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The Propositional Account of Effective Theories

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Abstract

I propose the "Propositional Account" of effective quantum field theories. According to the Propositional Account, each effective quantum field theory expresses propositions about various physical items: fields, interactions, and more. In addition, two effective quantum field theories are physically equivalent just in case they express the same propositions. As I explain, the Propositional Account is scientifically naturalistic, since it invokes terms and principles from the empirical science of linguistics. And the Propositional Account avoids problems faced by other accounts of the physical contents of effective theories.

1. Introduction

What do effective quantum field theories say about the physical world? And when are two effective quantum field theories physically equivalent? What, in other words, are the physical contents of effective theories, and when are two such theories physically the same?

In this paper, I present an account of effective quantum field theories—call it the "Propositional Account"—which answers these questions. As will become clear, there is much to like about the Propositional Account. It embodies a naturalistic approach to physical content, since it draws on the empirical science of linguistics. And it does not face problems that arise for other accounts of the contents of physical theories.

In section 2, I present the principle of the Propositional Account which describes effective theories' physical contents. In section 3, I present the principle of the Propositional Account which describes when one effective theory is physically equivalent to another. Finally, in section 4, I discuss some problems that alternative accounts of physical content and physical equivalence—based on privileged mappings—face.

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2. Physical Content

In this section, I formulate the principle which describes the physical contents of effective theories. To start, I summarize how effective theories work. Then I present and explicate the principle.

Following standard practice, I focus on effective theories formulated using a Lagrangian density. For example, take the following Lagrangian density, which is associated with a standard version of quantum electrodynamics:

$$\mathcal{L}_{\text{QED}} = \overline{\psi}\big(i\partial\!\!\!/ - m\big)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\overline{\psi}\gamma^\mu\psi A_\mu.$$

In this expression, the Einstein summation convention is being employed, the γ^μ are Dirac matrices which describe the structure of general relativistic spacetime, ψ is a bispinor field over the four-vectors of general relativity, $\bar{\psi} = \psi^\dagger\gamma^0$ is the Dirac adjoint, $\partial\!\!\!/ = \gamma^\mu\partial_\mu$ is basically a Lorentz-invariant derivative operator, the couplings m and e are the mass and charge of an electron, respectively, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$—where A_μ is the electromagnetic vector potential—is the electromagnetic field tensor.

There are at least two significant roles that Lagrangian densities play in effective theories. The first role is simple: when plugged into the corresponding Euler–Lagrange equations, a given Lagrangian density generates the equations of motion for various fields. For example, the Euler–Lagrange equations for $\mathcal{L}_{\text{QED}}$ can be used to derive equations of motion for the bispinor-valued field ψ and the electromagnetic vector potential A_μ. These equations describe how the values of those fields, at various regions of spacetime, constrain the values of those fields at other regions of spacetime.

The second role that Lagrangian densities play is more complicated. Through an intricate series of formal transformations, mathematical approximations, and so on, a given Lagrangian density can be used to calculate the transition amplitudes—that is, the probabilities of finding certain sorts of scattered particles in certain solid angles—which experiments can test. So Lagrangian densities encode empirically testable predictions.

Those derivations, though quite involved, are worth briefly summarizing, for they contribute to the contents which, according to the Propositional Account described below, effective theories have.[1] So very roughly put, one standard method for calculating transition amplitudes goes like this. To start, write the *S*-matrix—the basic operator which encodes transition amplitudes—as a time-ordered infinite series whose terms are integrals of various functions of quantized versions of the fields which appear in the Lagrangian density at issue; this is called "Dyson's expansion" of the *S*-matrix. Wick's theorem is used to compute the vacuum expectation values which Dyson's expansion contains. The result is a series of expressions, the integrals of which correspond to terms in Dyson's expansion; these

[1] For a summary of *S*-matrix theory, see Lancaster and Blundell (2014, 166–7); a more thorough introduction is in Eden et al. (1966, 182–278). For informal descriptions of time-ordering, see Lancaster and Blundell (2014, 156) and Peskin and Schroeder (1995, 31, 85); a rigorous definition of time-ordering is provided in Negele and Orland (1998, 49–50). Accessible overviews of Wick's theorem can be found in Lancaster and Blundell (2014, 171–4) and Peskin and Schroeder (1995, 88–90). And a thorough discussion of the basic features of incoming and outgoing states is provided in Taylor (1972, 25–34).

expressions can be conveniently summarized, and identified, using Feynman diagrams. Next, calculate those integrals: the results of those calculations are the S-matrix elements. Finally, use those S-matrix elements to compute transition amplitudes between incoming states and outgoing states.

An extremely important complication: many of the integrals mentioned above diverge, and so cannot be used to compute empirically testable transition amplitudes. To make those integrals convergent, various couplings and fields in the original Lagrangian densities—the ones used to compute the S-matrix elements—must be modified; this complicated procedure is often called "regularization and renormalization." The resulting Lagrangian densities, and the resulting expressions for various empirical quantities, describe physical phenomena up to specific energy scales only. That is why the quantum field theories corresponding to these Lagrangian densities are called "effective": they are only effective theories—they only accurately describe the physics—up to certain energies. Renormalization group equations relate these Lagrangian densities.

For example, given an energy level Λ, there is a modified version of the Lagrangian density $\mathcal{L}_{\text{QED}}$—denote it $\mathcal{L}_{\text{QED},\Lambda}$—which features a modified bispinor field ψ_Λ, a modified electromagnetic vector potential $A_{\mu,\Lambda}$, and modified coupling constants m_Λ and e_Λ. This Lagrangian density can be expressed as

$$\mathcal{L}_{\text{QED},\Lambda} = \overline{\psi}_\Lambda\big(i\not{\partial} - m_\Lambda\big)\psi_\Lambda - \frac{1}{4}(F_{\mu\nu,\Lambda})^2 - e_\Lambda\overline{\psi}_\Lambda\gamma^\mu\psi_\Lambda A_{\mu,\Lambda}.$$

The fields ψ_Λ and $A_{\mu,\Lambda}$, and the couplings m_Λ and e_Λ, are determined by regularization and renormalization.

So to summarize, Lagrangian densities play two significant roles in effective theories. First, when substituted into Euler–Lagrange equations, Lagrangian densities yield equations of motion for various fields; call this the "constraint role." Second, when used to generate integrals in expansions of S-matrix elements, and then subjected to regularization and renormalization, Lagrangian densities yield expressions for transition amplitudes between various states; call this the "transition role."

A terminological aside: Following standard practice, I often refer to any given Lagrangian density at an energy level as an effective theory. Strictly speaking, the effective theory is a combination of (i) the Lagrangian density, and (ii) a host of other assumptions which facilitate that density's use in the constraint role and in the transition role. So throughout what follows, read "the effective theory $\mathcal{L}_\Lambda$" as shorthand for "the effective theory consisting of a series of postulates which describe how $\mathcal{L}_\Lambda$ plays the constraint role and the transition role."

With all that as background, here is the first principle about the physical contents of effective theories.

Physical Content

For each energy level Λ and each effective theory $\mathcal{L}_\Lambda$ at that energy level, the physical propositions expressed by $\mathcal{L}_\Lambda$ are about physical fields, physical interactions, physical particles, physical transition amplitudes, and so on, at Λ. The mathematical terms for fields, couplings, and so on, which $\mathcal{L}_\Lambda$ features, refer to physically real fields, coupling interactions, and so on, in the world; likewise for many terms in quantities that $\mathcal{L}_\Lambda$—in its constraint role and transition role—is used to compute.

In other words, $\mathcal{L}_\Lambda$—insofar as it plays the constraint role and the transition role—expresses physical propositions like "This field interacts in thus-and-so way with this other field," "The probability of finding this sort of particle in this cross section is such-and-such," and so on. These physical propositions only invoke items—fields and interactions, for instance—to which symbols in $\mathcal{L}_\Lambda$, and symbols in calculations based on $\mathcal{L}_\Lambda$, refer. And the invoked items concern the specific energy level Λ in various important ways. So these fields, interactions, particles, and so on, are not fundamental. Nevertheless, they still exist.

Another terminological aside: For each energy level Λ and each effective theory $\mathcal{L}_\Lambda$ at that energy level, I often describe various fields, interactions, and so on—which figure in the propositions that $\mathcal{L}_\Lambda$ expresses—as being "at" Λ. This is just a convenient way of describing the specific fields, interactions, and so on, to which a given effective theory is committed. It should not be understood as implying that the propositions which $\mathcal{L}_\Lambda$ expresses are true or false "relative to energy levels." Nor should it be understood as implying that existence is somehow "level-relative," that the entities described by those propositions exist relative to some energy levels but not others. Expressions of the form "thus-and-so field at such-and-such energy level" are simply shorthand for more complicated expressions about the physical facts, connected to energies, which the corresponding Lagrangian densities express.

To illustrate Physical Content, consider a particular energy level Λ and the corresponding Lagrangian density $\mathcal{L}_{\mathrm{QED},\Lambda}$. The coupling constants, which quantify the strengths of various physical interactions, are m_Λ and e_Λ. The fields are ψ_Λ and $A_{\mu,\Lambda}$. The Euler–Lagrange equations for $\mathcal{L}_{\mathrm{QED},\Lambda}$ describe how those coupling constants and fields relate to each other when no measurements occur.[2] So for instance, those equations imply propositions like "Thus-and-so state of the bispinor field ψ_Λ constrains the state of the vector field $A_{\mu,\Lambda}$ in such-and-such ways." Regularization and renormalization techniques contribute to describing how m_Λ, e_Λ, ψ_Λ, and $A_{\mu,\Lambda}$, relate to each other when the system in question is measured. So for instance, the corresponding calculations imply propositions like "Thus-and-so incoming state is this-and-that likely to produce such-and-such outgoing state."

Physical Content is a somewhat schematic account of the physical contents of effective theories. Completely filling in that schema is, of course, far beyond the scope of this paper: there is nowhere near enough space to specify, for every single effective theory $\mathcal{L}_\Lambda$, the exact collection of physical propositions which $\mathcal{L}_\Lambda$ expresses. Nevertheless, Physical Content still provides an informative account of what many

[2] For present purposes, set the measurement problem aside. I assume that the measurement problem can be solved by adopting some interpretation or other. For instance, consider a version of the Everett interpretation which (i) takes the mathematical time evolution operator—in the interaction picture—to represent a physically real operator that evolves in accord with an interaction Hamiltonian, and (ii) takes the Born rule to be a law describing the centered chances of various indexical propositions obtaining (Wilhelm, 2022).

such propositions are. Just consider the propositions, described above, which effective theory $\mathcal{L}_{QED,\Lambda}$ expresses.[3]

Note that, according to Physical Content, the sorts of physical facts described by a true effective theory $\mathcal{L}_{\Lambda}$ are akin to the sorts of physical facts that non-fundamental special sciences describe. For instance, take evolutionary biology. Rabbits and wolves interact with one another in accord, roughly, with the Lotka–Volterra equations. These equations, and the interactions they describe, concern phenomena at a particular energy level: the level corresponding to medium-sized dry goods. So to put it roughly: rabbits and wolves interact at this level. They do not interact at other energy levels. The physical facts about their interactions concern phenomena at this particular energy level only.

Similarly for effective theories. A true effective theory $\mathcal{L}_{\Lambda}$ expresses facts like "The probability of the field transitioning from this state to that state is such-and-such." These facts concern objects and phenomena—in particular, physical field states and transition probabilities—at the particular energy level Λ. Other physical facts expressed by $\mathcal{L}_{\Lambda}$ concern particles at that energy level, and how they interact. So $\mathcal{L}_{\Lambda}$ expresses facts about physically real fields, physically real interactions, and so on, all at the specific energy level Λ.

3. Physical Equivalence

In this section, I present the second principle of the Propositional Account. This principle describes physical equivalence among effective theories. After presenting this principle, I defend it—and Physical Content—against objections. And I argue for these principles by invoking a methodological criterion which accounts of physical content should generally satisfy.

The physical equivalence principle is as follows; note that it implicitly invokes Physical Content.

Physical Equivalence

For each energy level Λ, and for all effective theories $\mathcal{L}_{1,\Lambda}$ and $\mathcal{L}_{2,\Lambda}$ at that energy level, $\mathcal{L}_{1,\Lambda}$ is physically equivalent to $\mathcal{L}_{2,\Lambda}$ if and only if $\mathcal{L}_{1,\Lambda}$ and $\mathcal{L}_{2,\Lambda}$ express exactly the same physical propositions.

In other words, effective theories are equivalent just in case the physical propositions they express—as given by Physical Content—are the same.

Physical Content and Physical Equivalence jointly form what I have been calling the "Propositional Account" of effective theories. Physical Content specifies those theories' physical contents. Physical Equivalence uses those contents to state the conditions under which one effective theory is physically equivalent to another.

The Propositional Account implies that effective theories are extremely fine-grained. In particular, according to the Propositional Account, many empirically

[3] For an account of the physical structures to which effective theories may refer, see Williams (2019, 220). For discussion of such structures in the case of classical mechanics in particular, see North (2009). For discussion of how to compare the structures of mathematical objects more generally, see Wilhelm (2021).

equivalent effective theories are physically inequivalent. For many effective theories make the same experimental predictions, and yet posit radically different fields, interactions, and so on. Effective theories formulated using canonical quantization, for instance, are physically different from effective theories formulated using path integrals, even if these two classes of theories make the same empirical predictions: while the former theories posit fields associating physical operator quantities with spacetime points, the latter theories posit fields associating physical scalar quantities with spacetime points instead.

Similarly, an effective theory with one set of renormalized constants, and one renormalized Lagrangian density, is physically different from an effective theory with a different set of renormalized constants and a different renormalized Lagrangian density. The empirical predictions of these theories may well be the same, of course. But these theories are physically different, because they posit different kinds of physical interactions, which are expressed by the different renormalized constants and densities.

It would be a mistake, however, to think that all this is some sort of problem for the Propositional Account. For all this is, basically, just a specific instance of the more general phenomenon of the underdetermination of theory by evidence. Underdetermination is extremely common in science generally, and in physics specifically. The fineness of grain, among effective theories, is just another case of this.

So the Propositional Account has an important lesson to teach: the physical contents of effective theories are more fine-grained than realists, or anti-realists, have often appreciated. And it is a feature of the Propositional Account, not a bug, that it draws such fine-grained distinctions: for many effective theories, despite generating the same empirical predictions, clearly posit different physical fields, interactions, and more. In many cases, those posits cannot be defined in terms of each other. So the effective theories really are distinct.

Relatedly, one might object that the Propositional Account contradicts intuitively plausible claims about physical equivalence among effective theories. By way of illustration, take the claim that Feynman's theory of quantum electrodynamics is physically equivalent to Schwinger's theory of quantum electrodynamics.[4] This claim seems true: after all, this claim is what Dyson (1949) seems to have famously shown.[5] And yet, one might object, the Propositional Account contradicts this. For some mathematical terms used in Feynman's theory are, one might claim, different from the mathematical terms used in Schwinger's theory. So plausibly, Physical Content implies that the propositions expressed by Feynman's theory posit physical items which are not posited by the propositions that Schwinger's theory expresses. Therefore, according to Physical Equivalence, Feynman's theory is not physically equivalent to Schwinger's theory.

There are two different problems with this objection. First, it is wrong to claim that the mathematical terms used in Feynman's theory are different from the

[4] For early presentations of important features of these theories, see Feynman (1948), Schwinger (1948), and Tomonaga (1946).

[5] The formalism for effective theories in this paper, which focuses on Lagrangian densities, can be adapted to fit Dyson's formalism, which focuses on Hamiltonian operators.

mathematical terms used in Schwinger's theory. As Dyson showed, the basic posits of both theories can be formulated using the same mathematical terms (1949, 491–2). The key difference between the two theories concerns the specific Hamiltonian operator from which they calculate various physical quantities, such as radiative corrections to the equations of motion of a single electron in an external field: whereas Feynman's theory focuses on a "mixed representation" operator $H_F(x_0)$, Schwinger's theory focuses on the effective external potential energy operator $H_T(x_0)$. The two operators are related, however, by

$$H_F(x_0) = S(\infty)H_T(x_0),$$

where $S(\infty)$ is, extremely roughly put, an operator which transforms a state of the system in the infinite past—for instance, a converging stream of particles—into the same state in the infinite future—for instance, after the particles have interacted. Both Feynman's theory and Schwinger's theory use $S(\infty)$ elsewhere in their calculations. So it is reasonable to suppose that both theories are committed to each of the mathematical operators $H_T(x_0)$ and $H_F(x_0)$ representing physically real energy densities. Of course, the different operators facilitate different sorts of calculations.[6] But that is irrelevant, for the purposes of evaluating the Propositional Account. Given the supposition that both theories are committed to physical correlates of the operators $H_F(x_0)$ and $H_T(x_0)$, and to the physical properties which those physical correlates have, Physical Content implies that both theories express the same propositions. And so, contrary to what the objection claims, Physical Equivalence—when supplemented with the supposition mentioned earlier—implies that Feynman's theory is physically equivalent to Schwinger's theory.

Second, the objection makes incorrect claims about what Dyson showed. Dyson never explicitly argues that Feynman's theory and Schwinger's theory are "physically equivalent." That phrase does not appear anywhere in Dyson's seminal paper. Instead, Dyson simply argues that Feynman's theory and Schwinger's theory are equivalent. The precise nature of the equivalence demonstrated is left open. Of course, according to one interpretation of Dyson's results, Feynman's theory is physically equivalent to Schwinger's theory. But this interpretation seems anachronistic: physical equivalence concerns physical interpretation of the sort which was famously eschewed by many physicists who originally formulated quantum electrodynamics, Feynman included. It is not as anachronistic, however, to interpret Dyson as showing that Feynman's theory is empirically equivalent, or calculationally equivalent, to Schwinger's theory.

Besides, empirical equivalence—or calculational equivalence—is clearly closer to what Dyson actually demonstrated. For what Dyson showed, in rough outline, is that the rules at the basis of Feynman's radiation theory can be derived from what is basically Schwinger's theory (Dyson 1949, 492–3). That sort of demonstration is much more like a demonstration of empirical equivalence, or calculational equivalence, than a demonstration of physical equivalence.

So this objection does not succeed. The Propositional Account is perfectly compatible with the view that Feynman's theory is physically equivalent to

[6] The infinite series expansion of $H_T(x_0)$ features unboundedly many nested commutators; that is what makes calculations based on Schwinger's theory so difficult.

Schwinger's theory. And in addition, it is wrong to read the notion of equivalence, in Dyson's paper, as this objection does: the equivalence at issue is more empirical or calculational, and less physical.

Here is another, extremely attractive feature of the Propositional Account. As will become clear, the Propositional Account satisfies the following methodological criterion for accounts of the contents of physical theories.

Content Naturalism Criterion

> Philosophical accounts of content should generally be formulated using the theoretical terms, and basic guiding principles, of our best science of content: namely, the naturalistic science of linguistics.

For example, here are some theoretical terms which practically all standard linguistic theories invoke: reference, denotation, expression, satisfaction, and propositions (Chomsky 1995, 24). And here is a guiding principle which the vast majority of linguistic theories posit: meaning is compositional, in that the meaning of a syntactically complex expression is determined by the meanings of its constituent expressions and their grammatical relations (Akmajian et al. 2001, 246). So the Content Naturalism Criterion says, roughly put, that philosophical accounts of content should (i) use terms like reference, denotation, expression, satisfaction, and propositions, and (ii) adhere to principles like the compositionality of meaning. For those are the basic tools of standard, empirically adequate, linguistic theories.

The motivating idea behind the Content Naturalism Criterion is as follows. Linguistics is a natural, empirical science of content (Schütze 2016). In particular, linguistics is the special science of the meanings which written inscriptions, spoken utterances, and so on, have.[7] So in order to be naturalistically respectable, philosophical accounts of content should generally defer to linguistics. Those accounts should be formulated using the terms, and principles, which linguists use: reference, denotation, expression, satisfaction, propositions, compositionality, and so on. Any philosophical account which does otherwise is non-naturalist. Such an account would eschew our best current approach to how written inscriptions, spoken utterances, and so on, meaningfully describe the world. Hence the Content Naturalism Criterion: philosophical accounts of content for any language whatsoever—including the languages in which effective theories are expressed—should be formulated using the theoretical terms which the natural, empirical science of linguistics provides.[8]

The Propositional Account is quite attractive, because it satisfies the Content Naturalism Criterion. For the Propositional Account is formulated in terms of reference, expression, propositions, and other standard posits of empirically adequate theories of contemporary linguistics. So the Propositional Account is naturalistic in the way that the Content Naturalism Criterion requires. And that is a very significant point in favor of the Propositional Account.

[7] Linguistics is, of course, also a science of syntax, grammar, morphological change, and so on. The Content Naturalism Criterion is compatible with that.

[8] One might object that the language of physics is different, somehow, from the languages on which linguists focus. But that is wrong. Physics is conducted using the sorts of formal and natural vocabulary studied by linguists.

The Content Naturalism Criterion deserves more discussion than it has received. Many accounts of contents, for physical theories in particular, rely exclusively on terms and principles drawn from mathematical physics. Now, there is nothing problematic about relying, in part, on terms and principles like that: physical theories are generally formulated using mathematics, and so it makes sense for accounts of those theories' contents to invoke mathematics as well. But relying entirely on those sorts of terms and principles—and in particular, failing to use standard linguistic tools like reference, propositions, and so on—is problematically non-naturalist. Linguistics is an empirical science like any other. It is our best science of content. So all philosophical accounts of content whatsoever, including accounts of the contents of even the most complex physical theories to date, should draw from contemporary linguistics. All philosophical accounts of content, that is, should satisfy the Content Naturalism Criterion. So the Content Naturalism Criterion articulates a methodology which philosophical accounts of content, including philosophical accounts of the contents of physical theories, should follow.

Correspondingly, think of the Propositional Account as suggesting a shift in the approach that philosophers take to questions about the contents of physical theories. Traditionally, philosophers have approached those questions by using tools and techniques drawn solely from the physical sciences. The Propositional Account suggests approaching those questions using tools and techniques drawn from linguistics as well. For again, linguistics is the science of content: so to be naturalistically respectable, accounts of the contents of physical theories should rely on the tools and techniques developed in linguistics.

4. Privileged Mappings

In this section, I discuss some alternative accounts of physical equivalence. These accounts use notions drawn from category theory (Weatherall 2016), formal logic (Barrett 2018), model theory (Dewar 2022), and more. After presenting these accounts, I raise three problems for them.

Though these accounts differ in various ways, they share a common core. Basically, they all endorse something roughly along the following lines.

Privileged Mappings

Theory T_1 is physically equivalent to theory T_2 if and only if T_1 and T_2 are related by certain privileged mappings.

In other words, theories are physically equivalent just in case certain sorts of mappings obtain between them.

Here is the first problem for Privileged Mappings: it does not satisfy the Content Naturalism Criterion. For Privileged Mappings is not formulated in terms of theoretical tools like reference, expression, propositions, and other standard posits of empirically adequate theories of linguistics. Instead, standard formulations of Privileged Mappings invoke theoretical tools like isomorphisms and map-based representations.[9] So most versions of Privileged Mappings in the literature do not draw from linguistics much at all. And so Privileged Mappings does not adhere to the naturalistic methodology which the Content Naturalism Criterion articulates.

Here is another way to put the point. Privileged Mappings, like all accounts of content and equivalence, should draw from the tools developed by those working on our best theories of meaning. Those tools come from the science—the empirical, naturalistic science—of linguistics. This fact, though often overlooked, is extremely important: linguistics is an empirical science like any other, and so philosophical accounts which are relevantly related to linguistics—accounts of content and equivalence, for instance—should draw heavily from it. To do otherwise, when formulating accounts of content and equivalence, is to eschew our best science of what content, and equivalence of content, ultimately are. So Privileged Mappings is scientifically non-naturalist. And that is a problem.

Now for the second problem: for effective theories in particular, there are no fully precise definitions of the mappings that Privileged Mappings invokes. In the physical theories on which the philosophical literature focuses—like classical mechanics, or general relativity—privileged mappings can be rigorously defined; those definitions invoke preservation of structure, commutativity with various operations, and more. But in the case of effective theories, no such clean, pristine definitions are available. For any proposed rigorous definition of the relevant mappings from one effective theory to another must be mathematically compatible with the formal techniques that physicists use to extract empirical predictions from effective theories. And many of those formal techniques are not mathematically well-defined; the Feynman path integral, for instance, invokes an ill-defined measure over all trajectories. So no rigorously defined mappings can be formally compatible with the mathematically unsound—though obviously physically sound—techniques that physicists actually use. Therefore, in the case of effective theories, the "certain privileged mappings" in Privileged Mappings cannot be defined in the ways that many philosophers have tried.

Here is the third problem: Privileged Mappings does not respect the fact that the same mathematical structure can be used to represent physically distinct situations. To see why, suppose that the "certain privileged mappings" in Privileged Mappings are taken to be isomorphisms of algebras, or diffeomorphisms of manifolds, or something like that. Then Privileged Mappings implies that the relevant theories corresponding to those algebras or those manifolds are physically equivalent. But that is implausible. One and the same mathematical algebra, for instance, can be used to represent many different physical systems.[10] Just as the grammatical structures of two natural languages could be isomorphic, and yet the corresponding words in those languages could still have different meanings, the formal structures of two scientific

[9] The same point applies to an account of content due to Wallace (2022). One of my main concerns about Wallace's account is that it fails to satisfy the Content Naturalism Criterion, for it eschews standard linguistic tools, like reference and denotation, in favor of non-standard tools like map-based representations (2022, 6). Relatedly, Wallace's account may fail to be compositional, though that will ultimately depend on details of the account that have yet to be filled in.

[10] Versions of this point are discussed in Belot (2013) and Teitel (2021).

theories could be isomorphic, and yet the corresponding terms in those theories could have different meanings as well.

5. Conclusion

There is much to like about the Propositional Account. It provides an account of the physical contents of effective theories. It provides an account of the conditions under which effective theories are physically equivalent. It embodies a naturalistic approach to content. And it avoids problems that alternative accounts face. The Propositional Account is worth taking seriously.

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