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ARTICLE

# Cycles and their important shocks: completing the investigation

Max Gillman[1,2,3] and Adrian Pagan[4]

[1]Department of Economics, University of Missouri-St. Louis, St. Louis, MO, USA
[2]Corvinus University of Budapest, Budapest, Hungary
[3]European Research University, Havirov, Czech Republic
[4]School of Economics, University of Sydney, Camperdown, Australia
**Corresponding author:** Max Gillman; Email: gillmanm@umsl.edu

**Abstract**

The paper looks at the question of measuring the importance of shocks to cycles. We consider two types of cycles - oscillations and those summarized by the NBER that require a study of growth in activity to establish turning points in the level of activity. The latter demarcate expansions and contractions. We establish a connection between these two concepts of cycles that shows shocks may have very different effects on each. As an application we look at a question that has often been asked over how important technology shocks are to cycles in activity? Some recent research concludes that total factor productivity (TFP) shocks are not important for oscillations and therefore models should be designed to reflect that. Using the same data we show that TFP shocks are very important to both types of cycles.

**Keywords:** Band pass filter; oscillations; business cycles; main shock

## 1. Introduction

Researchers in economics have long been interested in cycles and what drives them. Originally it was cycles in macroeconomic variables such as unemployment and production, but that broadened to investigations involving hog prices and production, equity prices, inventories, commodities, etc. In the beginning the focus was on describing the cycles, principally by graphical means. This involved locating periods in which the variable whose cycle was being investigated was either in an "up" or "down" state. In the past 70 years the focus has broadened to trying to answer the question of what accounts for these ups and downs that is cycles. Increasingly, the answer has been shocks. That then raises the question of what are the most important shocks for a cycle? To answer that one needs to construct a series from the original data that will capture the cycle and after that one needs to describe what the connection between this series and the shocks is. For that one needs to give some definition of a shock, and this is often done via a model. Thus, one often sees reference to technology, monetary, sentiment, etc., shocks, and some model is used to define them.

So what variable would one select if the focus is on macroeconomics and one wants to look at (say) cycles in economic activity? Early statistical work thought of a series representing activity as being the sum of trend plus cycle that is the "cycle" was present in the *deviation* of the variable ($y_t$) from a "trend," and so one wanted to construct a variable from the data, $y_t$, that eliminated a "trend." When the trend was deterministic a measure of the extent of the "ups" and "downs"—fluctuations—was the volatility of this detrended series. Once it was realized that there could be both stochastic and deterministic trends, in order to fit this definition the cycle was taken as present in the *deviation of $y_t$ from its permanent component*. Denoting this deviation by $\tilde{y}_t$ it is the transitory component of $y_t$.

Now the fundamental difficulty with this is that there is no unique permanent component, and so one needed to be more specific in its description. A popular approach was to assume that the trend followed an $I(2)$ process and was smooth. This was Hodrick and Prescott's (HP) solution, and it gave rise to a series $\tilde{y}_t^{HP}$ that would be the transitory component of $y_t$ with HP's specification of the trend. $\tilde{y}_t^{HP}$ could then be constructed using the HP filter. Another proposal was to difference $y_t$ to eliminate an $I(1)$ permanent component so that the cycle series would then be $\tilde{y}_t^D = \Delta y_t$. Objections were made to both of these approaches. It was shown that, if $y_t$ was $I(1)$, then $\tilde{y}_t^{HP}$ could have a peak in its spectrum that is a "spurious cycle" had been created (since the pure $I(1)$ process had none). The objection to using $\tilde{y}_t^D$ was not that any spurious cycle eventuated but that the high frequencies in $y_t$, often described as noise, would be boosted.

Early on with real business cycle (RBC) models $\tilde{y}_t^{HP}$ was the preferred series and $\tilde{y}_t^D$ was increasingly disregarded, although often the statistics of both, such as the variance, were still presented. Probably because of the spurious cycle issue with HP there has been an increasing tendency to construct a transitory component of $y_t$ by eliminating a permanent component and capturing only the *transitory component* that comes from *oscillations* between certain frequencies, such as 6 and 32 quarters (for quarterly data this range of frequencies is $\frac{\pi}{3}$ to $\frac{\pi}{16}$). This series is found by applying a band-pass (BP) filter to the $y_t$ to recover that component. In line with the above nomenclature this series will be called $\tilde{y}_t^{BP}$. Of course this does not exhaust the range of filters which might be applied to data $y_t$ to eliminate a permanent component and to produce a cycle series that is of interest. Thus there are filters that produce a series that capture "medium term cycles," that is they aim to capture a broader range of oscillations than $\frac{\pi}{3}$ to $\frac{\pi}{16}$, for example, Comin and Gertler (2006). There is also another filter applied to $y_t$ which yields a transitory component—the Beveridge and Nelson (1981) filter—$\tilde{y}_t^{BN}$. Because all these filters produce a common structure, we will focus upon the BP one that isolates oscillations between $\frac{\pi}{3}$ to $\frac{\pi}{16}$, as that seems to be the most commonly used approach today.

Now there is another alternative approach to cycles exemplified in Burns and Mitchell (1946) and the National Bureau of Economic Research (NBER) work on dating business cycles. This looks at "ups" and "downs" in $y_t$ that is the statistics they publish on what they term the business cycle relate to *the levels* of the series $y_t$.[1] As they say "The chronology identifies the dates of peak and trough months in economic activity. The peak is the month in which a variety of economic indicators reach their highest level, followed by a significant decline in economic activity. Similarly, a month is designated as a trough when economic activity reaches a low point and begins to rise again for a sustained period."[2]

These peaks and troughs are local maxima and minima in the series on economic activity. Therefore, in their work the business cycle reflects variation in the *level* of economic activity, and such variations may come from movements in either the permanent or transitory component of $y_t$. Peaks and troughs are *turning points* in economic activity. Turning points are what one sees in graphs of data and so it just formalizes what would have been visually apparent to earlier researchers. Once located, the turning points are used to describe *expansion* and *contraction* phases in the level of economic activity. Since turning points are just local maxima and minima of a series we know from elementary calculus that these are located by studying the *changes in the signs* of $\Delta y_t$. Hence to locate turning points, as the NBER do, we need to study $\tilde{y}_t^D$.

Unlike the early work on cycles mentioned above, *differencing is not being done to isolate a transitory component in $y_t$ but to capture the characteristics of the expansions and contractions in $y_t$*. As seen above, the NBER call the cycle seen in the levels of $y_t$ (which depends on the nature of $\tilde{y}_t^D$) the business cycle, and the turning points are published on their web page. The problem we face here is that the description "business cycle" is also used by writers to describe the behavior of $\tilde{y}_t^{BP}$. To make it clear what cycle is being referred to we will term what the NBER study, that is "ups" and "downs" in $y_t$, as the *activity cycle,* as it is what they are dating. We will refer to a study of $\tilde{y}_t^{BP}$ as looking at an *oscillations cycle*. The names are chosen to avoid any controversy over what is the "real" business cycle.

Section 2 looks at the connection between $\tilde{y}_t^{BP}$ and $\tilde{y}_t^{D} = \Delta y_t$, as the two main representatives of work on cycles today. It shows that the series $\tilde{y}_t^{BP}$ is a weighted average of the history of $\tilde{y}_t^{D}$ and not just the contemporaneous value. In general, the impact of a shock upon $\tilde{y}_t^{BP}$ need not be the same as that upon $\tilde{y}_t^{D}$, and so the impact of shocks on oscillations can be different to that on the level of activity. There seems no reason to focus solely upon $\tilde{y}_t^{BP}$ and to ignore results regarding shocks on $\tilde{y}_t^{D}$. Indeed, given the fact that NBER identification of recessions and expansions is so well ingrained in macroeconomic discussion, it would seem obvious that one would want to consider what the impact of shocks would be on $\tilde{y}_t^{D}$, as well as upon $\tilde{y}_t^{BP}$. That is the sense in which we are completing the investigation.

After settling on a variable to represent the cycle one then has to decide on some characteristic of that variable which can be used to answer the question regarding the importance of shocks. A popular one has been to ask how the shocks affect the variances of $\tilde{y}_t^{BP}$ and $\tilde{y}_t^{D}$. If a shock has a large contribution to the variance of these quantities it is deemed important. That is, a large contribution of a shock to the variance of $\tilde{y}_t^{BP}$ will make it important to the oscillatory cycle, while a large contribution to the variance of $\tilde{y}_t^{D}$ will mean it is important to the activity cycle. Because these series are different it is possible that a shock can be important to one cycle but not the other. Section 3 deals with this and outlines a method to measure the contribution to the variance of any series aiming to capture cycles that is $\tilde{y}_t^{BP}$ or $\tilde{y}_t^{D}$, that does not depend on how it is defined.

Section 4 looks at an application which studies the relative importance of technology shocks to the oscillatory and activity cycles using the data on total factor productivity (TFP) and nine other variables in Angeletos, Collard, and Dellas (2020) (ACD). ACD used this data to produce some influential work which concluded that "the data speak against theories that attribute the bulk of the business cycle to any of the following forces: technology shocks; financial, uncertainty, and other shocks that matter primarily by affecting aggregate TFP; news about medium to long-run productivity prospects; and inflationary demand shocks" (p. 3062). A core part of their methodology was to begin with a recursive structural vector autoregression (SVAR). This produces uncorrelated structural shocks. They then combine these together to produce a new set of structural shocks and ask whether a particular combination of them has a substantial effect on TFP. This particular combination is termed the "main shock." They find that the "main shock" has little influence on TFP.

This raises the issue of the exogeneity of TFP. In studies with DSGE models TFP shocks are generally taken to be strongly exogenous that is they are not influenced by other shocks reflecting demand and nominal magnitudes. So the appropriate question would seem to be *how much of the cycle is due to TFP shocks* rather than *how much of TFP is explained by what is termed the main shock*. To do this we place TFP first in a recursive SVAR, thereby enabling us to ask how much of the volatility of output is due to TFP shocks. We find that TFP is a major driver of the oscillatory cycle and a substantial driver of the activity cycle. Consequently, TFP shocks are important to both cycles and need to be allowed for in any model explaining cycles.

Finally, the structure employed in Section 4 (and by ACD) assumes the data is generated by a SVAR process. Because we have assumed that TFP is exogenous we can estimate its impact upon either the oscillatory or activity cycle by using a local projections approach that does not require a specific model class like a vector autoregression (VAR). This is done in Section 5 to check the robustness of our findings about the role of TFP shocks. We find that the conclusions of Section 4 are robust. TFP matters. Section 6 concludes.

## 2. Relations between the series used for cycle analysis

As we have set out we refer to the use of $\tilde{y}_t^{BP}$ as isolating an oscillatory cycle for study and $\tilde{y}_t^{D}$ as focussing upon an activity cycle. Let us look at oscillatory cycles. These work with a series $\tilde{y}_t^{BP}$

that capture oscillations in $y_t$. For this paper the oscillations are taken to range between 6 and 32 quarters. $\tilde{y}_t^{BP}$ is constructed by applying a BP filter to $y_t$. This has the form $\tilde{y}_t^{BP} = W(L)y_t$, where $W(L) = \sum_{j=-M}^{M} w_j L^j$, and the weights $w_j$ depend on the range of frequencies one wants.[3]

There are a number of BP filters. In Baxter and King (1999) $M$ is fixed to some value. Christiano and Fitzgerald (2003) have two versions depending on whether an $M$ is set or it changes as one gets closer to the end points of the data. This is examined in Kulish and Pagan (2021). To look at the nature of the filter in a simple context take the Baxter–King version and set $M = 3$. Then the weights are

$$\omega_0 = 0.165988, \omega_{\pm 1} = 0.108719, \omega_{\pm 2} = -0.027920, \omega_{\pm 3} = -0.163794$$

It is clear that $\sum_{k=-M}^{M} \omega_j = 0$ and this is true of any $M$ and for all BP filters.[4] It is also the case that $\omega_{-j} = \omega_j$. Therefore

$$\tilde{y}_t^{BP} = \omega_0 y_t + \omega_1 y_{t-1} + \omega_2 y_{t-2} + \omega_3 y_{t-3} + \omega_1 y_{t+1} + \omega_2 y_{t+2} + \omega_3 y_{t+3} \tag{1}$$

$$= \left(\sum_{j=-3}^{3} \omega_j\right) y_t - (\omega_1 \Delta y_t + \omega_2 \Delta_2 y_t + \omega_3 \Delta_3 y_{t-3}) + \omega_1 \Delta y_{t+1} + \omega_2 \Delta_2 y_{t+2} + \omega_3 \Delta_3 y_{t+3}, \tag{2}$$

where we have replaced $y_{t-j}$ in (1) with $y_t - \Delta_j y_t$ and $y_{t+j}$ with $y_t + \Delta_j y_{t+j}$. Now, as seen above, $\sum_{k=-3}^{3} \omega_j = 0$. The reason why this must be true is that $\tilde{y}_t^{BP}$ has to be $I(0)$ as it is transitory, and, if $\sum_{k=-3}^{3} \omega_j \neq 0$, there would be a permanent component left in it whenever $y_t$ was $I(1)$.

Subsequently, recognizing that $\Delta_j y_t = \sum_{k=0}^{j-1} \Delta y_{t-k}$, we get

$$\begin{aligned}\tilde{y}_t^{BP} &= -(\omega_1 \Delta y_t + \omega_2 \Delta y_t + \omega_2 \Delta y_{t-1} + \omega_3 \Delta y_t + \omega_3 \Delta y_{t-1} + \omega_3 \Delta y_{t-2}) \\ &\quad + \omega_1 \Delta y_{t+1} + \omega_2 \Delta y_{t+2} + \omega_2 \Delta y_{t+1} + \omega_3 \Delta y_{t+3} + \omega_3 \Delta y_{t+2} + \omega_3 \Delta y_{t+1} \\ &= -(\omega_1 + \omega_2 + \omega_3)\Delta y_t - (\omega_2 + \omega_3)\Delta y_{t-1} - \omega_3 \Delta y_{t-2} \\ &\quad + (\omega_1 + \omega_2 + \omega_3)\Delta y_{t+1} + (\omega_2 + \omega_3)\Delta y_{t+2} + \omega_3 \Delta y_{t+3},\end{aligned}$$

showing that $\tilde{y}_t^{BP}$ is a weighted average of $\Delta y_{t\pm j}$. We could write this in lag operator form as

$$\tilde{y}_t^{BP} = (-b_1 + b_1 L^{-1} - b_2 L + b_2 L^{-2} - b_3 L^2 + b_3 L^{-3})\Delta y_t \tag{3}$$

$$= B(L)\Delta y_t, \tag{4}$$

where

$$\begin{aligned} b_1 &= \omega_1 + \omega_2 + \omega_3 \\ b_2 &= \omega_2 + \omega_3 \\ b_3 &= \omega_3. \end{aligned}$$

One can see the pattern here to $b_j$. The above decomposition holds for any $M$ since it just involves replacing $y_{t\pm j}$ by a combination of $y_t$ and $\Delta y_{t\pm j}$. Accordingly, for any $M$, $\tilde{y}_t^{BP}$ will be constructed from $\Delta y_{t\pm j}$, and it is clear that the impact of a shock upon $\tilde{y}_t^{BP}$ (the oscillatory cycle) and $\Delta y_t$ (the activity cycle) can be quite different.

To capture the magnitude of any difference we would need a measure of the influence of shocks upon either $\tilde{y}_t^D$ or $\tilde{y}_t^{BP}$. The variance of $\tilde{y}_t^D$ or $\tilde{y}_t^{BP}$ has been a widely used index, particularly in the form of a variance decomposition of $\tilde{y}_t^D$ or $\tilde{y}_t^{BP}$ into the contributions from uncorrelated shocks. That gives a ranking of the importance of each of the shocks.

To see that there is a difference between the contributions of shocks to the variances of $\tilde{y}_t^D$ or $\tilde{y}_t^{BP}$ suppose that

$$\Delta y_t = 0.5\eta_{1t} + 0.5\eta_{2t-1} + 0.4\eta_{2t-2} + 0.3\eta_{2t-3} \tag{5}$$
$$= \Delta y_{1t} + \Delta y_{2t}, \tag{6}$$

where $\eta_{jt}$ are $n.i.d.(0, 1)$ uncorrelated shocks, $\Delta y_{1t} = 0.5\eta_{1t}$ and $\Delta y_{2t} = 0.5\eta_{2t-1} + 0.4\eta_{2t-2} + 0.3\eta_{2t-3}$. We might think of $\eta_{1t}$ as a supply side shock and $\eta_{2t}$ as a demand shock. The latter takes time to affect $y_t$. Notice that the data generating process in (5) can be converted to one for $\tilde{y}_t^{BP}$ by using (4)

$$\tilde{y}_t^{BP} = B(L)\Delta y_t = B(L)(\Delta y_{1t} + \Delta y_{2t}) = \Delta\tilde{y}_{1t}^{BP} + \Delta\tilde{y}_{2t}^{BP}.$$

Then the contribution of the first shock to $var(\tilde{y}_t^D)$ is 33%, since it equals $var(\Delta y_{1t})/var(\Delta y_t)$. Its contribution to $var(\tilde{y}_t^{BP})$ equals $var(\Delta\tilde{y}_{1t}^{BP})/var(\tilde{y}_t^{BP})$ and is 17.5% that is the supply side shock is far more important to the activity cycle than to oscillations. It should not be surprising that the contribution of $\eta_{1t}$ to the two variances will be different, as that for $\tilde{y}_t^{BP}$ depends on the parameters in $B(L)$ and that for $\tilde{y}_t^D$ does not. Whether this holds with the same force in actual data is what we will ask later.

How is one to compute a quantity like $var(\Delta\tilde{y}_{1t}^{BP})$? One possibility is to note that it is $var(0.5B(L)\eta_{1t})$ and this equals $0.5\sum_{j=1}^{\infty} b_j^2$. Another is to just simulate a long series on $y_{1t}$ from $\Delta y_{1t} = 0.5\eta_{1t}$, and then compute the variance of that simulated series. The latter is what we do. The reason for this choice lies in the fact mentioned earlier that the most widely used form of BP filter is that of Christiano and Fitzgerald (2003) and, with it, the weights $B(L)$ depend on where in the sample one is getting the estimate. So $\tilde{y}_1^{BP}$ and $\tilde{y}_{10}^{BP}$, for example, are computed with different $B(L)$, as one is at the start of the sample and the other within it. Kulish and Pagan (2021) discuss this. Thus it is simpler to simulate data from $\Delta y_{1t} = 0.5\eta_{1t}$ and then BP filter it with the Christiano and Fitzgerald filter to get $\tilde{y}_{1t}^{BP}$ and its variance than to derive an analytic form of the variance. Accordingly, we simulated 15,000 observations on $\Delta y_{1t}$, $\Delta y_t$ using (5) and then BP-filtered these variables over the 6–32 quarter oscillations to get $\tilde{y}_{1t}^{BP}$ and $\tilde{y}_t^{BP}$.

There is another way to get $var(\tilde{y}_{1t}^{BP})$. That comes from observing that this will be the area between the frequencies $\frac{\pi}{16}$ to $\frac{\pi}{3}$ (6 and 32 quarters) of the spectrum of $\tilde{y}_{1t}^{BP}$. This is used by ACD and is a frequency domain method. They are equivalent in large samples but we feel that the time domain method is simple to use and fits with what is generally done in macroeconomics.

To appreciate the issues regarding the two types of cycles a little more take quarterly per capita US Gross Domestic Product (GDP) as $y_t$ and date turning points in it. One finds that expansions would be 21.5 quarters long and contractions 4.3.[5] Magnitudes like this are familiar from NBER Business Cycle dating for US activity. Now suppose one looked at turning points in $\tilde{y}_t^{BP}$. We would find that the average duration between peaks would be 11.7 quarters so that is the length of the average cycle. Because the oscillations that are in $\tilde{y}_t^{BP}$ are between 6 to 32 quarters that might suggest the average would be 20 quarters. However, as set out in Kulish and Pagan (2021) and Harding and Pagan (2016), this is incorrect. To see why, suppose one had a series composed of the sum of an oscillation of six quarters and 24 quarters. Then their sum would have a turning point cycle of 6 quarters, simply because there are more turning points in the first oscillation than the second.

If we find the turning points in $y_t$ then there will be a number of peaks and troughs, say $N$. In NBER's description a cycle is a movement from one peak to the next peak so there will be $N$ cycles with durations $d_j$ $(j = 1, \ldots, N)$ (the period of time taken to go from one peak to the next). These durations were the basis of the 6–32 quarters parameters chosen by many researchers for BP filtering. Over the period 1955/1–2017/4 the durations for cycles in real per capita GDP $(y_t)$ range from 7 to 42 quarters. In contrast, the BP-filtered series $\tilde{y}_t^{BP}$ had turning points with a range of durations of 6–17 quarters, and there are none of length 32 quarters. Consequently, the two cycles coming from $\tilde{y}_t^{BP}$ and $\tilde{y}_t^{D}$ are likely to be different and to study just one of them seems to constitute an incomplete analysis, particularly given that the information from $\tilde{y}_t^{D}$ deals with NBER-type cycles.

## 3. Influence of shocks on cycles

Now one needs to express the series $y_t$ we wish to work with in terms of shocks, just as in (5). This might involve estimating a VAR. After estimating such a model one will have the same structural moving average representation as in (5), but now $y_t$ is an $n \times 1$ vector of variables:

$$y_t = F(L)\eta_t, \tag{7}$$

where $\eta_t$ is an $n \times 1$ vector of shocks that are $N(0, I_N)$. The contribution of each shock $\eta_{jt}$ to $\Delta y_t$, $\Delta y_t^{(j)}$, will be

$$\Delta y_t = \sum_{j=1}^{n} \Delta y_t^{(j)} = \sum_{j=1}^{n} (1-L)F_j(L)\eta_{jt} \tag{8}$$

$$= \sum_{j=1}^{n} \Phi_j(L)\eta_{jt} \tag{9}$$

$$= \sum_{j=1}^{n} \tilde{y}_t^{D(j)} \tag{10}$$

If one wants to look at NBER-type cycles in $y_t$ it is necessary to study $\tilde{y}_t^{D} = \Delta y_t$. Alternatively, if one wants to study the effect of a shock on a range of oscillations in $y_t$, applying the BP filter formula from (4) gives

$$\tilde{y}_t^{BP} = B(L)\Delta y_t \tag{11}$$

$$= \sum_{j=1}^{n} B(L)\Delta y_t^{(j)}$$

$$= \sum_{j=1}^{n} \tilde{y}_t^{BP(j)}. \tag{12}$$

Thus we have expressions for how any shock affects the two quantities involved in the different type of cycles.

Just as for the simple example of the last section, in order to assess the role of the $j'$th shock in the variance of 6–32 quarter oscillations requires the computation of $var(\tilde{y}_t^{BP(j)})/var(\tilde{y}_t^{BP})$. The same is true for the activity cycle, but now with $var(\tilde{y}_t^{D(j)})/var(\tilde{y}_t^{D})$ . To get these one needs $\tilde{y}_t^{D(j)}$ and $\tilde{y}_t^{BP(j)}$. As mentioned earlier a simple way to do that, given a $F(L)$ has been found by estimating the model used to isolate shocks, is to simulate a long series on $y_t$ and then either BP filter it to get $\tilde{y}_t^{BP}$ or difference it to get $\tilde{y}_t^{D}$. This will give us $var(\tilde{y}_t^{BP})$ and $var(\tilde{y}_t^{D})$. Afterward, performing another long simulation (and the same random numbers) where all shocks are set to zero, except for the $j'$th, provides $\tilde{y}_t^{BP(j)}$ and $\tilde{y}_t^{D(j)}$, from which their variances can be found. So we can compute the contribution of each shock to the oscillatory and the activity cycle simply by simulating enough observations. This makes any comparison of results a relatively simple thing to do.

Mostly the shocks $\eta_t$ are recovered by fitting a DSGE or SVAR model to the data $y_t$. These $\eta_t$ are generally given names. However there are cases where (say) the SVAR has shocks which might be described as technology, demand, etc., but these are combined in some way to produce an alternative set $\tilde{\eta}_t$ with names such as transitory or permanent, and then it is loosely said that the former are demand and the latter are supply.

In the same vein a recent proposal by Angeletos et al. (2020) has been to combine the basic shocks $\eta_t$ to produce what is termed a "main shock" $\tilde{\eta}^*_{jt}$, which would be the $j'$th one of $\tilde{\eta}_t$. Its construction begins with a recursive SVAR for a set of variables $y_t$ so as to generate shocks $\eta_t$. The latter are not given names but are simply a set of uncorrelated shocks. Then a new set of shocks $\tilde{\eta}_t$ is produced that are constructed as linear combinations of the $\eta_t$ that is $\tilde{\eta}_t = Q\eta_t$, where $Q$ is orthonormal that is $Q'Q = I = QQ'$. This choice of $Q$ ensures that the $\tilde{\eta}_t$ are uncorrelated with each other, just as the $\eta_t$ were. Consequently, we have the equivalent representation to (7)

$$y_t = F(L)\eta_t = F(L)QQ'\eta_t = \tilde{F}(L)\tilde{\eta}_t. \tag{13}$$

Now there are many orthonormal $Q$. To complete the analysis one needs a rule to settle on a single $Q$. To do this ACD propose that one of the $y_t$, (say) $y_{2t}$, is selected as a *target variable.* That variable can be written in terms of the $\tilde{\eta}_t$ as

$$y_{2t} = \tilde{F}_2(L)\tilde{\eta}_t.$$

This variable might be (say) unemployment. Applying the BP filter to produce the contribution to unemployment between 6 and 32 quarters will mean

$$\begin{aligned} \tilde{y}^{BP}_{2t} &= B(L)y_{2t} = B(L)\tilde{F}_2(L)\tilde{\eta}_t \\ &= \sum_{j=1}^{n} \Psi_j(L)\tilde{\eta}_{jt}. \end{aligned}$$

Given any $Q$ the contribution to $var(\tilde{y}^{BP}_{2t})$ of each of the shocks $\tilde{\eta}_{jt}$ can be determined. ACD vary $Q$ until they find one of the shocks (say $\tilde{\eta}_{kt}$, the $k'$th one of $\tilde{\eta}_t$) that maximizes the contribution to the volatility of the target variable.[6] Let this $Q$ be $Q^*$ with corresponding shocks $\tilde{\eta}^*_{jt}$. They then term this shock $\tilde{\eta}^*_{kt}$ the "main shock."

Although this establishes a "main" shock the variable whose cycle we wish to look at can be different from employment. It might, for example, be output. Let us call this variable of interest $z_t$. It will be one of the series in $y_t$ so we write $z_t = Sy_t$, where $S$ is a selection matrix. Therefore

$$z_t = S\tilde{F}^*(L)\tilde{\eta}_t = \sum_{j=1}^{n} \Phi_j(L)\tilde{\eta}_{jt} \tag{14}$$

and so one has an expression for $z_t$ in terms of the shocks $\tilde{\eta}_{jt}$ that ACD use, one of which is their "main shock." One can then ask how important is this constructed shock in explaining a cycle in output that is $z_t$. Since there are two cycles that can be examined for oscillations and activity, we form $\tilde{z}^{BP}_t$ and $\tilde{z}^D_t$ from $z_t$. ACD describe $\tilde{\eta}^*_{kt}$ as the main business cycle shock (MBS). However, because one can find a main shock with either $\tilde{z}^D_t$ or $\tilde{z}^{BP}_t$ using the method described above, we will distinguish between the *main oscillatory cycle shock (MOC)* found from $\tilde{z}^{BC}_t$, and the *main activity cycle shock (MAC )* found from $\tilde{z}^D_t$.

To derive the fraction of the variance of $\tilde{z}^{BC}_t$ due to any shock ACD work in the frequency domain with a spectral decomposition over $\frac{\pi}{16}$ to $\frac{\pi}{3}$, whereas, we do it by simulating a long history on $z_t$ and then computing $var(\tilde{z}^{BP}_t)$ from that with an appropriate BP filter. ACD argue that there is a difference between working in the frequency and time domain, as they found that targeting the volatility of a variable over 6–32 quarters did not produce the same shock as maximizing the forecast error variance decomposition (FEVD) over an horizon of 6–32 periods. In their work it turns out that maximizing the FEVD over a 4 quarter horizon is a better approximation. This however is not about the equivalence of the two domains, but rather about what the appropriate horizon of a FEVD would be to capture cycles. Pagan and Robinson (2014) showed why one would not look at the FEVD for long horizons when looking at turning points in the activity cycle. The rule for dating turning points which gives a good match to the NBER quarterly cycle results involves looking at the signs of variables that are combinations of $\Delta z_{t-2}$, $\Delta z_{t-1}$, $\Delta z_t$, $\Delta z_{t+1}$, $\Delta z_{t+2}$ that is it involves only the same information as an FEVD over a 4-period horizon, and not the 40 periods that is often used.

## 4. An example: are technology shocks important for cycles?

This might seem a strange question given the presence of technology shocks in almost all DSGE and macroeconomic models analyzing cycles. McGrattan (2020), for example, shows TFP shocks involving both tangible and intangible capital can resolve many puzzles in the literature about consumption and labor components. Models that focus on endogenous growth and human capital investment also rest upon TFP as a key shock driving the economy. What is interesting then is that ACD conclude that the "main shock" they isolate for the oscillatory cycle is not a TFP shock and that would suggest a lessened role for TFP shocks in driving cycles. So we want to look at this for both the oscillatory and activity cycles when TFP shocks are taken to be exogenous, so that the "main shock" has no effect on TFP and is not constructed from a TFP shock.

We work with ACD's data on the variables—the unemployment rate, logs of real per capita levels of GDP, investment and consumption, hours worked per person, labor productivity in the nonfarm business sector, utilization adjusted TFP, the labor share, the rate of change in the GDP deflator; and the Federal Funds rate. Like ACD we begin with a SVAR(2), but place TFP as the first variable in the system, followed by a recursive structure for the remaining variables in the order listed above. Then the TFP shock can be recovered from the first equation of the SVAR (the one that has $y_{1t}$ as dependent variable), as none of the other current shocks drive TFP.[7]

ACD include in their SVAR(2) a constant and a deterministic time trend and we do the same. One view of this is that the data is stationary around trend and so a variance decomposition can be performed to find the contribution of TFP shocks. Doing so shows that 51.4% of the variance of detrended GDP is accounted for by TFP shocks. On that basis TFP seems very important. However, care needs to be exercised since it may be that the log of GDP is not a stationary process, in which case computing a variance would be meaningless. Consequently, we move to look at the oscillatory and activity cycles, as these focus on variables $\tilde{y}_t^D$ and $\tilde{y}_t^{BP}$ whose variance exists when $y_t$ has a stochastic as well as deterministic trend.

### 4.1 What drives the oscillatory cycle?

In the recursive SVAR there are ten shocks, $\eta_t$, the first one, $\eta_{1t}$, being the TFP shock. To construct a "main shock" one selects a target variable (we choose unemployment as that was what ACD used in the body of their paper) and combine the nine SVAR shocks (not TFP) using generated $Q$ matrices. This produces nine constructed shocks $\tilde{\eta}_{jt}, = 2, \ldots, 10$. Given a $Q$ we can determine which of these nine shocks contributes most to the volatility of the BP-filtered target. For one $Q$, $Q^{(1)}$, it might be the second shock, and the contribution might be 50%. For another $Q$, $Q^{(2)}$, it could be the third shock, and the fraction might be 45%. Many $Q^{(j)}s$ are generated and the $Q^*$ that produces the absolute maximum for the share of the volatility of the target variable is selected. That $Q^*$ might be associated with the fourth of the shocks $\tilde{\eta}_t$ and that would then be called the "main shock." It is just a combination of the original SVAR shocks (excluding that for TFP).

Using the estimated SVAR(2) simulated data we generated 5000 $Q$s and found that the values defining the main shock gave substantially higher weights to the unemployment and GDP equation shocks in the corresponding SVAR(2) equations than the other seven SVAR(2) shocks, being 0.8420 and -0.4206. Thus the MOC shock might be interpreted as a negative demand shock which is consistent with what ACD's interpretation of it was based on its impulse responses.

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Now from (8) and (11) any variable of interest $z_t$ equaling $\tilde{y}_t^D$ or $\tilde{y}_t^{BP}$ can be written as the sum of moving averages of the shocks $\eta_t$. Therefore for any given $Q$ we can write the relevant $z_t$ as a moving average of the $\tilde{\eta}_t$. That is, once we have chosen the $Q^*$ that produces the main shock, $z_t$ will be a moving average of the TFP shock, the "main shock" and the "remaining" eight shocks. Because all the $\eta_t$ are uncorrelated with each other the variance of $z_t$ can be decomposed into the fractions due to each type of shock.

It is found for ACD's data set that the MOC accounts for 49% of the BP-filtered unemployment variance and 47% for GDP. TFP shocks account for 17% and 18% of the same two variables. They are the second biggest shock after the MOC. Consequently, although the MOC is certainly more important than TFP shocks for oscillations in GDP, the latter still has a major role in cycles and building models that exclude them seems unwise.

### 4.2 What drives the activity cycle?

We follow the same process as above but now work with $z_t = \Delta y_t$ rather than $\tilde{y}_t^{BP}$. The MAC accounts for 52% and 38% of the volatility of the unemployment rate change and the growth in GDP, respectively. TFP shocks account for 13% and 14%. Both the contribution of the main shock and TFP shocks for GDP growth are lower than for the oscillatory cycle. Indeed, the "remaining" shocks collectively account for more than either of these shocks. Of course these are a collective and not a single shock but it points to the fact that many shocks are needed to fully explain the activity cycle, one of which is TFP. TFP is the second biggest shock after the MAC.

## 5. A local projections approach to robustness of the conclusion about TFP shocks

In the subsections above, it was assumed that $y_t$ evolved as a VAR(2) and the contribution of TFP was recovered by simulating that. The VAR assumption was needed to determine the "main shock." However, this is not needed to determine the contribution of TFP shocks to cycles. The reason is that with TFP being exogenous a variable such as $\tilde{y}_t^D = \Delta y_t$ can be written as a moving average of the TFP shocks $\eta_{1t}$ and the contributions of other shocks

$$\tilde{y}_t^D = F_1(L)\eta_{1t} + u_t, \tag{15}$$

where $u_t$ is uncorrelated with $\eta_{1t}$, owing to the latter being uncorrelated with all leads and lags of the TFP shock. Now $\tilde{y}_t^D$ can be regressed against TFP shocks $\eta_{1t-j}$ to work out the contribution of those shocks to the volatility of $\tilde{y}_t^D$. The latter will be the $R^2$ from the regression. Although there is serial correlation in $u_t$ the regressors $\eta_{1t-j}$ are uncorrelated with it. The TFP shock $\eta_{1t}$ can be found by fitting a univariate model for TFP. There is little evidence that one needs a more general ARMA rather than an AR process for TFP, and we use an AR(2) to be consistent with the previous section.

It needs to be recognized though that one needs to set an order for the polynomial $F_1(L)$ that is the number of impulse responses needed to capture $\tilde{y}_t^D$. This number was set in ACD to $h = 40$, as seen from their impulse response calculations, so we could use that. One problem is that making the order of $F_1(L)$ high will produce a high $R^2$ from any regression due to the finite data set. Consequently, we will use the AIC model selection criterion starting with $h = 40$ in order to find the order of $F_1(L)$.

For BP-filtered data, one has the same equivalence:

$$\tilde{y}_t^{BP} = B(L)\tilde{y}_t^D = G_1(L)\eta_{1t} + B(L)u_t \tag{16}$$

and $G_1(L) = B(L)F_1(L)$. Clearly this now means there are forward lags in both the regressor $\eta_{1t}$ and $u_t$ due to the nature of $B(L)$. Earlier it was observed that $B(L)$ was constructed from the frequency information and $M$. It is often recommended that $M$ should be at least eight, so we will take that as the value. So the regression one needs to run has both backward and forward lags and the $R^2$ computed from this gives the contribution of TFP shocks to the oscillatory cycle.

Using the AIC model selection criterion, a maximum lag length for $h$ of 25 is found with BP-filtered data and this gave a share of BP-filtered GDP explained by TFP shocks of 19.5%. Consequently, compared to earlier results the contribution of TFP shocks to the oscillatory cycle is almost the same, providing robustness to that result.

The same exercise can be done for the activity cycle. In this case only a value for $h$ needs to be set. The AIC choice of $h$ is $h = 32$, and the fraction of the variance of GDP growth accounted for by TFP shocks is 23%. So the pattern for the contribution of TFP shocks to cycles seen in the previous sub-sections holds, although potentially there is a higher contribution to the activity cycle. So our results are robust when using a local projections approach.

## 6. Conclusion

It is hard for us to see why one would not want to ask about the influence of shocks on *both* oscillatory and activity cycles. If one wants to establish the primacy of the former it should first be asked why oscillations between 6 to 32 quarters are selected. Often it is motivated along the lines that there has been a history in empirical macroeconomics of identifying the business cycle with the fluctuations occurring in the 6–32 quarters range of frequencies. Now these were the range of cycle lengths found by the NBER (specifically Burns and Mitchell) for US aggregate activity *when using turning points*, and did not come from studying oscillations. So, apart from frequency of use, it is not entirely clear why this range of oscillations is selected. There is no reason one should not study them but the question is whether it is enough to do just that. As one might expect when looking at the relation between $\tilde{y}_t^{BP(1)}$ and $\tilde{y}_t^{D}$ above there can be differences between the impact of shocks on the two types of cycles, namely those found with turning points and those with oscillations. Drawing conclusions from just one of these definitions about what shocks are important for model design seems premature, and it is appropriate to provide a deeper investigation of the relation between the two types of cycles.

It is clear that the business cycle described by the NBER depends on the characteristics of the growth in activity $\Delta y_t$. Oscillations work with $\tilde{y}_t^{BP}$, and the BP filter eliminates the mean of $\Delta y_t$ and, being a linear combination of $\Delta y_{t\pm j}$, has different variance and serial correlation properties. Looking at the first modification, what happens to the cycle if the rate of growth of TFP declines? That is something that can be answered in relation to a cycle in $y_t$ but not in $\tilde{y}_t^{BP}$. This alone might suggest that one must look at more than one concept of a cycle and the series that needs to be examined to summarize it. Looking at the business cycle as defined by the NBER means one needs to focus on the process for $\Delta y_t$, where $y_t$ is the log of GDP. Therefore, to gauge the importance of shocks to the NBER business cycle we need to study what their impact is upon $\Delta y_t$. Studying growth rates $\Delta y_t$ is not being done to eliminate a permanent component from $y_t$, but because expansions and contractions in $y_t$ - what the NBER summarize with their cycle dates - are found from the signs of $\Delta y_t$.

This points to the fact that one wants to look at *all frequencies* of $\Delta y_t$ in order to capture the NBER activity cycle, and not just the range from 6–32 quarters in a weighted average of $\Delta y_t$ that constitutes the oscillatory cycle. What we have tried to do is to argue for the need to *complete any investigation* into the role of shocks so as to also cover the question of what is driving the expansions and contractions identified by institutions who use the methods of the NBER. Our empirical work showed similar conclusions if one does this. We found that TFP shocks were important. although capturing less than ACD's "main" shock for an oscillatory cycle of 6–32 quarters. The main shock is a composite of many shocks so this is probably not surprising. The same outcome was true of the activity cycle centered on what the NBER do. An implication of this is that we need models to be constructed that can capture *both sets of facts*, and not just one of them. One might observe that at present in the literature there is a tendency to draw conclusions from only one of the possible cycles. So one will see researchers concluding that demand is the driver of oscillatory cycles, while those working with growth will often find it is TFP. Both cycles are based on $\Delta y_t$, but one is a linear combination of its history and the other is not, so one can get different results. It would seem clear that one should look at *both* in order to complete an investigation of the impact of shocks on cycles.

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## Notes

**1** The NBER in the past have also mentioned a growth cycle which studied turning points in the transitory component coming from an application of their phase average filter, $\tilde{y}_t^{PA}$. This was used for cases like Germany and Japan in the post-WW2 period where the trend growth was so strong that there no "downs" in $y_t$.

**2** This is question 1 at the page https://www.nber.org/research/business-cycle-dating/business-cycle-dating-procedure-frequently-asked-questions

**3** The HP, medium term and Beveridge–Nelson filters all have this structure except that the latter is not two sided.

**4** It is true for the HP and Beveridge–Nelson filters as well.

**5** We use the turning point location algorithm add-on in EViews11 (BBQ) for locating them. BBQ needs to be provided with values for the minimum length of phases and a complete cycle. These are set at two and five quarters. BBQ is a simplified version of the Bry and Boschan (1971) algorithm used by NBER and other agencies to find turning points in data. There are many descriptions of it. One is in Harding and Pagan (2016). There it is shown that the BBQ algorithm gives a good fit to the dates on GDP turning points, as set out on the NBER's cycle dates page. The per capita GDP data comes from Angeletos et al. (2020).

**6** This is the max share approach to finding a dominant shock pioneered by Uhlig (2004).

**7** It should be noted that there is a difference between the influence of a shock and the influence of a component such as TFP. Suppose TFP follows a process of the form $\Delta \log TFP_t = \mu + \varepsilon_t$. Then $\Delta y_t$ has a mean that is related to $\mu$, and this will affect the signs of $\Delta y_t$ and so the location of turning points in $y_t$. Therefore, TFP can be important through this route, because a high mean rate of growth for TFP can mean few recessions in activity. This is not true for $\tilde{y}_t^{BP}$ as the BP filter would eliminate any stochastic and deterministic trends in $y_t$ that is $E(\tilde{y}_t^{BP}) = 0$.

## References

Angeletos, G.-M., F. Collard and H. Dellas. (2020) Business cycle anatomy. *American Economic Review* 110(10), 3030–3070.

Baxter, M. and R. G. King. (1999) Measuring business cycles: Approximate band-pass filters for economic time series. *The Review of Economics and Statistics* 81(4), 575–593.

Beveridge, S. and C. R. Nelson. (1981) A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle. *Journal of Monetary Economics* 7(2), 151–174.

Bry, G. and C. Boschan. (1971) *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*. New York: NBER.

Burns, A. F. and W. C. Mitchell. (1946) *Measuring Business Cycles*. New York: NBER.

Christiano, L. J. and T. J. Fitzgerald. (2003) The band pass filter. *International Economic Review* 44(2), 435–465.

Comin, D. and M. Gertler. (2006) Medium-term business cycles. *The American Economic Review* 96(3), 523–555.

Harding, D. and A. Pagan. (2016) *The Econometric Analysis of Recurrent Events in Macroeconomics and Finance*, Tinbergen Lectures. Princeton University Press.

Kulish, M. and A. Pagan. (2021) Turning point and oscillatory cycles: Concepts, measurement, and use. *Journal of Economic Surveys* 35(4), 977–1006.

McGrattan, E. R. (2020) Intangible capital and measured productivity. *Review of Economic Dynamics* 37(Supp. 1), S147–166.

Pagan, A. and T. Robinson. (2014) Methods for assessing the impact of financial effects on business cycles in macroeconometric models. *Journal of Macroeconomics* 41, 94–106.

Uhlig, H. (2004) What moves real GNP?" Econometric Society 2004 North American Winter Meetings, no. 636.

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