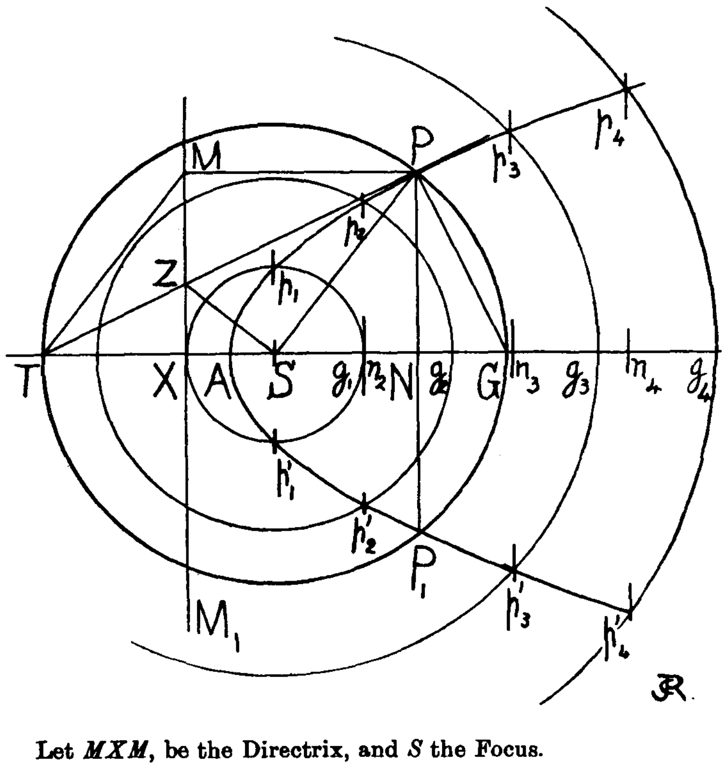
MATHEMATICAL NOTES

A Review of Elementary Mathematics and Science.

The Nor-Tan Circle of the Parabola.

The following was suggested by the constancy of the Sub-normal: and, so far as the writer is aware, is new.

It affords an accurate and rapid method of drawing a Parabola: it yields very simple proofs of certain fundamental properties: and also easy constructions for Tangent and Normal.



Let *MXM*, be the Directrix, and *S* the Focus.
Draw *XS* the Axis of the Parabola.

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With S as centre, and any radius greater than one-half XS describe a circle cutting the Axis in G.

Take a point N on the Axis such that $NG = XS$.

Through N draw the perpendicular PNP_1 meeting the circle in P and P_1.

These are points on the Parabola.

For, if PM is drawn perpendicular to the Directrix

$$PM = XN = XS + SN = SN + NG = SG = SP.$$

Now, the Tangent to a Parabola at any point P meets the Directrix in a point Z, such that angle ZSP is a Right Angle.

Hence if SZ is drawn perpendicular to SP, PZ is the Tangent at P.

Obviously the Triangles ZMP and ZSP are congruent: and ZP bisects Angle MPS. If PZ be produced to meet the Axis in T, T lies on the Circle PGP_1: for PZ is the locus of points equidistant from M and S, and $MPST$ is a Rhombus.

Since TG is a Diameter of the Circle, Angle TPG is a Right Angle: and PG is the Normal at P.

Hence the name Nor-Tan Circle: for this Circle determines the Tangent and Normal of the Parabola at the point P.

From the figure it is evident that

$$PN^2 = TN \,.\, NG$$

$$= 2AN \,.\, 2AS = 4AS \,.\, AN:$$ for A, the Vertex, is the mid-point of XS, and of TN.

To draw the Curve: the Focus and Directrix being given.

With S as Centre describe a number of concentric circles cutting the Axis in points $g_1 g_2 g_3 \ldots g_n$, and take points $n_1 n_2 n_3$ etc., so that, in each case, ng is equal to XS. Then through each point n draw a perpendicular pnp' to meet the appropriate circle in points p and p', which are points on the Parabola.

This construction emphasizes the fact that the "shape" of a Parabola is entirely dependent on the length of XS.

JAS. C. RITCHIE.