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ON r-GRAPHS AND r-MULTIHYPERGRAPHS WITH GIVEN MAXIMUM DEGREE

ZOLTÁN FÜREDI

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Abstract

It is well-known that if G is a multigraph (that is, a graph with multiple edges), the maximum number of pairwise disjoint edges in G is $\nu(G)$ and its maximum degree is $D(G)$, then $|E(G)| \le \nu\lfloor 3D/2 \rfloor$. We extend this theorem for r-graphs (that is, families of r-element sets) and for r-multihypergraphs (that is, r-graphs with repeated edges). Several problems remain open.

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1. Notations, preliminaries

A *multihypergraph* $\mathbf{H}$ is a pair $(V, \mathscr{E})$ where V is a (finite) set, the *vertex*-set, and $\mathscr{E}$ is a collection of subsets of V, the *edge*-set. If $\mathscr{E}$ does not contain multiple edges then $\mathbf{H}$ is called a *hypergraph.* For brevity we use the word "hypergraph" instead of "multihypergraph" if it does not cause ambiguity. The *rank* of $\mathbf{H}$ is the maximum cardinality of its edges, $r(\mathbf{H}) = \max\{|E| : E \in \mathscr{E}\}$. If all edges have r elements $\mathbf{H}$ is r-*uniform.* In this case $\mathbf{H}$ is also called an r-*graph* (or r-multihypergraph). The *degree* of a vertex v in $\mathbf{H}$ is denoted by $\deg_{\mathbf{H}}(v)$, or briefly by $\deg(v)$, and is the number

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of edges from $\mathscr{E}$ containing v. Let $D(\mathbf{H}) := \max\{\deg(v) : v \in V\}$, the maximum degree. A subset of edges $\mathscr{M} \subset \mathscr{E}$ is called a *matching* if every two numbers $M, M' \in \mathscr{M}$ are disjoint. The largest size of a matching in $\mathbf{H}$ is the matching number, $\nu(\mathbf{H})$. If $\nu(\mathbf{H}) = 1$ then $\mathbf{H}$ is *intersecting*.

Abbott, Hanson, Katchalski and Liu investigated the following problem in a series of papers ([1], [2], [4], [5]). Let r, ν, D be positive integers and put $N = N(r, \nu, D)$, the largest integer N for which there exists an r-uniform multihypergraph with N (not necessarily distinct) edges and having no independent set of edges of size greater than ν (that is, the matching number is at most ν) and no vertex of degree exceeding D. Such a family will be called an (r, ν, D)-*multihypergraph.*

The problem of evaluating $N(r, \nu, D)$ for all values of the parameters seems to be difficult. Nevertheless, the above authors established a couple of upper and lower bounds and obtained exact values of $N(r, \nu, D)$ for various infinite classes of values of r, ν and D. They proved

(1.0)[1] $$N(2, \nu, D) = \nu\lfloor \tfrac{3}{2}D\rfloor,$$

(1.1)[5] $$N(3, \nu, D) \le \tfrac{7}{3}\nu D \text{ with equality if } D \equiv 0 \pmod 3.$$

These theorems were also proved partly by Bollobás [9], [10]. (It follows from a theorem of Shannon (see, for example, [11]), that the *chromatic index* of a multigraph $\mathbf{G}$ is at most $\lfloor 3D/2\rfloor$.) (See the last section, Section 8.)

(1.2)[1] $$N(r, \nu, 2) = (r+1)\nu.$$

(1.3)[2] $$N(r, \nu, 3) = \begin{cases} (2r+1)\nu & \text{if } r \equiv 0, 1 \pmod 3, \\ 2r\nu & \text{if } r \equiv 2 \pmod 3. \end{cases}$$

(1.4)[1] $$N(r, \nu, D) \le \nu(r(D-1)+1),$$

and in (1.4) equality holds if and only if there exists an $S(n, D, 2)$ Steiner system over $n = r(D-1)+1$ vertices. Although (1.4) is trivial, it gives the exact values of $N(r, \nu, D)$ for several large classes of parameters. (A (multi) hypergraph $\mathbf{S}$ is an $S(n, D, 2)$ Steiner system if it is D-uniform, $|V(\mathbf{S})| = n$, and every two vertices are contained in exactly one edge.) It is well-known that if $S(n, D, 2)$ exists then $(n-1)/(D-1)$ and $\binom{n}{2}/\binom{D}{2}$ are integers, and these two constrains are sufficient for $n > n_0(D)$. (See Wilson [19].) In all these cases $r \ge D$. In this paper we concentrate on the case when D is large.

2. Fractional matchings and covers

To state our results we recall more definitions. An r-uniform hypergraph over $r^2 - r + 1$ vertices is called a *finite projective plane* of order $r-1$,

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denoted by $PG(2, r-1)$, if it is an $S(r^2 - r + 1, r, 2)$ Steiner system. Such planes are known to exist if $r - 1$ is a prime power or $r = 1, 2$.

A *cover* T of the hypergraph $\mathbf{H}$ is a finite set which intersects all its edges. The minimum size of a cover is the covering number, $\tau(\mathbf{H})$. For example, $\tau(PG(2, r-1)) = r$. A *fractional cover* t is a non-negative real-valued function $t : V(\mathbf{H}) \to \mathbf{R}^+$ such that

$$\sum_{x \in E} t(x) \geq 1$$

holds for all edges $E \in E(\mathbf{H})$. The *value* of t, $\|t\|$, is the sum $\sum t(x)$. The minimum value among all fractional covers is the fractional covering number, $\tau^*(\mathbf{H})$. The calculation of $\tau^*(\mathbf{H})$ is a linear programming problem, all coefficients are integer (0 or 1) and so the value of τ^* is always a rational number.

A *fractional matching* w of the hypergraph $\mathbf{H}$ is the real relaxation of matchings. It is a non-negative real-valued function over the edges of $\mathbf{H}$ such that

$$\sum_{E \ni x} w(E) \leq 1$$

hold for all $x \in V(\mathbf{H})$. The value of w, $\|w\|$, is the total sum $\sum w(E)$. The maximum of $\|w\|$ is the fractional matching number, $\nu^*(\mathbf{H})$. The calculation of τ^* and ν^* are dual linear problems, and hence $\nu^* = \tau^*$ holds for all hypergraphs.

It is easy to see that $\tau^*(PG(2, r-1)) = r - 1 + (1/r)$. In [17] the following theorem was proved: if the (multi)hypergraph $\mathbf{H}$ of rank r (where $r \geq 3$) does not contain $p + 1$ (pointwise) disjoint copies of $PG(2, r-1)$ then

$$\tau^*(\mathbf{H}) \leq \nu(r-1) + p/r. \tag{2.1}$$

This is a slight improvement on the trivial inequality

$$\tau^* \leq \tau \leq r\nu.$$

Let $\tau^*(r, \nu) = \sup\{\tau^*(\mathbf{H}) : r(\mathbf{H}) \leq r,\ \nu(\mathbf{H}) \leq \nu\}$. In [12] the following statement is proved: there exists a hypergraph $\mathbf{H}$ of rank r and matching number ν such that $\tau^*(\mathbf{H}) = \tau^*(r, \nu)$ and

$$|E(\mathbf{H})| \leq r\tau^*(r, \nu) \leq (r^2 - r + 1)\nu. \tag{2.2}$$

By (2.1) we have that $\tau^*(r, \nu) = (r - 1 + (1/r))\nu$ if and only if a $PG(2, r-1)$ exists. Otherwise $\tau^*(r, \nu) \leq (r-1)\nu$.

3. Multihypergraphs with bounded maximum degree

The following example is due to Bermond, Bond and Saclé [8].

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EXAMPLE 3.1. Suppose that there exists a projective plane of order $r-1$, $PG(2, r-1)$. Let L_0 be a line and $A_0 \subset L_0$ a set of $D - r\lfloor D/r \rfloor$ elements. Let $\mathbf{H}$ be the multihypergraph obtained form $PG(2, r-1)$ such that the multiplicity of a line L is

$$\begin{array}{ll} \lfloor D/r \rfloor & \text{if } L \cap A_0 = \varnothing, \\ \lceil D/r \rceil & \text{if } L \cap A_0 \neq \varnothing,\ L \neq L_0, \\ D-(r-1)\lceil D/r \rceil & \text{if } L = L_0. \end{array}$$

Then $\mathbf{H}$ is intersecting of rank r, maximum degree is D and $E(\mathbf{H}) = rD - (r-1)\lceil D/r \rceil$.

If we take ν disjoint copies of $\mathbf{H}$ we get

$$\nu(rD - (r-1)\lceil D/r \rceil) \leq N(r, \nu, D), \tag{3.1}$$

whenever a $PG(2, r-1)$ exists. Here we will prove

THEOREM 3.2. *For every* r, ν *and* D *one has*

$$\tau^*(r, \nu)D - r\tau^*(r, \nu) < N(r, \nu, D) \leq \tau^*(r, \nu)D.$$

THEOREM 3.3. *If* $D \geq (r-1)^2\nu$ *and a* $PG(2, r-1)$ *exists then*

$$N(r, \nu, D) = \nu(rD - (r-1)\lceil D/r \rceil).$$

Theorem 3.2 and (2.1) imply that

$$\lim_{D\to\infty} \frac{N(r, \nu, D)}{D} = \tau^*(r, \nu) \leq \nu\left(r - 1 + \frac{1}{r}\right). \tag{3.2}$$

In [2] it was proved that

$$\lim_{D\to\infty} \frac{N(r, 1, D)}{D} \leq r - 1 + \max_n \frac{n(r^2-r) - r^4 + 4r^3 - 6r^2 + 4r}{n^2 - n(2r+1) + r^3 - 2r^2 + 3r}. \tag{3.3}$$

Substituting $n = 2r^2 - r + 1$ one gets that the right-hand side of (3.3) is at least

$$(r-1) + \frac{1}{4} + \frac{11r^2 - 19r + 12}{4r(4r^2 - 7r + 3)}.$$

This is always larger than the bound in (3.2). In the case $\nu = 1$, Theorem 3.3 was conjectured by Bermond, Bond and Saclé [8] and in a slightly weaker form in [7]. They proved that equality holds in (3.1) for $\nu = 1$ and $r \leq 4$ for all D. Moreover they determined $N(r, 1, 3)$, (see (1.3)) and $N(r, 1, 4)$ for $r \not\equiv 3 \pmod 4$. This case was completed by Bermond and Bond [6]:

$$N(r, 1, r) = \begin{cases} 3r+1 & \text{if } r \equiv 0, 1 \pmod 4, \\ 3r & \text{if } r \equiv 2, 3 \pmod 4 \text{ but } r \neq 3, \\ 8 & \text{if } r = 3. \end{cases}$$

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4. The largest (r, ν, D)-hypergraphs

Denote by $f(r, \nu, D)$ the maximum number of r-tuples contained in an r-graph $\mathbf{F}$ with $\nu(\mathbf{F}) \le \nu$ and $D(\mathbf{F}) \le D$. Now multiple edges are not allowed. The function $f(2, \nu, D)$, that is, the case of graphs, was investigated by several authors ([3], [14], [18]). The determination of $f(2, \nu, D)$ was completed by Chvátal and Hanson [13]. In particular they proved that if $D > 2\nu$, then

$$f(2, \nu, D) = \nu D.$$

Bollobás [9] conjectured that this result has the following extension: suppose r is such that there exists a finite projective plane of order $r-2$, or $r = 2, 3$. If D is sufficiently large and divisible by $r-1$, then

$$f(r, \nu, D) = \frac{r^2 - 3r + 3}{r - 1}\nu D. \tag{4.1}$$

The lower bound in (4.1) is obtained as follows.

EXAMPLE 4.1. Take ν pointwise disjoint projective planes of order $r-2$ (or triangles, or points if $r = 3, 2$) each with $(r-2)^2 + (r-2) + 1 = r^2 - 3r + 3$ points and with $r-1$ points on each line. For each line of each plane take $D/(r-1)$ r-tuples in such a way that each of these r-tuples intersects these projective planes exactly in this line. (If $D > \nu(r^2 - 3r + 3)$, then a point not in these planes has degree less than D.)

Bollobás [10] proved his conjecture for $r = 3$ whenever $D > 72\nu^3$. In general (4.1) was proved in [16]. Here we give a more exact version which is valid if $D/(r-1)$ is not an integer, too.

THEOREM 4.2. *For any given r and ν there exists a real $c(r, \nu)$ such that*

$$\tau^*(r-1, \nu)D - c(r, \nu) \le f(r, \nu, D) \le \tau^*(r-1, \nu)D + c(r, \nu).$$

THEOREM 4.3. *If D is sufficiently large compared to r and ν, and there exists a finite projective plane $PG(2, r-2)$ (or $r = 2, 3$), then*

$$f(r, \nu, D) = N(r-1, \nu, D).$$

Here the value of $N(r-1, \nu, D)$ is

$$\nu\left((r-1)D - (r-2)\left\lceil\frac{D}{r-1}\right\rceil\right)$$

by Theorem 3.3.

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5. Proof of Theorem 3.2

Lower bound. Consider a hypergraph $\mathbf{H}$ of rank r and matching number ν such that $\tau^*(\mathbf{H}) = \tau^*(r, \nu)$. Such a hypergraph exists, and by (2.2) we may suppose that $|E(\mathbf{H})| \leq r\tau^*(\mathbf{H})$. Let $w : E(\mathbf{H}) \to \mathbf{R}^+$ be an optimal fractional matching. Multiply every edge E of $\mathbf{H}$ $\lfloor w(E)D \rfloor$ times. The obtained multihypergraph gives the lower bound.

Note that we obtained that (considering a rational w) equality holds in Theorem 3.2 for infinitely many values of D for any given r and ν.

Upper bound. Let $\mathbf{H}$ be an arbitrary multihypergraph. Then $w(E) = 1/D$ is a fractional matching with value $|E(\mathbf{H})|/D$, and hence we have

$$|E(\mathbf{H})| \leq D\tau^*(\mathbf{H}). \tag{5.1}$$

If $\mathbf{H}$ is an (r, ν, D)-multihypergraph then the right-hand-side of (5.1) is not larger than $D\tau^*(r, \nu)$.

6. Proof of Theorem 3.3

The lower bound for $N(r, \nu, D)$ is given by (3.1). To prove the upper bound let $\mathbf{H}$ be an (r, ν, D)-multihypergraph. Then (2.1) implies that either $\tau^*(\mathbf{H}) \leq \nu(r - 1 + 1/r) - 1/r$, or $\mathbf{H}$ contains ν disjoint $PG(2, r-1)$. In the first case $\mathbf{H}$ has at most $(\tau^*(r, \nu) - 1/r)D$ edges by (5.1), which is less than the left-hand-side of (3.1) for $D > \nu(r-1)^2$. In the latter case $\mathbf{H}$ has no edge which is not a line of a $PG(2, r-1)$. If a line L in a component of $\mathbf{H}$ has multiplicity at least $\lceil D/r \rceil$. Then that component consists of at most

$$\lceil D/r \rceil + \sum_{x \in L} (\deg_{\mathbf{H}}(x) - \lceil D/r \rceil) \leq rD - (r-1)\lceil D/r \rceil$$

edges. Otherwise, if each line has multiplicity at most $\lfloor D/r \rfloor$, then clearly a component of $\mathbf{H}$ has only $\leq \lfloor D/r \rfloor (r^2 - r + 1)$ edges.

7. Proof of Theorems 4.2 and 4.3

The lower bounds for $f(r, \nu, D)$ follow from the trivial inequality

$$f(r, \nu, D) \geq N(r-1, \nu, D),$$

and from Theorem 3.2 which yields

$$N(r-1, \nu, D) > \tau^*(r-1, \nu)D - \tau^*(r-1, \nu)r.$$

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To prove the upper bounds we need a definition and a lemma. The set-system $F_1, \dots, F_k$ is called a Δ-*system* with nucleus N if, for every $1 \le i < j \le k$, we have $F_i \cap F_j = N$. A well-known theorem of Erdős and Rado [14] is as follows.

(7.1) Suppose $r \ge 2$. If the set-system $\mathbf{H}$ is of rank r and $|E(\mathbf{H})| \ge k^r r!$, then it contains a Δ-subsystem consisting of k members.

PROOF OF THE UPPER BOUNDS. We follow the method of [16]. Suppose that the r-graph $\mathbf{F}$ has at most ν disjoint edges and its maximum degree is not more than D. We will prove that $c(r, \nu) \le (r\nu + 1)^r r!$, so without loss of generality we may suppose that $|E(\mathbf{F})| > (r\nu + 1)^r r!$. We define two hypergraphs $\mathbf{N}$ and $\mathbf{F}_0$ and a multihypergraph $\mathbf{F}_N$ with vertex set $V(\mathbf{F})$ as follows. Let $\mathbf{N}$ be a system of nuclei of those Δ-subsystems of $\mathbf{F}$ which contain at least $r\nu + 1$ different edges of $\mathbf{F}$. Clearly $\varnothing \notin E(\mathbf{N})$. Let $\mathbf{F}_0$ be the r-graph obtained from $\mathbf{F}$ by omitting those r-tuples that contain an edge of $\mathbf{N}$. Since $\mathbf{F}_0$ does not contain a Δ-system with $r\nu + 1$ members, we get by (7.1) that

$$|E(\mathbf{F}_0)| \le (r\nu + 1)^r r! . \tag{7.2}$$

Let us associate with each edge $F \in E(\mathbf{F}) - E(\mathbf{F}_0)$ a nucleus $N \in E(\mathbf{N})$ such that $N \subset F$. Denote by $\mathbf{F}_N$ the multihypergraph of the nuclei with these multiplicities, that is, the multihypergraph containing each member of $\mathbf{N}$ as many times as it has been associated. Note that since every member of $\mathbf{N}$ is a nucleus of a Δ-system of size at least $r\nu + 1$, we have $\nu(\mathbf{N}) \le \nu(\mathbf{F})$. Hence

$$\nu(\mathbf{F}_N) \le \nu(\mathbf{N}) \le \nu(\mathbf{F}) \le \nu . \tag{7.3}$$

Obviously,

$$\deg_{\mathbf{F}_N}(p) \le \deg_{\mathbf{F}}(p) \le D \tag{7.4}$$

holds for all vertex p. Apply (5.1) to $\mathbf{F}_N$; then we have by (7.2)–(7.4) that

$$|E(\mathbf{F})| \le D\tau^*(\mathbf{F}_N) + (r\nu + 1)^r r! . \tag{7.5}$$

As the rank of $\mathbf{F}_N$ is at most $r - 1$ we have $\tau^*(\mathbf{F}_N) \le \tau^*(r-1, \nu)$, which implies the upper bound in Theorem 4.2.

Now we prove the upper bound for $f(r, \nu, D)$ in Theorem 4.3 for $D > (r-1)(r\nu + 1)^r r! + \nu(r-1)(r-2)$. We distinguish two cases. Suppose first that

$$\tau^*(\mathbf{F}_N) \le \nu(r-2) + (\nu - 1)/(r-1) . \tag{7.6}$$

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Then (7.5) implies that for large enough D we have

$$|E(\mathbf{F})| \le \frac{r^2 - 3r + 3}{r - 1}\nu D - \frac{D}{r - 1} + (r\nu + 1)^r r! < N(r - 1, \nu, D).$$

If $\tau^*(\mathbf{F}_N)$ is larger than the right-hand-side of (7.6), then $\mathbf{N}$ contains ν pointwise disjoint projective planes of order $r - 2$ by (2.1). Then every r-tuple of $F \in E(\mathbf{F})$ contains a line of one of these planes, since otherwise we can find ν disjoint edges of $\mathbf{F}$ which are disjoint from F as well. That is, $E(\mathbf{F}_0) = \varnothing$. Then

$$|E(\mathbf{F})| = |E(\mathbf{F}_N)| \le N(r - 1, \nu, D).$$

8. Problems

(8.1) Clearly, $N(r, \nu, D) \ge \nu N(r, 1, D)$ and, by Theorem 3.3, equality holds if a $PG(2, r - 1)$ exists (at least whenever D is large). One can think that here equality holds for all r.

(8.2) The following is a slightly weaker conjecture than (8.1): for all r one has $\tau^*(r, \nu) = \nu\tau^*(r, 1)$.

(8.3) If $D \ge 3$ then $f(2, \nu, D) \ge \lfloor \frac{1}{2}(2\nu + 1)D \rfloor = \nu D + \lfloor D/2 \rfloor > \nu f(2, 1, D) = \nu D$. But one can think that in the case $r \ge 3$ there exists a $D_0 = D_0(r)$ such that for all ν, r and $D > D_0$ we have $f(r, \nu, D) = \nu f(r, 1, D)$.

(8.4) The *chromatic index* of a (multi)hypergraph $\mathbf{H}$ is the smallest integer $q = q(\mathbf{H})$ such that one can decompose $E(\mathbf{H})$ into q matchings. It is well-known that, for a 2-graph $\mathbf{G}$,

$$D \le q(\mathbf{G}) \le D + 1,$$

and for a 2-multigraph $\mathbf{G}$,

$$q(\mathbf{G}) \le \lceil \tfrac{3}{2} D \rceil.$$

(These are due to Vizing and Shannon, respectively. See, for example, [11].) Find the analogy of these theorems for r-(multi)hypergraphs. This question was proposed by Faber and Lovász [15] in 1972.

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Mathematical Institute
Hungarian Academy of Sciences
1364 Budapest POB 127
Hungary

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