

# Clumpiness of lens galaxies as a window on dark matter

Dorota Bayer 

Centre for Astrophysics & Supercomputing, Swinburne University of Technology,  
Hawthorn, Australia  
ARC Centre of Excellence for All Sky Astrophysics in 3 Dimensions (ASTRO 3D), Australia.  
email: [dorota.bayer@gmx.de](mailto:dorota.bayer@gmx.de)

**Abstract.** While the direct detection of the dark-matter particle remains very challenging, the physical properties of dark matter could potentially be constrained indirectly – by comparing the population characteristics of substructures in real galaxies with predictions from the phenomenological dark-matter models, such as cold, warm or self-interacting dark matter. Whereas these models are practically indistinguishable with respect to the expected abundance and statistical properties of massive galactic substructures, the critical difference lies in the low-mass regime  $\lesssim 10^8 M_\odot$ . Galaxy-galaxy strong gravitational lensing provides a unique opportunity to search for gravitational signatures of such low-mass substructures in galaxies acting as a strong gravitational lens on a more distant background galaxy, serendipitously located along the same line of sight. In Bayer et al. (2023a,b,c), we have recently introduced a novel approach to investigate the collective perturbative effect of low-mass substructures in the inner regions of massive elliptical lens galaxies and observationally constrain their power spectrum (on 1-10 kpc scales) based on the Fourier analysis of the associated surface-brightness anomalies in the lensed images (i.e., Einstein rings and gravitational arcs) observed with the Hubble Space Telescope. Here, we present a concise summary of the methodology, the encountered modelling challenges and the inferred observational constraints on the sub-galactic matter power spectrum. The future comparison of these results with predictions from hydrodynamical simulations might either verify the cold-dark-matter paradigm or require its substantial revision. Moreover, we demonstrate that the grid-based smooth lens modelling might model away surface-brightness anomalies caused by the presence of substructures in the lens galaxy and incorrectly interpret them as surface-brightness structure in the lensed background galaxy itself. If not properly understood and accounted for, such signal suppression might lead to severely biased constraints on the properties of substructures in galactic haloes.

**Keywords.** Cosmology, Dark matter, Galactic substructure, Gravitational lensing

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## 1. Introduction

One of the major unsolved puzzles in cosmology and the fundamental physics is the dark-matter problem – despite the essential role that dark matter plays in the cosmological structure-formation process, its nature and properties remain unknown. The standard cold dark matter (CDM) paradigm is still challenged by various alternative models, such as warm dark matter (WDM) or self-interacting dark matter (SIDM). While the direct detection of the dark-matter particle continues to pose a challenge, one of the most promising avenues to unveil the nature of dark matter is the study of the abundance and statistical properties of dark-matter substructures in galactic haloes. The competing dark-matter models, such as CDM, WDM or SIDM, lead to significantly different levels of mass clumpiness/smoothness on the sub-galactic scales. Whereas the CDM model

predicts galactic haloes to be inhabited by an abundant population of substructures on mass scales down to at least  $10^{-6}M_{\odot}$ , models with less massive or self-interacting dark-matter particle significantly suppress the formation of substructures, especially in the low-mass regime below  $\approx 10^8M_{\odot}$ .

Galaxy-galaxy strong gravitational lensing offers a unique opportunity to search for gravitational signatures of such low-mass dark-matter substructures in lens galaxies beyond the Local Group. Substructures in the lens galaxy (or along its line-of-sight) induce perturbations to the otherwise smoothly-distributed lensing potential. The resulting observable *surface-brightness anomalies* in the extended lensed images of the background galaxy (Einstein rings or gravitational arcs), i.e., deviations of the observed surface-brightness distribution from the prediction of a smooth-lens model, can be modelled and traced back to the underlying substructure in the lens galaxy. One of the most successful approaches in this respect is the *gravitational-imaging technique* (Koopmans 2005; Vegetti and Koopmans 2009), which allows one to constrain the projected position and mass of individual massive subhaloes in (massive elliptical) lens galaxies via their imprints on the lensed images. However, the current mass-detection threshold of this technique ( $\sim 10^8M_{\odot}$  for HST-observations of SLACS lenses) still lies above the mass regime in which the alternative dark-matter models could be distinguished, given the current sample sizes and data sensitivity.

In response to these observational limitations, in Bayer et al. (2023a,b,c), we have recently introduced an alternative statistical approach. This aims at investigating the overall clumpiness in the total (dark and baryonic) mass distribution of massive elliptical lens galaxies, i.e., the level of small-scale deviations from the best-fitting smooth elliptical power-law mass density model with external shear, and inferring observational constraints on the *sub-galactic matter power spectrum* from the *power spectrum of the collectively-induced surface-brightness anomalies* measured in highly magnified galaxy-scale Einstein rings and gravitational arcs. Here, we present a concise summary of the methodology, the encountered modelling challenges and the most important results.

## 2. Methodology

In Bayer et al. (2023a), we have presented a methodology allowing us to reliably measure the power spectrum of surface-brightness anomalies in extended lensed images of galaxy-galaxy strong gravitational lens systems (i.e., Einstein rings or gravitational arcs), observed with the Hubble Space Telescope (HST/WFC3/F390W), through the following data-analysis steps:

- (1) modelling and subtraction of the lens-galaxy light;
- (2) simultaneous reconstruction of the best-fitting smoothly-varying lensing potential and the intrinsic unlensed surface-brightness distribution of the background galaxy;
- (3) statistical quantification of the residual surface-brightness fluctuations in the lensed images in terms of the azimuthally-averaged power spectrum  $P_{\delta I}(k)$ ;
- (4) estimation and correction for the noise power spectrum in the HST-observations.

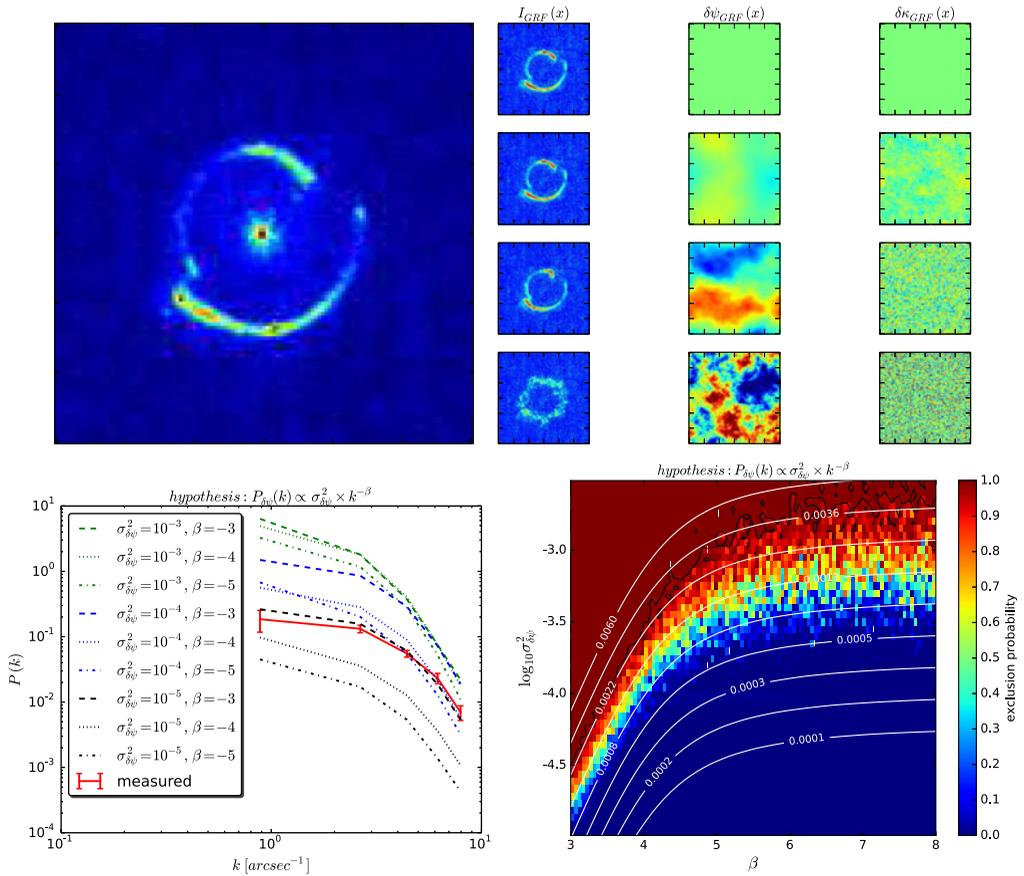
The noise correction of the residual surface-brightness fluctuations requires a particularly careful consideration due to the noise correlations caused by the combined effect of the drizzling procedure and the charge transfer inefficiency of the HST/WFC3/UVIS-CCDs. To this end, we have developed a methodology to estimate the noise power spectrum in the analysed science images based on a set of blank-sky cutouts from the full field of view that we overlay with the Poisson noise of the specific observation.

In Bayer et al. (2023b), we have extended this methodology to trace the revealed surface-brightness anomalies back to the underlying population of small-scale density variations in the lens galaxy, which can arise not only from the low-mass dark-matter substructures and line-of-sight haloes but also from density inhomogeneities in the baryonic mass distribution, such as e.g. globular clusters or tidal streams. As proposed by Chatterjee and Koopmans (2018), we model the overall density variations as Gaussian Random Field (GRF) potential perturbations  $\delta\psi_{\text{GRF}}(\mathbf{x})$  superposed on a smoothly-varying lensing potential and assume them to be uniquely characterised by a power-law power spectrum  $P_{\delta\psi}(\sigma_{\delta\psi}^2, \beta; k)$  with two free parameters: the total variance  $\sigma_{\delta\psi}^2$  and the power-spectrum slope  $\beta$ . The latter describes the distribution of the integrated variance over the different length scales (i.e.,  $k$ -modes) and, thus, quantifies the scale dependence of the hypothetical small-scale density variations.

To interpret the real measurement, we have performed a systematic study of simulated surface-brightness anomalies induced by low-mass substructures with varying statistical properties. The investigation is performed in the parameter space spanned by  $\sigma_{\delta\psi}^2$  and  $\beta$ . For each point on this grid, referred to as a *matter-power spectrum model*, we have generated realizations of Gaussian potential perturbations, superposed them on the best-fitting smooth lensing potential inferred for the real data, computed the resulting perturbed lensed images and, finally, determined the power spectrum of the induced surface-brightness anomalies. This catalogue of mock power spectra can be directly compared to the actual measurement, which allows us to determine exclusion probabilities for all considered matter-power spectrum models, as demonstrated in Fig. 1. A valuable insight for future studies is that the fundamental quantity determining the exclusion probability of the matter-power-spectrum models is the integrated variance of the perturbations in the deflection angle  $\sigma_{\delta\alpha}^2$  resulting from the induced potential perturbations  $\delta\psi_{\text{GRF}}(\mathbf{x})$ . The exclusion probability is almost insensitive to the slope of the power spectrum of the deflection angles  $P_{\delta\alpha}(k)$ .

### 3. Observational constraints on the sub-galactic matter power spectrum

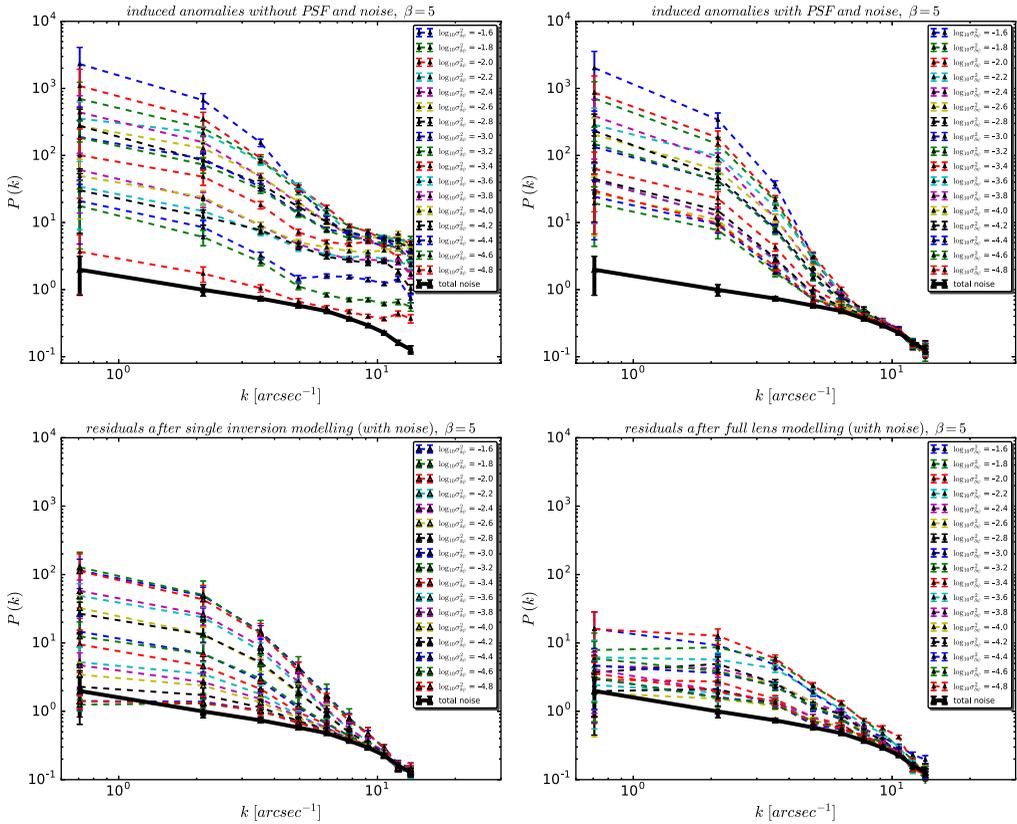
The pilot application of the complete methodology to HST/WFC3/F390W-observations of the lens system SDSS J0252+0039 from the SLACS Survey has allowed us to infer the first observational constraints on the power spectrum of low-mass sub-galactic structures on 1-10 kpc scales, as is presented in Fig. 1. Based on this pilot study, we have ruled out (with the exclusion probability of at least 99 per cent) potential perturbations  $\delta\psi_{\text{GRF}}(\mathbf{x})$  with the variance  $\sigma_{\delta\psi}^2$  exceeding  $\sim 10^{-2.5}$  in the analysed field of view of  $\approx 20$  kpc on a side for a wide range of power-spectrum slopes  $\beta$ . In order to make these results less dependent on the chosen field of view, we have also derived the corresponding constraints on the dimensionless convergence power spectrum  $\Delta_{\delta\kappa}^2(\lambda)$  for three different sub-galactic scales. Accordingly, we rule out matter-power-spectrum models with  $\Delta_{\delta\kappa}^2(0.5 \text{ kpc}) > 1$  on 0.5-kpc scale,  $\Delta_{\delta\kappa}^2(1 \text{ kpc}) > 0.1$  on 1-kpc scale and  $\Delta_{\delta\kappa}^2(3 \text{ kpc}) > 0.01$  on 3-kpc scale at the 99 per cent confidence level. Alternatively, these constraints can be also translated into upper limits on the standard deviation of the aperture mass (i.e., the total mass within a cylinder with the diameter  $\lambda$  projected onto the lens plane) in the proximity to the Einstein radius of the lens galaxy:  $\sigma_{AM}(0.5 \text{ kpc}) < 0.8 \times 10^8 M_{\odot}$  inside the circular aperture of diameter  $\lambda = 0.5$  kpc,  $\sigma_{AM}(1 \text{ kpc}) < 1 \times 10^8 M_{\odot}$  for  $\lambda = 1$  kpc and  $\sigma_{AM}(3 \text{ kpc}) < 3 \times 10^8 M_{\odot}$  for  $\lambda = 3$  kpc. Due to the encountered degeneracy between the real surface-brightness anomalies induced by mass structure in the lens galaxy and the complex intrinsic surface-brightness distribution of the star-forming background galaxy itself, the inferred constraints can be expressed only in terms of an upper limit.



**Figure 1. Upper-limit constraints on the sub-galactic matter power spectrum in the massive elliptical lens galaxy SDSS J0252+0039.** *Top left panel:* HST/WFC3/F390W imaging of the strong gravitational lens system SDSS J0252+0039 with the side length of 4.48 arcsec. *Top right panel:* Mock surface-brightness anomalies induced in the lensed images by Gaussian potential perturbations  $\delta\psi_{\text{GRF}}(\mathbf{x})$  in the lens galaxy. These are assumed to be fully characterized by the power spectrum  $P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}$  with the integrated variance  $\sigma_{\delta\psi}^2$  and the power-law slope  $\beta$ . *Bottom left panel:* The power spectrum of surface-brightness anomalies measured in real observations (red line) in comparison to the power spectra of mock surface-brightness anomalies induced by Gaussian potential perturbations with specific values of  $\sigma_{\delta\psi}^2$  and  $\beta$  (dashed lines). *Bottom right panel:* Exclusion probabilities for the integrated variance  $\sigma_{\delta\psi}^2$  and the power-law slope  $\beta$ . The white contour lines connect points corresponding to the same variance of the resulting deflection-angle perturbations  $\sigma_{\delta\alpha}^2$  in the analysed field of view; based on Bayer et al. (2023a,b).

### 4. Signal suppression in grid-based smooth lens modelling

A critical issue in our analysis has been the degeneracy between the real surface-brightness anomalies due to mass structure in the lens galaxy and the complex intrinsic surface-brightness distribution of the star-forming background galaxy itself. Grid-based source modelling can model away some of the anomalies caused by mass structure in the lens galaxy and partially even the observational noise. Due to numerical effects, this signal suppression is particularly severe when the lens modelling is performed in a narrow field of view with the highest source-grid resolution. A practical solution to this



**Figure 2. Signal suppression in grid-based smooth lens modelling.** *Top:* Power spectra of mock surface-brightness anomalies induced in the lensed images of SDSS J0946+1006 by GRF-potential perturbations characterised by the power spectrum  $P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}$  with different levels of the total variance  $\sigma_{\delta\psi}^2$  and a fixed power-spectrum slope  $\beta = 5$  before (*left panel*) vs after (*right panel*) accounting for the observational effects (i.e., convolution with the PSF and adding a realistic noise realization) for a set of 17 mocks. *Bottom:* Suppressed power spectra of mock surface-brightness anomalies after a single-inversion reconstruction of the source galaxy with no optimisation of the lensing potential (*left panel*) vs the *full* grid-based smooth lens modelling of the mock lensed images, i.e., simultaneous optimization of the lensing potential and reconstruction of the background source on a grid (*right panel*); based on Bayer et al. (2023c).

problem is to lower the resolution of the source reconstruction, while keeping fixed the best-fitting parameter values of the lensing potential obtained with the highest resolution. Alternatively, this problem can be also partly alleviated by the choice of a larger mask covering not only the lensed images but also additional pixels located close to the edge of the analysed cutout. These carry additional information about the noise level in the analysed image and provide more constraints for the Bayesian lens-modelling procedure, which improves the quality of the resulting lens models.

Our most recent work (Bayer et al. 2023c) is fully devoted to alleviating this degeneracy and paving the way for a future unbiased *detection*. With this goal in mind, we propose and test a more accurate version of the methodology to model the simulated perturbed lensed images in exactly the same manner as the real data. The modification is introduced gradually in two steps. First, we perform the less computationally expensive

single-inversion modelling (i.e., reconstruction of the pixelized surface-brightness distribution of the background galaxy, while keeping fixed the parameter values of the lensing potential obtained with the highest resolution), which is suitable for studies of large samples. Second, we carry out the much more accurate full grid-based smooth lens modelling (i.e., simultaneous modelling of the lens and source galaxy). The latter approach allows us to compare the observed and the simulated lensed images in a fully consistent manner, however, it is significantly more time-consuming and introduces another layer of complexity and modelling choices to be made.

To quantify the effect of signal suppression and test the feasibility of the improved methodology, we apply it to deep high-signal-to-noise HST/ACS/F814W-observations of the lens system SDSS J0946+1006, commonly known as the Double Einstein Ring, and a set of 17 mock perturbed lensed images mimicking SDSS J0946+1006. The results presented in Fig. 2 show a striking suppression of the true surface-brightness anomalies in the course of the full grid-based smooth lens modelling (up to  $\sim 2$  orders of magnitude for the most perturbed lens systems), especially for the lowest considered  $k$ -values (i.e., largest scales). This finding implies that the surface-brightness anomalies measured in the real data might be significantly suppressed as well and the analysed lens system could in reality be considerably more perturbed. Therefore we conclude that signal suppression in the grid-based smooth lens modelling is an important issue that requires more attention from the lensing community. If not properly understood and accounted for, it might lead to severely biased results with respect to both the level of substructure in lens galaxies and the properties of lensed high-redshift sources.

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