$1 - m^{-\frac{1}{\delta}} > \frac{1}{\delta}\left(1 - \frac{1}{m}\right).$$

When $m = 1$ the two sides of the inequality are zero, but for evasion to be expedient $m > 1$. In this case, the derivative on the left-hand side w.r.t. m (i.e. $\frac{1}{\delta} m^{-\frac{1}{\delta}-1}$) is higher than the same derivative on the right-hand side (i.e. $\frac{1}{\delta} m^{-2}$) if $\delta > 1$, and, accordingly, c is higher.

The optimal capital dynamics is

$$\frac{dk_t^*}{k_t^*} = \frac{(1 - \phi\tau)\, g^{\beta} A - g - \rho + \frac{\tau}{\eta(\tau)} - \lambda}{\delta} dt - \left(1 - \left(\frac{\eta\,(\tau)\,\lambda}{\tau}\right)^{\frac{1}{\delta}}\right) d\Pi_t. \tag{10}$$

Should Government set the audit parameter to erase evasion (i.e. $\lambda = \tau/\eta\,(\tau)$), the optimal growth with $e_t^* = 0$ would be

$$\left.\frac{dk_t^*}{k_t^*}\right|_{\lambda = \frac{\tau}{\eta(\tau)}} = \frac{(1 - \phi\tau)\, g^{\beta} A - g - \rho}{\delta} dt. \tag{11}$$

From these results we see that the optimal consumption is an affine transformation of the capital growth rate

$$\frac{c_t^*}{k_t} = \rho + (\delta - 1)\,\frac{1}{dt}\mathbb{E}_t\left[\frac{dk_t^*}{k_t^*}\right] + \lambda\left(\delta - (\delta - 1)\left(\frac{\eta\,(\tau)\,\lambda}{\tau}\right)^{\frac{1}{\delta}} - \left(\frac{\eta\,(\tau)\,\lambda}{\tau}\right)^{\frac{1-\delta}{\delta}}\right).$$

In particular, we can conclude that the optimal consumption is proportional to the expected capital growth. In other words, any policy that maximizes the expected growth of capital, will also maximize optimal consumption. Furthermore, since utility is a monotonic function of consumption, maximizing consumption also coincides with utility maximization.

Proposition 3. *The government expenditure levels that maximize the agent's welfare, the expected optimal economic growth, and the consumer's consumption ratio, all coincide:*

$$\arg\max_g \frac{1}{dt}\mathbb{E}_t\left[\frac{dk_t^*}{k_t^*}\right] = \arg\max_g \mathbb{E}_t\left[\int_t^{\infty} U\left(c_s^*\right) e^{-\rho(s-t)} dt\right]$$
$$= \arg\max_g \frac{c_t^*}{k_t}.$$

3.2 Mean reverting conditions

The optimal debt/GDP ratio has a dynamics whose form is like in the following stochastic differential equation:

$$d\left(\frac{B_t}{y_t}\right) = p_0\left(p_1 - \frac{B_t}{y_t}\right) dt + \text{stoch. var.},$$

where p_0 and p_1 are highly nonlinear combinations of all model parameters (see Appendix B). If the parameters p_0 is positive, then the process converges toward a long-term equilibrium given by p_1. Instead, if $p_0 < 0$, the process is divergent (independently of the value of p_1). The higher p_0, the faster the convergence toward the long-term equilibrium value. Thus, we can conclude what follows.

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Proposition 4. *The optimal debt/GDP is mean reverting if and only if*

$$\frac{(1-\phi\tau-\delta(1-\tau))\,g^{\beta}A-g-\rho}{\delta}+\lambda\underbrace{\left(-\frac{m-1}{\phi}\left(1-m^{-\frac{1}{\delta}}\right)+\frac{\delta-1}{\delta}+\frac{m}{\delta}-m^{\frac{1}{\delta}}\right)}_{=:\Lambda(m,\delta,\phi)}>0, \quad (12)$$

in which $m:=\frac{\tau}{\eta(\tau)\lambda}$. *In this case, the long-term equilibrium of debt/GDP is*

$$\frac{\delta}{g^{\beta}A}\frac{g-(1-\phi)\,\tau g^{\beta}A+\frac{1-\phi}{\phi}\lambda\left(m^{1-\frac{1}{\delta}}-1\right)\left(m^{\frac{1}{\delta}}-1\right)}{((\delta-\phi)\,\tau-(\delta-1))\,Ag^{\beta}-g-\rho+\delta\lambda\left(\frac{1-m}{\phi}\left(1-m^{-\frac{1}{\delta}}\right)+\frac{\delta-1}{\delta}+\frac{m}{\delta}-m^{\frac{1}{\delta}}\right)}. \quad (13)$$

Proof. *See Appendix B.* □

We recall that tax evasion is expedient if $m>1$, while for $m=1$ the optimal tax evasion is zero. Since $\Lambda(1,\delta,\phi)=0$, while the function Λ is always negative for any $m>1$ (as shown in Appendix B), then the second term of (12) undermines the mean reversion property of the debt/GDP ratio.

Public expenditure g may contribute to both convergence and growth, but in a different way so that both objectives may not be simultaneously pursued.

We can provide a strong interpretation of the debt/GDP equilibrium value in (13) if we take into account the case without evasion (i.e. $m=1$). In this case it is easy to show that (13) becomes

$$\frac{\frac{g-(1-\phi)\tau g^{\beta}A}{g^{\beta}A}}{\frac{(1-\phi\tau)Ag^{\beta}-g-\rho}{\delta}+(\tau-1)\,Ag^{\beta}}.$$

The first fraction at the denominator coincides with the optimal capital growth (without evasion) as shown in (10). Let us call γ^* this optimal growth rate. If we multiply and divide by k_t the fraction in the numerator we get

$$\frac{\frac{G_t-(1-\phi)\tau y_t}{y_t}}{\gamma^*-(1-\tau)\,Ag^{\beta}}.$$

This ratio can be seen as the present value of a perpetual annuity as follows:

$$\int_t^{\infty}\frac{G_s-(1-\phi)\,\tau y_s}{y_s}e^{-\left(\gamma^*-(1-\tau)Ag^{\beta}\right)(s-t)}ds,$$

if the optimal growth rate $\gamma^*$is higher than the (net of tax) total factor productivity, which is the only convergence condition in this framework with optimal zero tax evasion.

Thus, we can conclude that the debt/GDP ratio converges to the discounted value of all the future deficit/GDP ratios and the discount rate is given by the optimal capital growth, reduced by the (net of tax) total factor productivity.[5] In this formula we see that the risk aversion parameter δ affects only the discount rate in the growth of capital γ^*. Instead, if tax evasion is expedient, risk aversion enters both the cash flows and the rate of discount and, thus, affects in a nontrivial way government debt sustainability (as shown in the numerical simulations).

3.3 Social welfare, tax evasion, and debt dynamics

In the previous sections we have shown that there exists a level of public expenditure that maximizes consumption, growth, and agent's welfare simultaneously, but such level may not be compatible with debt sustainability, that is, with the convergence of the debt/GDP ratio toward an equilibrium value.

Thus, in order to guarantee that the debt/GDP is convergent over time, the government should set g to the level that maximizes one of the welfare measures, under the constraint that the mean reversion strength of the debt/GDP dynamics is positive. If we call $J(t, k_t; g)$ the value function of the agent (as defined in (17)), the Government problem can be written as follows:

$$\max_g J(t, k_t; g) \tag{14}$$

s.t.

$$\frac{(1-\phi\tau)\, g^\beta A - g - \rho}{\delta} - (1-\tau)\, Ag^\beta + \lambda\left(\frac{1-m}{\phi}\left(1 - m^{-\frac{1}{\delta}}\right) + \frac{\delta-1}{\delta} + \frac{m}{\delta} - m^{\frac{1}{\delta}}\right) > 0.$$

This problem makes sense because public debt allows Government to disentangle the tax rate τ from the public expenditure g. In fact, without debt, the amount g should satisfy a budget constraint. If we call T_t the net income of the government, its dynamics without debt would be

$$dT_t = \left((1-\phi)\, \tau\left(1 - e_t^*\right) y_t - gk_t\right) dt + (1-\phi)\, \eta\, (\tau)\, e_t^* y_t d\Pi_t,$$

whose expected value is

$$\mathbb{E}_t\left[dT_t\right] = \left((1-\phi)\left(\tau - \tau e_t^* + \eta\, (\tau)\, e_t^* \lambda\right) y_t - gk_t\right) dt,$$

and, accordingly, the level g should satisfy the condition $\mathbb{E}_t\left[dT_t\right] = 0$.

Furthermore, in our framework the Government may be tempted to tolerate evasion for allowing a higher expected economic growth, according to the following result.

Proposition 5. *The expected economic growth with evasion is always higher than that achieved without evasion:*

$$\frac{1}{dt}\mathbb{E}_t\left[\frac{dk_t^*}{k_t^*}\right] > \frac{1}{dt}\mathbb{E}_t\left[\left.\frac{dk_t^*}{k_t^*}\right|_{e^*=0}\right].$$

Proof. *If we substitute the optimal economic growth from (10), we get*

$$\frac{(1-\phi\tau)\, g^\beta A - g - \rho + \frac{\tau}{\eta(\tau)} - \lambda}{\delta} - \lambda\left(1 - \left(\frac{\eta\,(\tau)\,\lambda}{\tau}\right)^{\frac{1}{\delta}}\right) > \frac{(1-\phi\tau)\, g^\beta A - g - \rho}{\delta},$$

which becomes

$$\frac{1}{\delta}\frac{\tau}{\eta\,(\tau)\,\lambda} + \left(\frac{\eta\,(\tau)\,\lambda}{\tau}\right)^{\frac{1}{\delta}} > \frac{1}{\delta} + 1,$$

and it is easy to show that this inequality always holds for any $\tau > \eta\,(\tau)\,\lambda$. □

However, while evasion may increase the economic growth, it also reduces government revenue; the optimal debt/GDP ratio is likely to increase with tax evasion and its dynamics may explode.

If the Government is not constrained by the mean reverting condition on debt/GDP, the public expenditure rate that maximizes growth/value function/consumption is

$$g_{growth}^* = (A\beta(1-\phi\tau))^{\frac{1}{1-\beta}}\,, \tag{15}$$

which is easily obtained from (10). If this is not the case the level of public expenditure compatible with mean reversion is the one for which the constraint in (14) is still greater than zero.

The algebraic solution to Problem (14) does not give any true policy insight since it is a highly nonlinear combination of the model parameters. Thus, in the following section we propose a numerical simulation.

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Table 1. Values of the parameters for the baseline simulation

Parameter	Value	Source
Tax inefficiency	$\phi = 0.1$	OECD (2011)
Marginal productivity of G_t	$\beta = 0.25$	Kamps (2006)
Tax rate	$\tau = 0.347$	Bernasconi et al. (2020)
Total factor productivity	$A = 0.26$	Chosen to replicate GDP growth rate*
Audit frequency	$\lambda = 0.137$	Bernasconi et al. (2020)
Audit fine	$\eta = 2.5$	Bernasconi et al. (2020)
Risk aversion	$\delta = 1.05$	Levy (2024)
Subjective discount rate	$\rho = 0.01$	Standard

*For an emerging economy see https://www.imf.org/external/datamapper/NGDP_RPCH@WEO/OEMDC

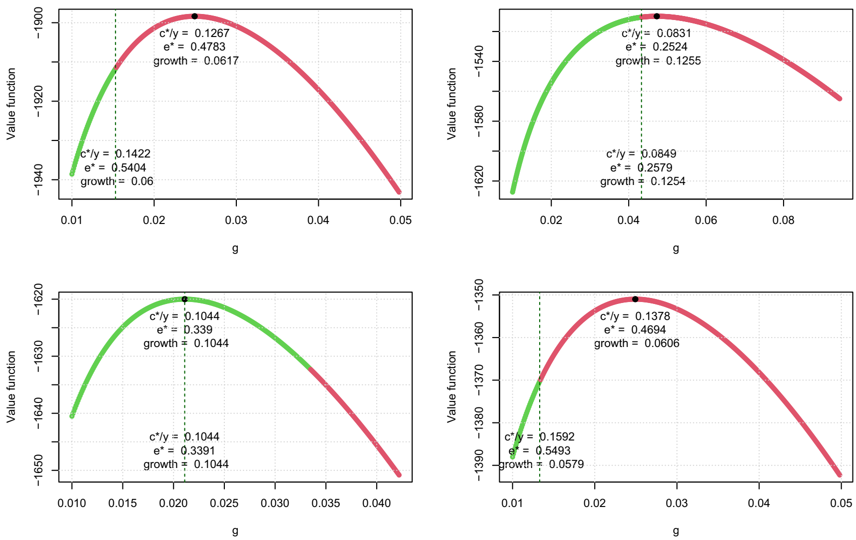

Figure 1. Value function $J(t, k_t; g)$ as a function of public expenditure g. In the upper left graph the base case is drawn using the parameters in Table 1. In the upper right graph $A = 0.42$. In the lower left graph, $\beta = 0.15$. In the lower right graph, $\delta = 1.07$. In the green (red) section of the curve, the debt/GDP ratio is convergent (divergent).

3.4 Numerical simulation

Some further insights into the relationship between growth, tax evasion and debt sustainability may be gained through some numerical simulations that allow to better understand the role of the different structural parameters in this relationship.

Table 1 shows the values of the parameters used for the benchmark case.

As a measure of tax inefficiency we have used some recent estimates of tax collection activities (OECD 2011) which are somehow a lower bound estimate of these costs. The marginal productivity of public expenditure is derived from recent OECD estimates while the fiscal and total productivity parameters are in line with Bernasconi et al. 2020 while A in the benchmark has been chosen to replicate the growth of emerging economies, which rely more than mature ones on public expenditure as a drive for growth. Figure 1 shows the value of $J(t, k_t; g)$ as a function of g, the ratio of public expenditure to income/production. The curve is green if the value of g satisfied the mean reverting condition and red when this condition is not met.

In the baseline case (upper left) the economic system is not strong enough to support tax evasion. The highest level of public expenditure for which debt is sustainable is much lower than

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the level that maximizes the value function (about 40% less). On the contrary, evasion is over 6 point percent higher (0.504 against 0.478) than the level that would allow to get a sustainable debt. Growth as well as welfare is lower as expected. It would be necessary to increase the productivity A in order to get a level of public expenditure that both maximizes growth and secures debt sustainability. In our example the value of 0.42 (almost double the baseline) would reduce the gap between optimal and sustainable public expenditure, even if the mean reverting constraint is still binding.

The bottom left graph shows the role of the private capital productivity $(1-\beta)$. A value of $\beta = 0.15$ (i.e. an economy that relies less on public expenditure to grow, as it would be the case of a more mature economy) would allow to set expenditure to its optimal level and the debt to be mean reverting.

Finally, in the bottom right graph we show the role of risk aversion: for lower levels of risk aversion, the debt/GDP is not mean reverting.

If debt/GDP dynamic is not mean reverting, Government will have to reduce public expenditure to get back on track. From (12) we see that, if $\delta < \frac{1-\phi\tau}{1-\tau}$, the public expenditure rate that maximizes convergence, that is, the public expenditure that makes debt/GDP to converge as quick as possible, is:

$$g^*_{conv} = \left(A\beta(1-\phi\tau-\delta(1-\tau))\right)^{\frac{1}{1-\beta}}, \tag{16}$$

and we can immediately conclude that

$$g^*_{conv} < g^*_{growth},$$

which means that the level of public expenditure that maximizes growth may not be compatible with debt/GDP convergence.

Optimal government expenditure g^*_{growth} depends neither on the audit parameters, nor on tax evasion. This is an interesting result of our model: the presence of the debt allows Government to set public expenditure independently on the level of tax evasion, but this does not prevent evasion from having undesired effects.

3.5 The effect of evasion on an equilibrium budget

In a model without evasion the public expenditure which maximizes the objectives in Proposition 3 is given by[6]

$$g_E = \left((1-\phi\tau)\,\beta A\right)^{\frac{1}{1-\beta}},$$

and the tax rate τ_E which guarantees the equilibrium of the Government budget balance (without debt) solves

$$\tau_E y_t = g_E k_t,$$

which becomes

$$\tau_E A\left(g_E\right)^{\beta} k_t = g_E k_t,$$

and whose solution is

$$\tau_E = \frac{\beta}{1+\phi\beta}.$$

This is the tax rate that we will use in the numerical simulations as a basic framework. In Figure 2 we show the same simulations we performed in the previous subsection, but with the tax defined in this way. The tax rate resulting from applying the values presented in Table 1 is lower than the basis value used in the previous simulations. Thus, we decided to increase the value of λ (and take $\lambda = 0.097$) in order to keep a reasonable value for the optimal evasion.

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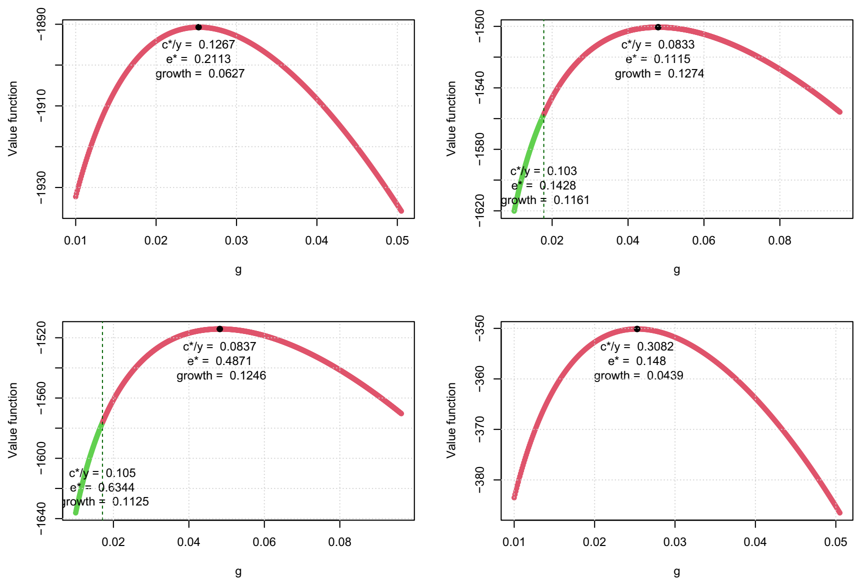

Figure 2. Value function $J(t, k_t; g)$ as a function of public expenditure g. In the upper left graph the base case is drawn with parameters in Table 1, but with $\tau = \frac{\beta}{1+\phi\beta}$, and $\lambda = 0.097$. In the upper right graph $A = 0.42$. In the lower left graph, $\beta = 0.255$ and $A = 0.42$. In the lower right graph, $\delta = 1.5$. In the green (red) section of the curve, the debt/GDP ratio is convergent (divergent).

All the graphs of the figure show a narrower range of public expenditure g that allows for debt/GDP convergence. In particular, the converge happens at a cost of a smaller value function or growth. Thus, we can conclude that the presence of the public debt allows the Government to increase the public expenditure by tolerating a level of evasion without preventing the debt/GDP to be stable over time.

4. Conclusions

Debt sustainability, its effect of debt on economic growth, and its relationship with tax evasion have become one of the most well-researched topics recently. We show that debt allows Governments to set the level of expenditure that maximizes growth also in the presence of tax evasion. However, in a general economic framework (where the equilibrium interest rate depends on the factor productivity), we show that tax evasion always reduces the range of parameters for which the debt/GDP ratio is mean reverting. Furthermore, mean reversion can be achieved with a sufficiently high total factor productivity whose increment also increases welfare (production is more efficient), reduces tax evasion and increases investments (consumption increases at a lower rate than income). Thus, evasion shows to be a problem especially for low-growth economies, which should try to reduce tax evasion as much as possible. This result is quite interesting in the light of recent estimates (Loayza 2016) showing that the size of the informal sector (a proxy for tax evasion) dampens economic growth. In this respect our model could be either interpreted as theoretical support for those findings or as the signal of a more structural problem. We have shown that, in order to preserve the debt/GDP sustainability, a relatively big informal sector leads to less growth. Furthermore, it is possible to use public expenditure and other fiscal policies to increase growth only if Government reduces its size.

From a policy perspective, our model reveals that while tolerating tax evasion may enhance expected optimal economic growth, it simultaneously undermines the stability of government

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debt. Therefore, tax evasion should be minimized as much as possible, particularly in economies that depend heavily on public expenditure to drive economic growth.

For future research, we plan to take into account the opportunity for the Government to sell its debt abroad in order to reduce the importance of the link between evasion and debt stability.

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Notes

1 Most models in this literature assume budget balance and study the effect of tax evasion on growth. See Dzhumashev et al. (2023) for a review.
2 As shown by Barro (1990) for an *Ak* technology fiscal illusion does not alter the optimal government expenditure and for this reason we have ruled it out so that in our model any departure from the optimal expenditure can be interpreted in terms of debt sustainability/tax evasion consequences.
3 We recall that the first and the second moment of the Poisson process coincide: $\mathbb{E}_t\left[d\Pi_t\right]=\mathbb{V}_t\left[d\Pi_t\right]=\lambda dt$.
4 One of the most commonly used form is an affine transformation of the tax rate $\eta(\tau)=\eta_0+\eta_1\tau$. In particular, when $\eta_1=0$, the fine is computed on evaded income as in Allingham and Sandmo (1972), while with $\eta_0=0$, it is computed on evaded tax as in Yitzhaki (1974).
5 See Arai (2011) for a comparison with traditional literature
6 It is sufficient to maximize (11).

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Appendix A. Proof of Proposition 1

Given the optimization problem, we can define the value function $J(t, k_t)$ as follows:

$$J(t, k_t) = \max_{\{c_s, e_s\}_{s\in[t,\infty[}} \mathbb{E}_t \left[\int_t^{\infty} U(c_s)\, e^{-\rho(s-t)} ds \right], \tag{17}$$

which must solve the following partial differential equation (so-called Hamilton–Jacobi–Bellman (HJB) equation):

$$\begin{aligned} 0 = J_t - (\rho + \lambda)\, J + J_k\big((1-\phi\tau)\, g^{\beta} A k_t - g k_t\big) \\ + \max_{c_t} \left\{ \frac{c_t^{1-\delta}}{1-\delta} - J_k c_t \right\} \\ + \max_{e_t} \left\{ J_k \phi \tau\, e_t g^{\beta} A k_t + \lambda J\big(k_t - \phi\eta(\tau) e_t y_t\big) \right\}, \end{aligned}$$

in which the lower scripts on J indicate partial derivatives. We take the guess function for J in the following form:

$$J = F^{\delta} \frac{k_t^{1-\delta}}{1-\delta},$$

where F is assumed to be constant such that the following equations is solved:

$$0 = -(\rho+\lambda) F^{\delta} \frac{k_t^{1-\delta}}{1-\delta} + F^{\delta} k_t^{-\delta} \left((1-\phi\tau) g^{\beta} A k_t - g k_t \right) + \max_{c_t} \left\{ \frac{c_t^{1-\delta}}{1-\delta} - F^{\delta} k_t^{-\delta} c_t \right\} + \max_{e_t} \left\{ F^{\delta} k_t^{-\delta} \phi\tau e_t g^{\beta} A k_t + \lambda F^{\delta} \frac{\left(k_t - \phi\eta(\tau) e_t y_t\right)^{1-\delta}}{1-\delta} \right\}.$$

The first-order condition on consumption is

$$c_t^* = \frac{k_t}{F},$$

while the first-order condition on evasion is

$$e_t^* = \frac{1}{\phi\eta(\tau) g^{\beta} A} \left(1 - \left(\frac{\eta(\tau)\lambda}{\tau} \right)^{\frac{1}{\delta}} \right).$$

Once these optimal values are substituted into the HJB equation, we get

$$0 = -(\rho+\lambda) F^{\delta} \frac{k_t^{1-\delta}}{1-\delta} + F^{\delta-1} \frac{k_t^{1-\delta}}{1-\delta} + F^{\delta} k_t^{1-\delta} \left((1-\phi\tau) g^{\beta} A - g \right) - F^{\delta-1} k_t^{1-\delta} + F^{\delta} k_t^{1-\delta} \frac{\tau}{\eta(\tau)} \left(1 - \left(\frac{\eta(\tau)\lambda}{\tau} \right)^{\frac{1}{\delta}} \right) + \lambda F^{\delta} k_t^{1-\delta} \frac{\left(\frac{\eta(\tau)\lambda}{\tau} \right)^{\frac{1-\delta}{\delta}}}{1-\delta},$$

which gives

$$\frac{1}{F} = (\rho+\lambda)\frac{1}{\delta} + \frac{\delta-1}{\delta} \left((1-\phi\tau) g^{\beta} A - g + \frac{\tau}{\eta(\tau)} \right) - \lambda \left(\frac{\eta(\tau)\lambda}{\tau} \right)^{\frac{1-\delta}{\delta}}.$$

When F is substituted into the optimal consumption the result of the proposition is obtained.

Appendix B. Proof of Proposition 4

After plugging the optimal consumption and evasion in the debt/GDP dynamics we obtain the following expected value:

$$\mathbb{E}_t \left[d\left(\frac{B_t}{y_t} \right) \right] = \left(\frac{g^{1-\beta}}{A} - (1-\phi)\tau + \frac{1-\phi}{\phi} \frac{\lambda}{g^{\beta} A} \left(\left(\frac{\eta(\tau)\lambda}{\tau} \right)^{\frac{1}{\delta}-1} - 1 \right) \left(\left(\frac{\eta(\tau)\lambda}{\tau} \right)^{-\frac{1}{\delta}} - 1 \right) \right) dt - \frac{B_t}{y_t} \left[\frac{(1-\phi\tau) g^{\beta} A - g + \frac{\tau}{\eta(\tau)} - (\rho+\lambda)}{\delta} + \lambda \left(1 - \left(\frac{\eta(\tau)\lambda}{\tau} \right)^{-\frac{1}{\delta}} \right) - r \right] dt,$$

which is a mean reverting process if the coefficient of $\frac{B_t}{y_t}$ in the square brackets on the right-hand side is positive. Let us now investigate the general equilibrium solution by substituting the value of the interest rate r with its equilibrium value:

$$r^* = (1-\tau) A g^{\beta} + \frac{\tau - \eta(\tau)\lambda}{\phi\eta(\tau)} \left(1 - \left(\frac{\eta(\tau)\lambda}{\tau} \right)^{\frac{1}{\delta}} \right), \tag{18}$$

which is obtained by setting the interest rate to the level of the expected marginal product of private capital.

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We recall that the expected production net of taxes is given by

$$y_t - \tau y_t\left(1 - e_t^*\right) - \lambda\eta\left(\tau\right) e_t^* y_t,$$

and after substituting the optimal evasion and computing the derivative w.r.t. k_t, we get the result shown in the text.

After substituting for the equilibrium value of the interest rate (18), the debt/GDP dynamics becomes

$$\begin{aligned}\mathbb{E}_t\left[d\left(\frac{B_t}{y_t}\right)\right] = &\left(\frac{g^{1-\beta}}{A} - (1-\phi)\,\tau + \frac{1-\phi}{\phi}\frac{\lambda}{g^{\beta}A}\left(m^{1-\frac{1}{\delta}}-1\right)\left(m^{\frac{1}{\delta}}-1\right)\right)dt\\ &-\frac{B_t}{y_t}\left[\frac{(1-\phi\tau)\,g^{\beta}A - g - \rho}{\delta} - (1-\tau)\,Ag^{\beta}\right.\\ &\left.+\lambda\underbrace{\left(\frac{1-m}{\phi}\left(1-m^{-\frac{1}{\delta}}\right)+\frac{\delta-1}{\delta}+\frac{m}{\delta}-m^{\frac{1}{\delta}}\right)}_{=:\Lambda(m,\delta,\phi)}\right]dt,\end{aligned}$$

in which, for the sake of simplicity, we set $m := \frac{\tau}{\eta(\tau)\lambda}$, and the sustainability condition is the one written in the proposition.

If this process is mean reverting, then it converges to the ratio of the first term and the coefficient of $\frac{B_t}{y_t}$. Furthermore, it is easy to show that

$$\Lambda(1,\delta,\phi) = 0,$$

$$\left.\frac{\partial\Lambda}{\partial m}\right|_{m=1} = -\frac{1}{\phi}\left(1-m^{-\frac{1}{\delta}}\right) - \frac{1}{\delta}\frac{m-1}{\phi}m^{-\frac{1}{\delta}-1} + \frac{1}{\delta} - \frac{1}{\delta}m^{\frac{1}{\delta}-1}\Bigg|_{m=1} = 0,$$

$$\begin{aligned}\left.\frac{\partial^2\Lambda}{\partial m^2}\right|_{m=1} &= -\frac{1}{\delta}\frac{1}{\phi}m^{-\frac{1}{\delta}-1} - \frac{1}{\delta}\frac{1}{\phi}m^{-\frac{1}{\delta}-1} - \frac{1}{\delta}\frac{m-1}{\phi}\left(-\frac{1}{\delta}-1\right)m^{-\frac{1}{\delta}-2} - \frac{1}{\delta}\left(\frac{1}{\delta}-1\right)m^{\frac{1}{\delta}-2}\Bigg|_{m=1}\\ &= \frac{1}{\delta^2}\frac{1}{\phi}\left((\phi-2)\,\delta-\phi\right) < 0.\end{aligned}$$

The sign of the first derivative is negative for $\phi = 1$ and, thus, it is also negative for any lower value of ϕ. Finally, we can conclude that $\Lambda(m,\delta,\phi)$ is always negative in the domain, decreasing and concave.

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