30, ruling out $s = 60, 120$. Similarly one can rule out $s = 20$ (and its multiple $s = 40$) and $s = 16$ (and its multiple $s = 48$). This leaves the seven values

$$s = 2, 4, 6, 8, 10, 12, 24,$$

and, for each of these, $\cos(2\pi / s) = \cos(\pi / b)$ does indeed have surd form. Thus $\cos(k\pi / b)$ has surd form if. and only if, $b = 1, 2, 3, 4, 5, 6, 12$. This proves that the only rational multiples of π between 0 and $\pi / 2$ whose cosines have surd form are $\pi / 3$, $\pi / 4$, and $\pi / 6$ (whose cosines are well known) together with

$$\begin{aligned} \cos(5\pi / 12) &= \sin(\pi / 12) = (\sqrt{6} - \sqrt{2}) / 4, \\ \cos(2\pi / 5) &= \sin(\pi / 10) = (\sqrt{5} - 1) / 4, \\ \cos(\pi / 5) &= \sin(3\pi / 10) = (\sqrt{5} + 1) / 4, \\ \text{and} \quad \cos(\pi / 12) &= \sin(5\pi / 12) = (\sqrt{6} + \sqrt{2}) / 4. \end{aligned}$$

If $\theta + \phi = \pi / 2$, then $\sin\phi = \cos\theta$ and ϕ is acute if, and only if, θ is acute. So Bob Burn had already got a complete list of the angles he sought.

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DEAR EDITOR,

An alternative form for χ^2.

The usual formula for χ^2 is: $X^2 = \sum \frac{(O_i - E_i)^2}{E_i}$. Expanding the brackets in the usual way and remembering that both the O_i and E_i sum to the total frequency, the following is obtained:

$$X^2 = \sum \frac{O_i^2}{E_i} - \text{total frequency}.$$

I recently came across a student using this formula, so clearly it is not new, but I thought it might not be well known.

Yours sincerely,

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