

ROBUST H_∞ STABILISATION WITH DEFINITE ATTENUANCE OF AN UNCERTAIN IMPULSIVE SWITCHED SYSTEM

HONGLEI XU¹, XINZHI LIU² and KOK LAY TEO³

(Received 3 December, 2004)

Abstract

In this paper, we study the problem of robust H_∞ stabilisation with definite attenuation for a class of impulsive switched systems with time-varying uncertainty. A norm-bounded uncertainty is assumed to appear in all the matrices of the state model. An LMI-based method for robust H_∞ stabilisation with definite attenuation via a state feedback control law is developed. A simulation example is presented to demonstrate the effectiveness of the proposed method.

1. Introduction

Robust H_∞ stability and control problems of dynamic systems have attracted considerable attention for several decades. The main focus has been on H_∞ problems for linear systems [2, 10], nonlinear systems [9], and systems without delay as well as with delays [2, 5, 9, 10] and so on. In recent years, interest has been extended to the robust H_∞ stability problem of impulsive dynamic systems.

Some typical examples of impulsive systems can be found in [11]. One such system is used to model the population of a certain kind of insect via introducing its natural enemies at certain time instances. Another system models control of the reaction process of a chemical reactor by adding chemicals at certain time instances. In [7], it is pointed out that the dynamical behaviour of the total stock value of a particular investor can be described by an impulsive system. Some other examples can be found in [3] and [6]. In [3] and [4], various issues concerning the stability and robust stability of impulsive dynamic systems are studied using Lyapunov functions. In particular, the

¹Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, Hubei, 430074, P. R. China.

²Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1 Canada.

³Department of Mathematics and Statistics, Curtin University of Technology, Perth WA 6845, Australia; e-mail: K.L.Teo@curtin.edu.au.

© Australian Mathematical Society 2005, Serial-fee code 1446-1811/05

impulsive switched system with norm-bounded time-varying uncertainty has attracted much attention.

In this paper, we consider the robust H_∞ control problem with definite attenuation of a class of uncertain impulsive switched systems. The problem is to design a feedback control law such that the closed loop impulsive system is asymptotically stable and robustly stable with definite attenuation and H_∞ performance. Using the LMI approach, we derive a set of sufficient conditions for ensuring the existence of such a feedback control law. This feedback control law can then be obtained by solving a set of linear matrix inequalities.

The rest of the paper is organised as follows. In Section 2, a general class of uncertain impulsive switched systems with time-varying uncertainty is presented and some useful definitions are given. In Section 3, some sufficient conditions for robust H_∞ stability with definite attenuation for the impulsive switched system are derived. A constructive method for constructing the corresponding feedback control law is then given. In Section 4, a numerical example is presented to illustrate how such a feedback control law is constructed. Finally, Section 5 concludes the paper.

2. Problem statement

Consider the following class of uncertain impulsive switched systems:

$$\begin{cases} \dot{x}(t) = (A_{i_k} + \Delta A_{i_k}(t))x(t) + B_{i_k}e^{-\lambda t}w(t) + C_{i_k}u(t), & t \neq t_k, \\ \Delta x(t) = I_k(t, x) = D_k x(t), & t = t_k, \\ x(t) = 0, & t = t_0 = 0. \end{cases} \quad (2.1)$$

Here $x \in R^n$ is the state, $u \in R^m$ is the control input and $w \in R^p$ is an uncertainty. Also $A_{i_k} \in R^{n \times n}$, $B_{i_k} \in R^{n \times p}$, $C_{i_k} \in R^{n \times m}$ and $D_k \in R^{n \times n}$ are constant real matrices, λ is a positive constant, $k = 1, 2, \dots$, $i_k \in \{1, 2, \dots, s\}$ and s is a positive integer. For each $i_k \in \{1, 2, \dots, s\}$, $\Delta A_{i_k}(\cdot)$ is an unknown real norm-bounded matrix function representing the time-varying parameter uncertainty. An admissible uncertainty is assumed to be of the form

$$\Delta A_{i_k}(t) = F_{i_k} \Xi_{i_k}(t) H_{i_k}, \quad (2.2)$$

where F_{i_k} and H_{i_k} are known real constant matrices and $\Xi_{i_k}(t)$, $i_k \in \{1, 2, \dots, s\}$, are unknown real time-varying matrices satisfying $\|\Xi_{i_k}(t)\| < 1$. Here $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^-) = x(t_k) = \lim_{h \rightarrow 0^+} x(t_k - h)$, $x(t_k^+) = \lim_{h \rightarrow 0^+} x(t_k + h)$, t_k is an impulsive switched point, $k = 1, 2, \dots$, and $t_0 < t_1 < t_2 < \dots < t_k < \dots$ ($t_k \rightarrow \infty$ as $k \rightarrow \infty$).

Define the variable $\xi(t) = e^{\lambda t}x(t)$ as the state variable of the impulsive switched system (2.1) and $z(t) = E_{i_k}\xi(t)$, $E_{i_k} \in R^{q \times n}$, the control output. Let $\tilde{u}(t) = e^{\lambda t}u(t)$. Then system (2.1) can be written as:

$$\begin{cases} \dot{\xi}(t) = (\lambda I + A_{i_k} + \Delta A_{i_k}(t))\xi(t) + B_{i_k}w(t) + C_{i_k}\tilde{u}(t), & t \neq t_k, \\ \Delta\xi(t) = I_k(t, \xi) = D_k\xi(t), & t = t_k, \\ z(t) = E_{i_k}\xi(t), \\ \xi(t) = 0, & t = t_0 = 0, \end{cases} \quad (2.3)$$

where $\xi \in R^n$ is the state, $\tilde{u} \in R^m$ is the control input, $w \in R^p$ is the uncertainty and $z \in R^q$ is the control output. Other parameters are the same as those defined for system (2.1).

DEFINITION 1. The uncertain impulsive switched system (2.1) is said to be robustly stable if the trivial solution $x(t, t_0, x_0) = 0$ of the functional differential equation (2.1) with $u(t) = 0$ is asymptotically stable with respect to all admissible uncertainties.

DEFINITION 2. The uncertain impulsive switched system (2.1) is said to be robustly stabilisable if there exists a linear state feedback control law $u(t) = \Gamma_{i_k}x(t)$ with $\Gamma_{i_k} \in R^{m \times n}$ such that the resulting closed loop system is robustly stable in the sense of Definition 1.

DEFINITION 3. For given scalars $\gamma > 0$ and $\lambda > 0$, the uncertain impulsive switched system (2.1) is said to be robustly H_∞ stable with definite attenuation under any given switching law if the uncertain impulsive system has H_∞ performance, that is, $\|z(t)\|_\tau < \gamma \|w(t)\|_\tau$ is satisfied, and the trivial solution $\xi(t, t_0, x_0) = 0$ of the functional differential equation (2.3) with $u(t) = 0$ is asymptotically stable with respect to all admissible uncertainties.

DEFINITION 4. For given scalars $\gamma > 0$ and $\lambda > 0$, the uncertain impulsive switched system (2.1) is said to be robustly H_∞ stabilisable with definite attenuation if there exists a state feedback control law such that for any admissible uncertainty, the following conditions are satisfied under any given switching law:

- (i) Robust stability: The resulting closed-loop system of the impulsive switched system (2.3) is asymptotically stable;
- (ii) H_∞ performance: When a positive constant γ is given as the objective performance, $\|z(t)\|_\tau < \gamma \|w(t)\|_\tau$.

In this paper, we shall derive a set of sufficient conditions to guarantee robust H_∞ stabilisation with definite attenuation for a class of uncertain impulsive switched systems. More specifically, for given scalars $\gamma > 0$ and $\lambda > 0$, our objective is

to obtain sufficient conditions to ensure robust stability with definite attenuation and, at the same time, achieve H_∞ performance. Furthermore, they can be used to find a stabilising state feedback control law $u(t) = \Gamma_{i_k} x(t)$ such that the resulting closed loop impulsive system is robustly H_∞ stable with definite attenuation. This state feedback control law can be obtained via solving a set of linear matrix inequalities.

3. Main results

First, we give several lemmas which are needed for the proofs of our main theorems.

LEMMA 3.1 ([8]). *Let $G \in R^{p \times q}$ be a matrix such that $G^T G \leq I$. Then*

$$2x^T G y \leq x^T x + y^T y \quad (3.1)$$

for all $x \in R^p$ and $y \in R^q$. In the case $G = I$, (3.1) reduces to $2x^T y \leq x^T x + y^T y$.

LEMMA 3.2. *Let $P \in R^{n \times n}$ be a positive definite matrix and $Q \in R^{n \times n}$ a symmetric matrix. Then*

$$\lambda_{\min}(P^{-1} Q) x(t)^T P x(t) \leq x(t)^T Q x(t) \leq \lambda_{\max}(P^{-1} Q) x(t)^T P x(t)$$

for all $x(t) \in R^n$.

LEMMA 3.3 ([5]). *Let A , F , Ξ and H be real matrices of appropriate dimensions with $\|\Xi\| \leq 1$. Then for any scalars $\varepsilon > 0$,*

$$F \Xi H + H^T \Xi^T F^T \leq \varepsilon^{-1} F F^T + \varepsilon H^T H.$$

To proceed further, we assume that the following condition is satisfied:

ASSUMPTION 3.4. $\|w(t)\| \leq L \|\xi(t)\|$.

We now present our main results on robust H_∞ stabilisation with definite attenuation.

THEOREM 3.5. *Let β_k be the largest eigenvalue of the matrix*

$$P_{i_{k-1}}^{-1} (I + D_k)^T P_{i_k} (I + D_k).$$

Suppose that Assumption 3.4 is satisfied. Then, for any given switching law, system (2.1) is robustly H_∞ stable with definite attenuation if there exists a symmetric positive

definite matrix P_{i_k} , positive scalars $\gamma, \varepsilon, \lambda$ and $0 \leq \beta_k < 1, k \in N$, such that the following linear matrix inequalities are satisfied:

$$\begin{bmatrix} S(\lambda, P_{i_k}, \varepsilon) & \varepsilon^{-1/2} P_{i_k} F_{i_k} & P_{i_k} B_{i_k} & L I \\ \varepsilon^{-1/2} F_{i_k}^T P_{i_k} & -I & & \\ B_{i_k}^T P_{i_k} & & -I & \\ L I & & & -I \end{bmatrix} < 0 \quad \text{and} \quad (3.2)$$

$$\begin{bmatrix} S(\lambda, P_{i_k}, \varepsilon) & \varepsilon^{-1/2} P_{i_k} F_{i_k} & \gamma^{-1} P_{i_k} B_{i_k} & E_{i_k}^T \\ \varepsilon^{-1/2} F_{i_k}^T P_{i_k} & -I & & \\ \gamma^{-1} B_{i_k}^T P_{i_k} & & -I & \\ E_{i_k} & & & -I \end{bmatrix} < 0, \quad (3.3)$$

where

$$S(\lambda, P_{i_k}, \varepsilon) = (\lambda I + A_{i_k}^T) P_{i_k} + P_{i_k} (\lambda I + A_{i_k}) + \varepsilon H_{i_k}^T H_{i_k}. \quad (3.4)$$

PROOF. For $t \in (t_k, t_{k+1}]$, define the following Lyapunov function for system (2.3) with $u(t) = 0$:

$$V(t) = \xi^T(t) P_{i_k} \xi(t), \quad (3.5)$$

where P_{i_k} is a positive definite symmetric matrix. Taking the time derivative of $V(t)$ along the solution of (2.3) with $u(t) = 0$, and then making use of (2.2), we obtain

$$\begin{aligned} \dot{V}(t) &= \dot{\xi}^T(t) P_{i_k} \xi(t) + \xi^T(t) P_{i_k} \dot{\xi}(t) \\ &= [\lambda \xi^T(t) + \xi^T(t) (A_{i_k} + \Delta A_{i_k})^T + w^T(t) B_{i_k}^T] P_{i_k} \xi(t) \\ &\quad + \xi^T(t) P_{i_k} [\lambda \xi(t) + (A_{i_k} + \Delta A_{i_k}) \xi(t) + B_{i_k} w(t)] \\ &= \xi^T(t) [(\lambda I + A_{i_k}^T) P_{i_k} + P_{i_k} (\lambda I + A_{i_k}) + H_{i_k}^T \Xi_{i_k} (P_{i_k} F_{i_k})^T + P_{i_k} F_{i_k} \Xi_{i_k} H_{i_k}] \xi(t) \\ &\quad + w^T(t) (\xi^T(t) P_{i_k} B_{i_k})^T + \xi^T(t) P_{i_k} B_{i_k} w(t). \end{aligned} \quad (3.6)$$

By Lemma 3.3, we have, for any $\varepsilon > 0$,

$$H_{i_k}^T \Xi_{i_k} (P_{i_k} F_{i_k})^T + P_{i_k} F_{i_k} \Xi_{i_k} H_{i_k} \leq (1/\varepsilon) P_{i_k} F_{i_k} F_{i_k}^T P_{i_k} + \varepsilon H_{i_k}^T H_{i_k} \quad (3.7)$$

and, for any $\gamma > 0$,

$$\begin{aligned} &w^T(t) (\xi^T(t) P_{i_k} B_{i_k})^T + \xi(t) P_{i_k} B_{i_k} w(t) \\ &\leq \gamma^{-2} \xi^T(t) P_{i_k} B_{i_k} B_{i_k}^T P_{i_k} \xi(t) + \gamma^2 w^T(t) w(t). \end{aligned} \quad (3.8)$$

Applying (3.7) and (3.8) with $\gamma = 1$ to (3.6), and using Assumption 3.4, it follows that

$$\begin{aligned} \dot{V}(t) &\leq \xi^T(t) [(\lambda I + A_{i_k}^T) P_{i_k} + P_{i_k} (\lambda I + A_{i_k}) + \varepsilon^{-1} P_{i_k} F_{i_k} F_{i_k}^T P_{i_k} + \varepsilon H_{i_k}^T H_{i_k}] \xi(t) \\ &\quad + \xi^T(t) P_{i_k} B_{i_k} B_{i_k}^T P_{i_k} \xi(t) + w^T(t) w(t) \\ &\leq \xi^T(t) [S(\lambda, P_{i_k}, \varepsilon) + \varepsilon^{-1} P_{i_k} F_{i_k} F_{i_k}^T P_{i_k} + P_{i_k} B_{i_k} B_{i_k}^T P_{i_k} + L^2 I] \xi(t), \end{aligned} \quad (3.9)$$

where $S(\lambda, P_{i_k}, \varepsilon)$ is defined by (3.4).

To ensure that

$$\dot{V}(t) < 0 \quad \text{for } t \in (t_{k-1}, t_k], \tag{3.10}$$

we choose $\varepsilon > 0$ such that

$$S(\lambda, P_{i_k}, \varepsilon) + \varepsilon^{-1} P_{i_k} F_{i_k} F_{i_k}^T P_{i_k} + P_{i_k} B_{i_k} B_{i_k}^T P_{i_k} + L^2 I < 0. \tag{3.11}$$

Let us now look at the function $V(t)$, defined by (3.5), at the switching points t_k . Then we have

$$\begin{aligned} V(t_k^+) &= \xi^T(t_k^+) P_{i_k} \xi(t_k^+) = \xi^T(t_k^-) (I + D_k)^T P_{i_k} (I + D_k) \xi(t_k^-) \\ &= \xi^T(t_k^-) P_{i_{k-1}} [P_{i_{k-1}}^{-1} (I + D_k)^T P_{i_k} (I + D_k)] \xi(t_k^-) \\ &\leq \beta_k V(t_k^-) < V(t_k^-), \end{aligned} \tag{3.12}$$

where $\beta_k, 0 \leq \beta_k < 1$, is the maximum eigenvalue of $P_{i_{k-1}}^{-1} (I + D_k)^T P_{i_k} (I + D_k)$.

Combining (3.10) and (3.12), it follows that system (2.3) with $u = 0$ is asymptotically stable.

It remains to show that the uncertain impulsive system has H_∞ performance. For this, it follows from (3.6)–(3.8) that

$$\begin{aligned} \dot{V}(t) &\leq \xi^T(t) [(\lambda I + A_{i_k}^T) P_{i_k} + P_{i_k} (\lambda I + A_{i_k}) + \varepsilon^{-1} P_{i_k} F_{i_k} F_{i_k}^T P_{i_k} \\ &\quad + \varepsilon H_{i_k}^T H_{i_k}] \xi(t) + \gamma^{-2} \xi^T(t) P_{i_k} B_{i_k} B_{i_k}^T P_{i_k} \xi(t) + \gamma^2 w^T(t) w(t) \\ &= \xi^T(t) [S(\lambda, P_{i_k}, \varepsilon) + \varepsilon^{-1} P_{i_k} F_{i_k} F_{i_k}^T P_{i_k} + \gamma^{-2} P_{i_k} B_{i_k} B_{i_k}^T P_{i_k} + E_{i_k}^T E_{i_k}] \xi(t) \\ &\quad - \xi^T(t) E_{i_k}^T E_{i_k} \xi(t) + \gamma^2 w^T(t) w(t). \end{aligned}$$

Now, by imposing

$$S(\lambda, P_{i_k}, \varepsilon) + \varepsilon^{-1} P_{i_k} F_{i_k} F_{i_k}^T P_{i_k} + \gamma^{-2} P_{i_k} B_{i_k} B_{i_k}^T P_{i_k} + E_{i_k}^T E_{i_k} < 0, \tag{3.13}$$

we obtain $\dot{V}(t) < -\|E_{i_k} \xi(t)\|^2 + \gamma^2 \|w(t)\|^2 = -\|z(t)\|^2 + \gamma^2 \|w(t)\|^2$ and hence

$$\|z(t)\|^2 < -\dot{V}(t) + \gamma^2 \|w(t)\|^2. \tag{3.14}$$

Integrating both sides of (3.14), we have

$$\int_0^\tau \|z(t)\|^2 dt < -\int_0^\tau \dot{V}(t) dt + \gamma^2 \int_0^\tau \|w(t)\|^2 dt, \quad \tau \in (t_k, t_{k+1}]. \tag{3.15}$$

From (3.5) we see that $V(0) = 0$ and $V(t_k) > 0$. Since $0 \leq \beta_k < 1$, it follows that

$$\begin{aligned} \int_0^\tau \dot{V}(t) dt &= \int_0^{t_1} \dot{V}(t) dt + \int_{t_1}^{t_2} \dot{V}(t) dt + \dots + \int_{t_{k-1}}^{t_k} \dot{V}(t) dt + \int_{t_k}^\tau \dot{V}(t) dt \\ &= V(t_1) - V(0) + V(t_2) - V(t_1^+) + \dots + V(t_k) - V(t_{k-1}^+) \\ &\quad + V(\tau) - V(t_k^+) \\ &\geq \sum_{i=1}^k (1 - \beta_i) V(t_i) + V(\tau) > 0. \end{aligned} \tag{3.16}$$

Combining (3.15) and (3.16), we have

$$\|z(t)\|_\tau^2 = \int_0^\tau \|z(t)\|^2 dt < \gamma^2 \int_0^\tau \|w(t)\|^2 dt = \gamma^2 \|w(t)\|_\tau^2.$$

Thus the H_∞ performance is satisfied. Therefore system (2.1) is robustly H_∞ stable with definite attenuation.

Finally, using the Schur complementary theorem [1], the inequalities (3.11) and (3.13) are equivalent to those of (3.2)–(3.3). This completes the proof.

In the case when there is no uncertainty in system (2.1) (or system (2.3)), that is, $\Delta A_{ik} = 0$, we have the following result.

COROLLARY 3.6. *Consider system (2.1) (or (2.3)) with $F_{ik} = 0$, $H_{ik} = 0$. Assume that Assumption 3.4 is satisfied. Let β_k be the largest eigenvalue of $P_{ik}^{-1}(I + D_k)^T P_{ik}(I + D_k)$. Then the uncertain impulsive switched system (2.1) is robustly H_∞ stable with definite attenuation if there exists a symmetric positive definite matrix P_{ik} , positive scalars γ, λ and $0 \leq \beta_k < 1$, and $k \in N$ such that the following linear matrix inequalities are satisfied:*

$$\begin{bmatrix} (\lambda I + A_{ik}^T)P_{ik} + P_{ik}(\lambda I + A_{ik}) & P_{ik}B_{ik} & LI \\ B_{ik}^T P_{ik} & -I & \\ LI & & -I \end{bmatrix} < 0 \quad \text{and} \tag{3.17}$$

$$\begin{bmatrix} (\lambda I + A_{ik}^T)P_{ik} + P_{ik}(\lambda I + A_{ik}) & \gamma^{-1}P_{ik}B_{ik} & E_{ik}^T \\ \gamma^{-1}B_{ik}^T P_{ik} & -I & \\ E_{ik} & & -I \end{bmatrix} < 0. \tag{3.18}$$

THEOREM 3.7. *Let β_k be the largest eigenvalue of the matrix $P_{ik}^{-1}(I + D_k)^T P_{ik}(I + D_k)$. For any given switching law, consider the uncertain impulsive switched system (2.1) and assume that Assumption 3.4 is satisfied. Then system (2.1) is robustly H_∞ stable with definite attenuation if there exists a positive definite symmetric matrix*

P_{ik} , positive scalars $\gamma, \varepsilon, \lambda$ and $0 \leq \beta_k < 1$, and $k \in N$ such that the following linear matrix inequalities are satisfied:

$$\begin{bmatrix} S(\lambda, P_{ik}, \varepsilon) & \varepsilon^{-1/2} P_{ik} F_{ik} & P_{ik} & LI \\ \varepsilon^{-1/2} F_{ik}^T P_{ik} & -I & & \\ P_{ik} & & 2C_{ik} C_{ik}^T - B_{ik} B_{ik}^T & \\ LI & & & -I \end{bmatrix} < 0 \quad \text{and} \quad (3.19)$$

$$\begin{bmatrix} S(\lambda, P_{ik}, \varepsilon) & \varepsilon^{-1/2} P_{ik} F_{ik} & P_{ik} & E_{ik}^T \\ \varepsilon^{-1/2} F_{ik}^T P_{ik} & -I & & \\ P_{ik} & & 2C_{ik} C_{ik}^T - \gamma^{-2} B_{ik} B_{ik}^T & \\ E_{ik} & & & -I \end{bmatrix} < 0, \quad (3.20)$$

where $S(\lambda, P_{ik}, \varepsilon) = (\lambda I + A_{ik}^T) P_{ik} + P_{ik} (\lambda I + A_{ik}) + \varepsilon H_{ik}^T H_{ik}$. Moreover, the feedback control law is given by $u(t) = -C_{ik}^T P_{ik} x(t)$, which is a robust H_∞ stabilising controller.

PROOF. For the given control law

$$u(t) = \Gamma_{ik} x(t), \quad (3.21)$$

where $\Gamma_{ik} = -C_{ik}^T P_{ik}$. Then $\tilde{u}_{ik}(t) = \Gamma_{ik} \xi(t)$. Define, for $t \in (t_k, t_{k+1}]$, the following Lyapunov function for (2.3): $V(t) = \xi^T(t) P_{ik} \xi(t)$, where P_{ik} is a positive definite symmetric matrix. Taking the time derivative of $V(t)$ along the solution of (2.3) with $u(t)$ given by (3.21), we obtain

$$\begin{aligned} \dot{V}(t) &= \dot{\xi}^T(t) P_{ik} \xi(t) + \xi^T(t) P_{ik} \dot{\xi}(t) \\ &= [\lambda \xi^T(t) + \xi^T(t) (A_{ik} + \Delta A_{ik})^T + w^T(t) B_{ik}^T + \tilde{u}^T(t) C_{ik}^T] P_{ik} \xi(t) \\ &\quad + \xi^T(t) P_{ik} [\lambda \xi(t) + (A_{ik} + \Delta A_{ik}) \xi(t) + B_{ik} w(t) + C_{ik} \tilde{u}(t)] \\ &= \xi^T(t) [(\lambda I + A_{ik}^T) P_{ik} + P_{ik} (\lambda I + A_{ik}) + H_{ik}^T \Xi_{ik} (P_{ik} F_{ik})^T + P_{ik} F_{ik} \Xi_{ik} H_{ik}] \xi(t) \\ &\quad + w^T(t) (\xi^T(t) P_{ik} B_{ik})^T + \xi^T(t) P_{ik} B_{ik} w(t) - 2\xi^T(t) P_{ik} C_{ik} C_{ik}^T P_{ik} \xi(t). \end{aligned}$$

Then it follows that

$$\begin{aligned} \dot{V}(t) &\leq \xi^T(t) [(\lambda I + A_{ik}^T) P_{ik} + P_{ik} (\lambda I + A_{ik}) + \varepsilon^{-1} P_{ik} F_{ik} F_{ik}^T P_{ik} + \varepsilon H_{ik}^T H_{ik}] \xi(t) \\ &\quad + \xi^T(t) (P_{ik} B_{ik} B_{ik}^T P_{ik} - 2P_{ik} C_{ik} C_{ik}^T P_{ik}) \xi(t) + w^T(t) w(t) \\ &\leq \xi^T(t) [S(\lambda, P_{ik}, \varepsilon) + \varepsilon^{-1} P_{ik} F_{ik} F_{ik}^T P_{ik} + P_{ik} (B_{ik} B_{ik}^T - 2C_{ik} C_{ik}^T) P_{ik} + L^2 I] \xi(t), \end{aligned}$$

where $S(\lambda, P_{ik}, \varepsilon)$ is defined by (3.4).

Now, using an argument similar to that given for (3.11), we choose $\varepsilon > 0$ such that

$$S(\lambda, P_{ik}, \varepsilon) + \varepsilon^{-1} P_{ik} F_{ik} F_{ik}^T P_{ik} + P_{ik} B_{ik} B_{ik}^T P_{ik} - 2P_{ik} C_{ik} C_{ik}^T P_{ik} + L^2 I < 0. \quad (3.22)$$

Furthermore, using an argument similar to that given for (3.13), we impose that

$$S(\lambda, P_{i_k}, \varepsilon) + \varepsilon^{-1} P_{i_k} F_{i_k} F_{i_k}^T P_{i_k} + \gamma^{-2} P_{i_k} B_{i_k} B_{i_k}^T P_{i_k} - 2 P_{i_k} C_{i_k} C_{i_k}^T P_{i_k} + E_{i_k}^T E_{i_k} < 0. \quad (3.23)$$

Hence, using the Schur complementary theorem, the inequalities (3.22)–(3.23) can be written equivalently as (3.19)–(3.20).

4. A numerical example

Consider the uncertain impulsive switched system given by

$$\begin{cases} \dot{x}(t) = (A_1 + \Delta A_1(t))x(t) + B_1 e^{-\lambda t} w(t) + C_1 u(t), & t \neq t_k, \\ \Delta x(t) = I_k(t, x) = D_k x(t), & t = t_k, \\ z(t) = E_1 e^{\lambda t} x(t), \\ x(t) = 0, & t = t_0 = 0, \end{cases} \quad (4.1)$$

$$\begin{cases} \dot{x}(t) = (A_2 + \Delta A_2(t))x(t) + B_2 e^{-\lambda t} w(t) + C_2 u(t), & t \neq t_k, \\ \Delta x(t) = I_k(t, x) = D_k x(t), & t = t_k, \\ z(t) = E_2 e^{\lambda t} x(t), \\ x(t) = 0, & t = t_0 = 0 \end{cases} \quad (4.2)$$

with $A_1 = \begin{bmatrix} 0.8 & 0.5 \\ 0 & -1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 & 1 \\ 0.8 & 0.5 \end{bmatrix}$, $C_1 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$, $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.5 & 0.5 \\ 0 & -1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0.5 & 1 \\ 1 & 1.5 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 & 1.5 \\ 1 & 1.5 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D_k = -0.5$ and $\Delta A_{i_k}(t)$ is the uncertainty matrix satisfying $\|\Delta A_{i_k}(t)\| \leq 0.2$. Then this system is in the form of system (2.1) (or (2.3)) with

$$F_{i_k} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix} \quad \text{and} \quad H_{i_k} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Choose $\gamma = \varepsilon = L = \lambda = 1$. Then, by solving the corresponding versions of (3.19) and (3.20), we obtain, under any given switching law, positive definite symmetric matrices:

$$P_1 = \begin{bmatrix} 2.6756 & 0.7351 \\ 0.7351 & 0.0941 \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} 2.6887 & 0.2045 \\ 0.2045 & 0.0823 \end{bmatrix}$$

such that the conditions of Theorem 3.7 are satisfied. Hence the required state feedback control law is: $u(t) = \Gamma_{i_k} x(t)$, where

$$\Gamma_1 = \begin{bmatrix} -3.0432 & -0.7822 \\ -2.0729 & -0.4617 \end{bmatrix} \quad \text{and} \quad \Gamma_2 = \begin{bmatrix} -2.8932 & -0.2868 \\ -2.9954 & -0.3979 \end{bmatrix}.$$

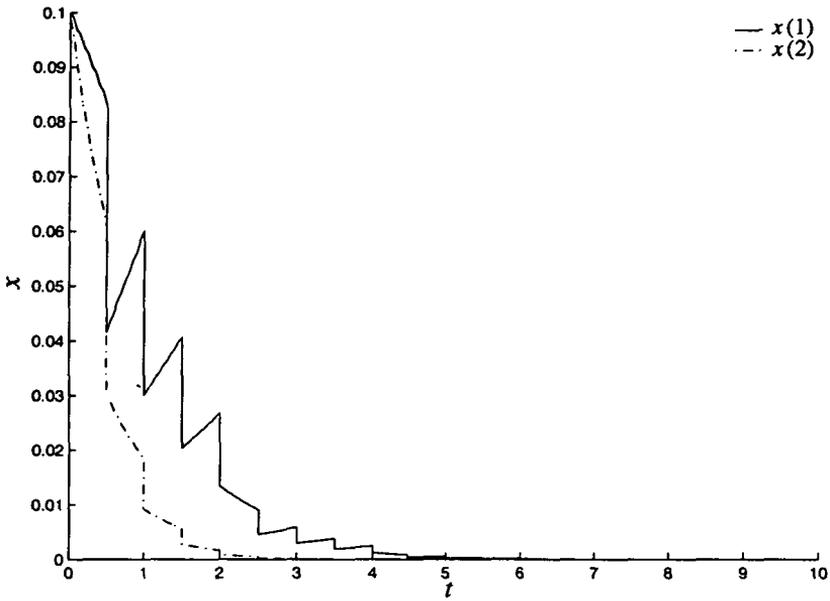


FIGURE 1. Behaviour of the state $x(t)$ when $\Xi_1(t) = \Xi_2(t) = \sin(2\pi * 10t) * I$ and $w(t) = 0$.

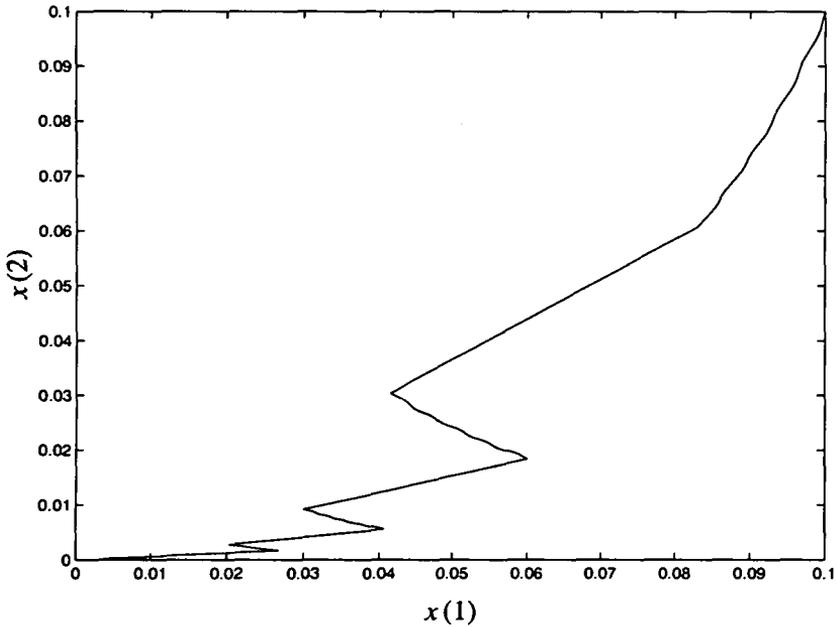


FIGURE 2. Phase portrait of the uncertain impulsive switched system when $\Xi_1(t) = \Xi_2(t) = \sin(2\pi * 10t) * I$ and $w(t) = 0$.

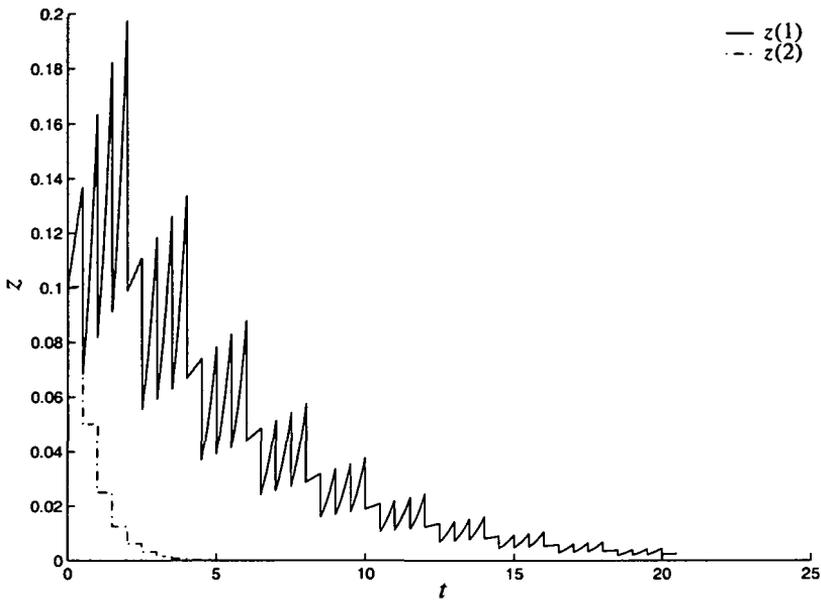


FIGURE 3. Behaviour of the state $\xi(t)$ when $\Xi_1(t) = \Xi_2(t) = \sin(2\pi * 10t) * I$ and $w(t) = 0$.

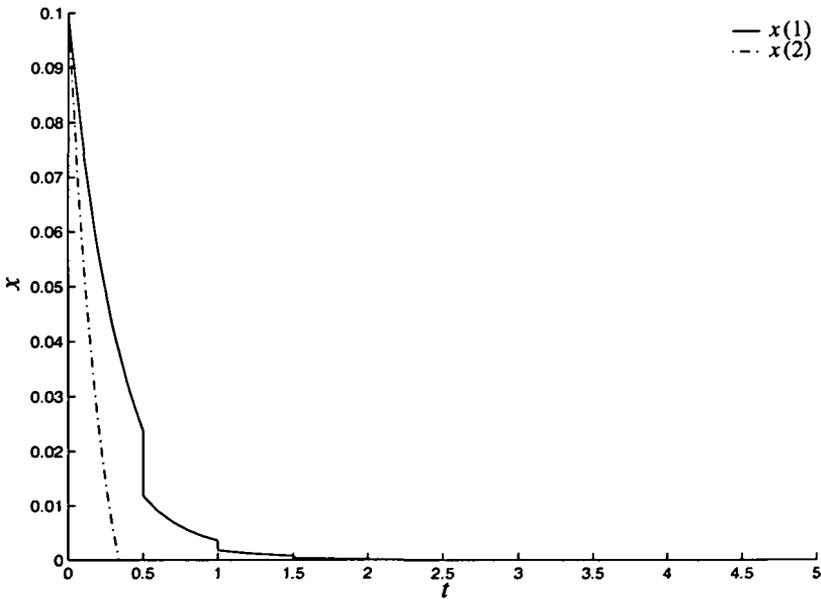


FIGURE 4. Behaviour of the state $x(t)$ with feedback control law $u(t) = \Gamma_{14}x(t)$ when $\Xi_1(t) = \Xi_2(t) = \sin(2\pi * 10t) * I$ and $w(t) = 0$.

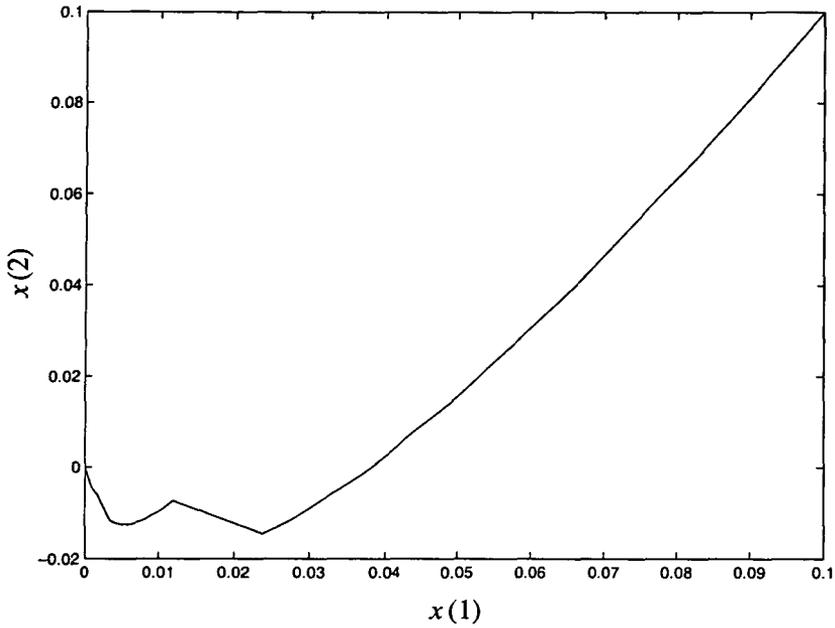


FIGURE 5. Phase portrait of the uncertain impulsive switched system with feedback control law $u(t) = \Gamma_{i_k}x(t)$ when $\Xi_1(t) = \Xi_2(t) = \sin(2\pi * 10t) * I$ and $w(t) = 0$.

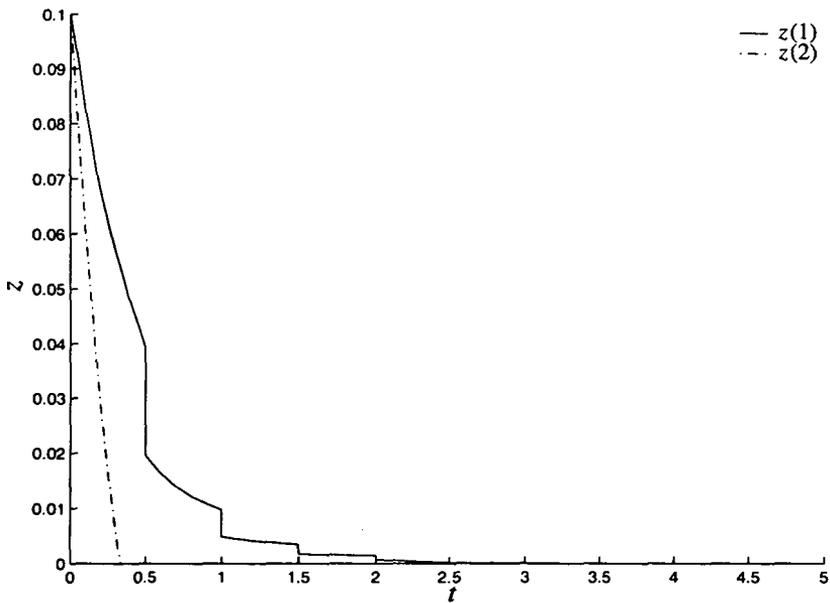


FIGURE 6. Behaviour of the state $\xi(t)$ with feedback control law $u(t) = \Gamma_{i_k}x(t)$ when $\Xi_1(t) = \Xi_2(t) = \sin(2\pi * 10t) * I$ and $w(t) = 0$.

This state feedback control law asymptotically stabilises the impulsive switched system with norm-bounded time-varying uncertainty. Furthermore, it also guarantees robust stability with definite attenuation and has H_∞ performance.

Assume that the switching law alternates the state of system (2.3) between (4.1) and (4.2). The numerical simulations are depicted in Figures 1–6. These figures show that the impulsive switched system with time-varying uncertainty is robustly stable. Furthermore, under the feedback control law obtained, the corresponding closed loop system is not only robustly stable with definite attenuation $\lambda = 1$ but also has H_∞ performance.

5. Conclusion

We have derived a set of readily computable conditions in terms of linear matrix inequalities for a class of impulsive switched systems with time-varying uncertainty. Based on a positive definite solution of linear matrix inequalities, we can construct a robust H_∞ state feedback control law, which guarantees robust stability with definite attenuation and gives rise to robust H_∞ performance for the class of impulsive switched systems with norm-bounded time-varying uncertainty. An illustrative example was solved so as to demonstrate the effectiveness of the proposed approach.

6. Acknowledgement

This research was partially supported by NSERC-Canada, and by a research grant (PolyU5197/03E) from the Research Grants Council of Hong Kong.

References

- [1] S. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan, *Linear matrix inequalities in system and control theory* (SIAM, Philadelphia, 1994).
- [2] P. P. Khargohekar, I. R. Petersen and K. Zhou, "Robust stabilization of uncertain linear systems: quadratic stabilizability and H_∞ control theory", *IEEE Trans Automat. Contr.* **35** (1990) 356–361.
- [3] V. Lakshmikantham, D. D. Bainov and P. S. Simeonov, *Theory of impulsive differential equations* (World Scientific Press, Singapore, 1989).
- [4] V. Lakshmikantham and X. Z. Liu, *Stability analysis in terms of two measures* (World Scientific Press, Singapore, 1994).
- [5] X. Li and C. E. Souza, "Criteria for robust stability and stabilization of uncertain linear system with state delay", *Automatica* **33** (1997) 1657–1662.
- [6] X. Z. Liu, "Impulsive stabilization and applications to population models", *Rocky Mountain J. Math.* **25** (1995) 381–395.

- [7] X. Z. Liu and K. L. Teo, "Impulsive control of chaotic system", *Internat. J. Bifur. Chaos Appl. Sci. Eng.* **12** (2002) 1181–1190.
- [8] I. R. Peterson, "A stabilization algorithm for a class of uncertain linear systems", *Systems Control Lett.* **8** (1987) 351–357.
- [9] T. Shen and K. Tamura, "Robust H_∞ control of an uncertain nonlinear system via state feedback", *IEEE Trans. Automat. Contr.* **40** (1995, 1987) 766–768.
- [10] L. Xie and C. E. Souza, "Robust H_∞ control for linear time-invariant systems with norm bounded uncertainty in the input matrix", *Systems Control Lett.* **14** (1990) 389–396.
- [11] T. Yang, "Impulsive control", *IEEE Trans. Automat. Contr.* **44** (1999) 1081–1083.