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Solar and Stellar Dynamo Waves Under Asymptotic Investigation

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Abstract. The main magnetic activity of the Sun can be visualised by Maunder butterfly diagrams which represent the spatio-temporal distribution of sunspots. Besides sunspots there are other tracers of magnetic activity, like filaments and active regions, which are observable over a wider latitudinal range of the Sun. Both these phenomena allow one to consider a complete picture of solar magnetic activity, which should be explained in the framework of one relatively simple model.

A kinematic $\alpha\omega$-dynamo model of the magnetic field's generation in a thin convection shell with nonuniform helicity for large dynamo numbers is considered in the framework of Parker's migratory dynamo. The obtained asymptotic solution of equations governing the magnetic field has a form of a modulated travelling dynamo wave. This wave propagates over the most latitudes of the solar hemisphere equatorwards, and the amplitude of the magnetic field first increases and then decreases with the propagation. Over the subpolar latitudes the dynamo wave reverses, there the dynamo wave propagates polewards and decays with latitude. Butterfly diagrams are plotted and analyzed.

There is an attractive opportunity to develop a more quantitatively precise model taking into account helioseismological data on differential rotation and fitting the solar observational data on the magnetic field and turbulence, analyzing the helicity and the phase shift between toroidal and poloidal components of the field.

1. Parker's Migratory Dynamo

We are dealing with Parker's migratory dynamo (Parker 1955). This model is a strong simplification of dynamo processes of the solar interior. It is enough to remember that Parker's migratory dynamo gives only a kinematic description of the solar dynamo wave and does not consider nonlinear processes. Nevertheless, it happens that this simple model yields for the value under consideration the results which are comparable with observations.

Mean field magnetohydrodynamics in a thin, differentially rotating convection shell for the axisymmetric case and extremely large dynamo numbers gives

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the following equations governing a dynamo wave (Parker 1955, see also Stix 1989):

$$\frac{\partial A}{\partial t} = \alpha(\theta)B + \frac{\partial^2 A}{\partial \theta^2}, \tag{1a}$$

$$\frac{\partial B}{\partial t} = -DG(\theta)\cos\theta\frac{\partial A}{\partial \theta} + \frac{\partial^2 B}{\partial \theta^2}. \tag{1b}$$

Here B is the azimuthal component of the mean magnetic field, A is proportional to the azimuthal component of the magnetic potential, D is the dimensionless dynamo number which characterizes the intensity of the sources of the magnetic field generation. We consider here the case $D < 0$, which gives a basically equatorward dynamo wave. θ is the latitude in the shell that is measured from the solar equator, $\alpha(\theta)$ is the mean helicity, and $G(\theta)$ the radial gradient of the angular rotation, which are normalized with respect to maximum of their values, α_* and G_*, respectively.

An asymptotic solution of equations (1) for $|D| \gg 1$ has been obtained by Kuzanyan and Sokoloff (1995) using the **WKBJ method.** At leading order with respect to small parameter $|D|^{-1/3}$, it has the following form:

$$\begin{pmatrix} A \\ |D|^{-2/3}B \end{pmatrix} = \exp\left(i|D|^{1/3}S + |D|^{2/3}\Gamma_0 t + \ldots\right)(f_0 + \ldots), \tag{2}$$

where S and vector f_0 are complex functions of latitude θ.

The asymptotic solution under consideration should enable the dynamo wave to decay at locations remote from maximum of the sources of generation, which are in our model represented by function $\hat{\alpha}(\theta) = \alpha(\theta)\cos\theta$. So the solution should vanish remote from the domain where $\hat{\alpha}$ is maximum. In other words, we consider the dynamo wave, decaying at the pole and the equator.

2. Properties of the Asymptotic Solution

1. The **maximum of the solution** is, however, situated at point $\theta_1 < \theta_0$ at which $Im\,S(\theta)$ is minimum and for which

$$\hat{\alpha}_1 = \frac{\hat{\alpha}(\theta_1)}{\hat{\alpha}_*} = \frac{9\sqrt{3}}{16\sqrt{2}\sqrt{\sqrt{3}-1}} \approx 0.81\,,$$

   where $\hat{\alpha}_* = \hat{\alpha}(\theta_0)$ is maximum of function $\hat{\alpha}(\theta)$ (the sources of generation). For $\alpha(\theta) = \sin\theta$ we have $\theta_0 = \pi/4 = 45°$ and $\theta_1 \approx 27°$. So, it is **shifted** from point θ_0 **towards the equator.**

2. The dynamo wave propagates mostly equatorwards. However, at point $\theta_2 > \theta_0$, for which

$$\hat{\alpha}_2 = \frac{\hat{\alpha}(\theta_2)}{\hat{\alpha}_*} = \frac{9\sqrt{6}}{64} \approx 0.35,$$

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quantity $Re\, S'(\theta)$ changes its sign, and **the dynamo wave reverses.** For $\alpha(\theta) = \sin\theta$ we have $\theta_2 \approx 80°$, *i. e.*, **near the pole.** This reversal is really observable over the Sun.

Let us note, the location of points θ_1 and θ_2 entirely depends on function $\hat{\alpha}(\theta)$ but not the dynamo number $|D|$. For comparision of these properties with observations see Kuzanyan and Sokoloff (1995, 1997).

3. Comparison with Observations

The following definition for the dynamo number is used in equation (1b):

$$|D| = R_0^4 \frac{\alpha_* G_*}{\beta^2},$$

where $R_0 \approx 7 \cdot 10^{10}\, cm$ is the solar radius, β the turbulent magnetic diffusivity. For a crude estimation we use $G_* \approx \Omega / R_0$, where $\Omega \approx 2.7 \cdot 10^{-6}\, s^{-1}$ is the angular velocity of the Sun. Then the dynamo number is $|D| = R_0^3 \alpha_* \Omega / \beta^2$. Mixing length theory allows us to estimate turbulent diffusivity $\beta \sim 2 \cdot 10^{12}\ cm^2 s^{-1}$ and mean helicity $\alpha_* \sim 10^2\, cm\, s^{-1}$ Taking into account that for large $|D|$ period of the solar cycle $T_o \approx 22\,$yr according with Parker's migratory dynamo's approach is (in dimensional units)

$$T_o = \frac{2\pi}{Im\Gamma_0} |D|^{-2/3} \frac{R_0^2}{\beta}, \tag{3}$$

we accept below $|D| = 10^3$.

Let us calculate now the location and the half-width of the zone of increased value of the toroidal field (the zone of the dynamo wave's maximum) for different instants of time.

Assuming that the generation of the magnetic field is somehow suppressed by nonlinear or/and diffusion processes, we introduce the following function that is proportional to the magnitude of the toroidal field in the leading order of asymptotic expansion:

$$F(\theta, t) = Re \left\{ \exp \left[i|D|^{1/3} S(\theta) + i|D|^{2/3} Im\Gamma_0 t \right] \right\}, \tag{4}$$

where t is the dimensionless time measured in units of diffusion time, R_0^2/β. This function is plotted in Figure 1. The picture is qualitatively the same as the well-known Maunder butterfly diagrams. Further analysis of this function reveals agreement with observations (Kuzanyan and Sokoloff 1997).

4. Conclusion and Perspectives

1. The $\alpha\omega$-dynamo problem, which includes inhomogenity in localization of the sources of the magnetic field's generation without meridional circulation and further complications, reveals a **qualitative** picture of the dynamo wave propagation over the convection zone in one hemisphere which is **consistent with observations.**

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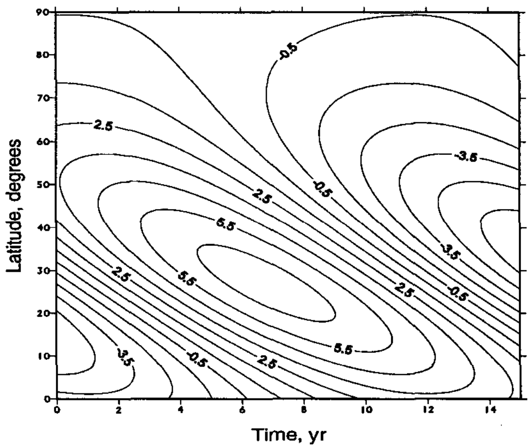

Figure 1. Values of function $F(\theta, t)$ in formula (4) measured in relative units over the solar cycle. Reproduced from Kuzanyan and Sokoloff (1997), Figure 2, with permission from Kluwer Academic Publishers.

2. It is possible to develop a more quantitatively precise model which includes nonlinearity and some higher order effects to enable one to **reconstruct** the profile αG against θ with **fitting** the solar **observational data** on the magnetic field. Such a model may use:
   (a) helioseismological data of the differential rotation,
   (b) observational data of the turbulence and so, helicity and diffusion,
   (c) information on the phase shift between toroidal and poloidal fields,
   (d) magnetic field observations in sunspots, and active regions.

   We may solve an **inverse problem** of reconstruction of (c) and (d) using of (a) and (b) as an initial guess.

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