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# PERMUTATION REPRESENTATION OF GROUPS WITH BOOLEAN ORTHOGONALITIES

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**Abstract**

Reduced rings and lattice-ordered groups are examples of groups with Boolean orthogonalities. In this note we show that any group with a Boolean orthogonality satisfying a finiteness condition introduced by Stewart is isomorphic with a group of homeomorphisms of a topological space, in which two homeomorphisms are orthogonal if and only if they have disjoint supports.

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## Introduction

Groups with Boolean orthogonalities were introduced by Davis (1971a) principally to obtain representation and extension theorems for lattice-ordered groups and reduced rings in a uniform way. Since then Boolean orthogonalities on groups and rings have been studied by Davis (1971b), (1975) and Stewart (1975). A universal algebraic study of algebras with Boolean orthogonalities has been undertaken by Cornish (1975) and Cornish and Stewart (1977).

In this note we show that every group $G$ with a Boolean orthogonality $\perp$ satisfying a certain finiteness condition can be faithfully represented as a group of permutations $\hat{G} = \{\hat{g}: g \in G\}$ of a set $X$ in such a way that $g \perp h$ holds if and only if $\{x \in X: x\hat{g} = x\} \cup \{x \in X: x\hat{h} = x\} = X$, for all $g\ h \in G$.





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## Boolean orthogonalities on permutation groups

Let $G$ denote a group and $\perp$ a binary relation on $G$. For a non-empty subset $S$ of $G$ we define $S^{\perp} = \{g \in G: g \perp h \text{ for all } h \in S\}$ and $S^{\perp\perp} = (S^{\perp})^{\perp}$. If $S = \{h\}$ is a singleton set then we denote $S^{\perp}$ and $S^{\perp\perp}$ by $h^{\perp}$ and $h^{\perp\perp}$ respectively.

The relation $\perp$ is a *Boolean orthogonality* on $G$ if for each non-empty subset $S$ of $G$ the set $S^{\perp}$ is a subgroup of $G$ and, in addition,

(1) $\perp$ is a symmetric relation,

(2) if $g \perp g$ then $g = 1$,

(3) if $g \perp h$ then $k^{-1}gk \perp k^{-1}hk$ for all $k \in G$,

(4) if $g^{\perp\perp} \cap h^{\perp\perp} = 1$ then $g \perp h$.

THEOREM 1 (Davis (1971b)). *Let $G$ be a group with a Boolean orthogonality $\perp$. Then $B(G) = \{S^{\perp}: S$ is a non-empty subset of $G\}$, ordered by inclusion, is a complete Boolean algebra with $A \wedge B = A \cap B$ and $A \vee B = (A^{\perp} \cap B^{\perp})^{\perp}$ for all $A, B \in B(G)$.*

Let $G$ be a subgroup of the symmetric group on a set $X$. The support of $g \in G$ is the set $\operatorname{supp}(g) = \{x \in X: gh \neq x\}$. We shall say that $G$ is a *d-group* (for "directed group") if whenever $g_1, g_2 \in G$ and $\varnothing \neq \operatorname{supp}(g_1) \cap \operatorname{supp}(g_2)$ there is an $h \in G$ with $\varnothing \neq \operatorname{supp}(h) \subseteq \operatorname{supp}(g_1) \cap \operatorname{supp}(g_2)$.

PROPOSITION 2. *Let $G$ be a subgroup of the symmetric group on $X$. Define the binary relation $\perp$ on $G$ by $g \perp h$ if $\operatorname{supp}(g) \cap \operatorname{supp}(h) \neq \varnothing$. If $G$ is a d-group then $\perp$ is a Boolean orthogonality on $G$. Conversely if $\perp$ is a Boolean orthogonality and for each $(x, g) \in X \times G$ with $xg = x$ there is an $h \in G$ with $x \in \operatorname{supp}(h)$ and $\operatorname{supp}(g) \cap \operatorname{supp}(h) = \varnothing$, then $G$ is a d-group.*

PROOF. Suppose $G$ is a $d$-group. The relation $\perp$ is clearly symmetric and satisfies (2) of the definition of a Boolean orthogonality. Furthermore each $S^{\perp}$ is clearly a subgroup of $G$ and condition (3) follows from the fact that $\operatorname{supp}(k^{-1}gk) = \operatorname{supp}(g)k$ for all $g, k \in G$. Suppose that $g, h \in G$ and $g \perp h$ does not hold. Then $\operatorname{supp}(g) \cap \operatorname{supp}(h) \neq \varnothing$ so there is a $k \in G$ with $\varnothing \neq \operatorname{supp}(k) \subseteq \operatorname{supp}(g) \cap \operatorname{supp}(h)$. If $g' \in g^{\perp}$ then $\operatorname{supp}(g') \cap \operatorname{supp}(k) \subseteq \operatorname{supp}(g') \cap \operatorname{supp}(g) = \varnothing$ so $g' \in k^{\perp}$. That is $g^{\perp} \subseteq k^{\perp}$. Similarly $h^{\perp} \subseteq k^{\perp}$ so $1 \neq k \in g^{\perp\perp} \cap h^{\perp\perp}$, and therefore $\perp$ is a Boolean orthogonality.

Suppose conversely that $\perp$ is a Boolean orthogonality and for all $(x, g) \in X \times G$ with $xg = x$ there is a $g' \in G$ with $x \in \operatorname{supp}(g')$ and $\operatorname{supp}(g) \cap \operatorname{supp}(g') = \varnothing$. Assume that $g, h \in G$ and $\operatorname{supp}(g) \cap \operatorname{supp}(h) \neq \varnothing$. Since $\perp$ is a Boolean orthogonality we can find $1 \neq k \in g^{\perp\perp} \cap h^{\perp\perp}$.

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Suppose $x \notin \operatorname{supp}(g)$. Then there is a $g' \in G$ with $x \in \operatorname{supp}(g')$ and $\operatorname{supp}(g) \cap \operatorname{supp}(g') = \varnothing$, so $\operatorname{supp}(g') \cap \operatorname{supp}(k) = \varnothing$ and therefore $x \notin \operatorname{supp}(k)$. Similarly $x \notin \operatorname{supp}(h)$ implies $x \notin \operatorname{supp}(k)$ so $\varnothing \neq \operatorname{supp}(k) \subseteq \operatorname{supp}(g) \cap \operatorname{supp}(h)$. That is, $G$ is a $d$-group.

Let $G$ be a group of permutations of $X$. If $G$ is a $d$-group then we shall equip $G$ with the Boolean orthogonality $\perp$ defined by $g \perp h$ if and only if $\operatorname{supp}(g) \cap \operatorname{supp}(h) = \varnothing$. We let $\Sigma_G$ denote the topology on $X$ having the sets $\operatorname{supp}(g)$, $g \in G$, as sub-basic open sets. The condition that $G$ be a $d$-group is almost the condition that the sets $\operatorname{supp}(g)$, $g \in G$, are *basic* open sets for the $\Sigma_G$-topology. We give a name to this latter phenomenon: $G$ is a $d^*$-group if the sets $\operatorname{supp}(g)$, $g \in G$, form a base for the $\Sigma_G$-topology on $X$. Thus, $G$ is a $d^*$-group if and only if

(1) for all $g, h \in G$, if $x \in \operatorname{supp}(g) \cap \operatorname{supp}(h)$ then there is a $k \in G$ with $x \in \operatorname{supp}(k) \subseteq \operatorname{supp}(g) \cap \operatorname{supp}(h)$,

(2) no point of $x$ is fixed by all $g \in G$.

LEMMA 3. *Let $G$ be a $d^*$-group on the set $X$. For all $g \in G$, $g^{\perp\perp} = \{k \in G\colon \operatorname{supp}(k) \subseteq \overline{\operatorname{supp}(g)}\}$, where $\bar{S}$ denotes the closure of $S \subseteq X$ for the $\Sigma_G$-topology.*

PROOF. Take $g \in G$ and suppose $\operatorname{supp}(k) \subseteq \overline{\operatorname{supp}(g)}$. If $g \perp h$ then $\operatorname{supp}(g) \cap \operatorname{supp}(h) = \varnothing$ so $\operatorname{supp}(h) \cap \overline{\operatorname{supp}(g)} = \varnothing$ and therefore $\operatorname{supp}(h) \cap \operatorname{supp}(k) = \varnothing$. That is, $k \in g^{\perp\perp}$.

Suppose on the other hand that $k \in g^{\perp\perp}$. If $\overline{\operatorname{supp}(g)} = X$ there is nothing to show. Suppose otherwise that $O$ is a non-empty open set contained in $X \setminus \overline{\operatorname{supp}(g)}$. Then $O = \bigcup\{\operatorname{supp}(h)\colon \operatorname{supp}(h) \cap \operatorname{supp}(g) = \varnothing\}$ so $\operatorname{supp}(k) \cap O = \bigcup\{\operatorname{supp}(k) \cap \operatorname{supp}(h)\colon \operatorname{supp}(h) \cap \operatorname{supp}(g) = \varnothing\} = \varnothing$. That is, $\operatorname{supp}(k) \subseteq \overline{\operatorname{supp}(g)}$.

COROLLARY 4. *Let $G$ be a $d^*$-group on the set $X$. Then the following are equivalent, for all $g, h \in G$:*

(1) $g \perp h$,

(2) $\operatorname{supp}(g) \cap \operatorname{supp}(h) = \varnothing$,

(3) $\overline{\operatorname{supp}(g)} \cap \overline{\operatorname{supp}(h)}$ *has empty interior for the $\Sigma_G$-topology.*

We note that every group is a $d$-group in its right regular representation. The Boolean orthogonality defined in proposition is, however, trivial in this case in the sense that $g \perp h$ holds if and only if $g = 1$ or $h = 1$.

We note also that not every permutation group is a $d$-group. For instance if $G$ acts sharply doubly transitively on $X$ then $G$ is not a $d$-group. Indeed, since no permutation can have a one-element support the symmetric group on three or more letters is not a $d$-group.

***Permutation representation***

Throughout this section $G$ will denote a group with a Boolean orthogonality $\perp$.

A subgroup $H$ of $G$ is a *$\pi$-subgroup* if $h_1^{\perp\perp} \vee \cdots \vee h_n^{\perp\perp} \subseteq H$ for each finite subset $\{h_1, \ldots, h_n\}$ of $H$.

The $\pi$-subgroups of $G$ ordered by inclusion clearly form a complete lattice.

LEMMA 5. *Let $H$ be a $\pi$-subgroup of $G$ and $g$ an element of $G$. Let $\langle H \cup \{g\}\rangle$ be the smallest $\pi$-subgroup of $G$ containing $H \cup \{g\}$. Then $\langle H \cup \{g\}\rangle = \bigcup \{F^{\perp\perp} \vee g^{\perp\perp}\colon F \subseteq G, F$ finite$\}$.*

PROOF. If $F = \{h_1, \ldots, h_n\}$ is a finite subset of $H$ then $F^{\perp\perp} = h_1^{\perp\perp} \vee \cdots \vee h_n^{\perp\perp}$ so $F^{\perp\perp} \vee g^{\perp\perp} \subseteq \langle H \cup \{g\}\rangle$. If $k_1, k_2 \in G$ then $k_1^{\perp} \cap k_2^{\perp} \subseteq (k_1k_2^{-1})^{\perp}$, since sets of the form $S^{\perp}$ are subgroups of $G$, so $k_kk_2^{-1} \in k_1^{\perp\perp} \vee k_2^{\perp\perp}$. Consequently if $k_1 \in F_1^{\perp\perp} \vee g^{\perp\perp}$ and $k_2 \in F_2^{\perp\perp} \vee g^{\perp\perp}$, where $F_1$ and $F_2$ are finite subsets of $H$, then

$$k_1k_2^{-1} \in k_1^{\perp\perp} \vee k_2^{\perp\perp} \subseteq F_1^{\perp\perp} \vee F_2^{\perp\perp} \vee g^{\perp\perp} = (F_1 \cup F_2)^{\perp\perp} \vee g^{\perp\perp}.$$

Hence $\bigcup\{F^{\perp\perp} \vee g^{\perp\perp}\colon F \subseteq G, F \text{ finite}\}$ is a subgroup of $G$, and therefore by its definition a $\pi$-subgroup, contained in $\langle H \cup \{g\}\rangle$, so equality holds.

We call the meet-irreducible elements of the lattice of $\pi$-subgroups of $G$ the *prime $\pi$-subgroups of $G$*. Thus, a $\pi$-subgroup $P$ of $G$ is a prime $\pi$-subgroup if and only if for all $\pi$-subgroups $H_1$, $H_2$ of $G$, $H_1 \cap H_2 \subseteq P$ implies $H_1 \subseteq P$ or $H_2 \subseteq P$. Equivalently, a $\pi$-subgroup $P$ is prime if and only if for all $g, h$ in $G$, $g^{\perp\perp} \cap h^{\perp\perp} \subseteq P$ implies $g \in P$ or $h \in P$.

We shall now see that when $G$ satisfies a certain finiteness condition for $\perp$, the prime $\pi$-subgroups of $G$ intersect in 1. This fact allows us to represent such groups as $d^*$-groups of homeomorphisms.

DEFINITION. The Boolean orthogonality $\perp$ on $G$ is *finite* if for all $g, h \in G$ there is a finite subset $F$ of $G$ with $g^{\perp\perp} \cap h^{\perp\perp} = F^{\perp\perp}$.

In the context of Boolean orthogonalities on rings this finiteness condition was introduced by Stewart (1975).

PROPOSITION 6. *Let $G$ be a group with a finite Boolean orthogonality $\perp$ and suppose $g_0 \neq 1$ in $G$. Then there is a subgroup of $G$ maximal with respect to being a $\pi$-subgroup of $G$ not containing $g_0$. Any such $\pi$-subgroup is prime.*

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PROOF. The set of $\pi$-subgroups of $G$ is plainly inductive so if $g_0 \neq 1$ then there is a subgroup $H$ of $G$ maximal with respect to being a $\pi$-subgroup not containing $g_0$. Assume that $H$ is not prime. Then there exist $g, k \notin H$ such that $g^{\perp\perp} \cap k^{\perp\perp} \subseteq H$. Since $g_0 \in \langle H \cup \{g\}\rangle \cap \langle H \cup \{k\}\rangle$ there are finite subsets $F_1$, $F_2$ of $H$ such that

$$\begin{aligned} g_0 &\in (F_1^{\perp\perp} \vee g^{\perp\perp}) \cap (F_1^{\perp\perp} \vee k^{\perp\perp}) \\ &= (F_1^{\perp\perp} \cap F_2^{\perp\perp}) \vee (F_1^{\perp\perp} \cap k^{\perp\perp}) \vee (F_2^{\perp\perp} \cap g^{\perp\perp}) \vee (g^{\perp\perp} \cap k^{\perp\perp}). \end{aligned}$$

Since $\perp$ is finite, there is a finite subset $F$ of $G$ with $F^{\perp\perp} = g^{\perp\perp} \cap k^{\perp\perp}$. Since $g^{\perp\perp} \cap k^{\perp\perp} \subseteq H$ then $F \subseteq H$ and $g_0 \in H$–a contradiction, so $H$ is a prime $\pi$-subgroup.

COROLLARY 7. *If $G$ is a group with a finite Boolean orthogonality $\pi$ then $\bigcap \{P \subseteq G$: $P$ is a prime $\pi$-subgroup$\} = 1$.*

It is easily seen from condition (3) for a Boolean orthogonality that each conjugate of a (prime) $\pi$-subgroup of $G$ is again a $\pi$-subgroup of $G$.

Let $G$ and $H$ be groups with Boolean orthogonalities, both denoted by $\perp$. A group homomorphism $f\colon G \to H$ is a $\perp$*-homomorphism* if $g \perp h$ in $G$ implies $gf \perp hf$ in $H$. A $\perp$*-isomorphism* $f\colon G \to H$ is a bijection such that both $f$ and $f^{-1}$ are $\perp$-homomorphisms.

We now let $X = \bigcup \{G/P$: $P$ is a proper prime $\pi$-subgroup of $G\}$ be the disjoint union of left cosets of proper prime $\pi$-subgroups of $G$.

If $H$ is a subgroup of the symmetric group on $X$ and also a $d$-group we shall equip $H$ with the Boolean orthogonality $\perp$ defined by $g \perp h$ if and only if $\operatorname{supp}(g) \cap \operatorname{supp}(h) = \varnothing$.

THEOREM 8. *Let $G$ be a group with a finite Boolean orthogonality $\perp$. Then $G$ is $\perp$-isomorphic with a $d^*$-group of homeomorphisms of a topological space.*

PROOF. Given $g \in G$ we define $\hat{g}\colon X \to X$ by $(Pk)\hat{g} = Pkg$. Then $\hat{G} = \{\hat{g}$: $f \in G\}$ is a subgroup of the symmetric group on $X$, and $g \to \hat{g}$ is an isomorphism onto $\hat{G}$ since the intersection of all prime $\pi$-subgroups of $G$ is 1. We now see that $g \perp h$ in $G$ if and only if $\operatorname{supp}(\hat{g}) \cap \operatorname{supp}(\hat{h}) = \varnothing$. Suppose $g \perp h$ in $G$. If $(Pk)\hat{g} \neq Pk$ then $kgk^{-1} \notin P$ That is, $g \in k^{-1}pk$ and so, since $k^{-1}$ is a prime $\pi$-subgroup and $g^{\perp\perp} \cap h^{\perp\perp} \subseteq k^{-1}pk$, we have $h \in k^{-1}pk$. That is, $(Pk)\hat{h} = Pk$ so $\operatorname{supp}(\hat{g}) \subseteq X \setminus \operatorname{supp}(\hat{h})$.

Suppose conversely that $\operatorname{supp}(\hat{g}) \cap \operatorname{supp}(\hat{h}) = \varnothing$. Let $P$ be a prime $\pi$-subgroup of $G$. Then $Pg = P$ or $Ph = p$ so $g^{\perp\perp} \subseteq P$ or $h^{\perp\perp} \subseteq P$. In any case, $g^{\perp\perp} \cap h^{\perp\perp} \subseteq P$. Since the prime $\pi$-subgroups of $G$ meet in 1 we have $g^{\perp\perp} \cap h^{\perp\perp} = 1$ and therefore $g \perp h$.

Thus the relation $\perp$ on $\hat{G}$ defined by $\hat{g} \perp \hat{h}$ if and only if $\operatorname{supp}(g) \cap \operatorname{supp}(h) = \varnothing$ is a Boolean orthogonality. Suppose that $g, h \in G$ and $Pk_0 \in \operatorname{supp}(\hat{g}) \cap \operatorname{supp}(\hat{h})$. Then $g, h \in k_0^{-1}Pk_0$. Take $k \in g^{\perp\perp} \cap h^{\perp\perp} \setminus k_0^{-1}Pk_0$, so that $Pk_0 \in \operatorname{supp}(k)$. If $P^1k_1 \in \operatorname{supp}(k)$ then $k \notin k_1^{-1}P^1k_1$ so $g^{\perp\perp} \cap h^{\perp\perp} \not\subseteq k_1^{-1}P^1k_1$. In this case $g, h \notin k_1^{-1}P^1k_1$ so $P^1k_1 \in \operatorname{supp}(g) \cap \operatorname{supp}(h)$. That is, $\hat{G}$ is a $d^*$-group on $X$. Clearly each $\hat{g} \in \hat{G}$ is a homeomorphism of $X$ for the $d^*$-topology.

## Transitive representations and examples

We have seen that every group $G$ with a finite Boolean orthogonality $\perp$ is $\perp$-isomorphic with a $d^*$-group of homeomorphisms of a topological space.

THEOREM 9. *Let $G$ be a group with a finite Boolean orthogonality $\perp$. Then $G$ is $\perp$-isomorphic with a transitive $d^*$-group $\hat{G}$ of homeomorphisms of a topological space equipped with the $\Sigma_{\hat{G}}$-topology if and only if $G$ has a prime $\pi$-subgroup $P$ such that $\bigcap\{k^{-1}Pk\colon k \in G\} = 1$.*

PROOF. Suppose that $G$ has a prime $\pi$-subgroup $P$ whose conjugates meet in 1. Then by proceeding as in Theorem 8 with the set of all prime $\pi$-subgroups of $G$ replaced by conjugates of $P$ we see that $G$ has a transitive representation as a $d^*$-group of permutations on $X = \bigcup\{G/k^{-1}Pk\colon k \in G\}$.

Suppose conversely that $G$ is ($\perp$-isomorphic with) a $d^*$-group of permutations acting transitively on $X$. Take $x \in X$. We see that the point stabilizer $G_x = \{g \in G\colon xg = x\}$ contains a prime $\pi$-subgroup of $G$. The set of $\pi$-subgroups of $G$ contained in $G_x$ is inductive so let $H$ be maximal with respect to being a $\pi$-subgroup of $G$ contained in $G_x$. Suppose that $g, h \in H$ but $g^{\perp\perp} \cap h^{\perp\perp} \subseteq H$. Then $\langle H \cup \{g\}\rangle$ contains some $g'$ with $xg' \neq x$ and $\langle H \cup \{h\}\rangle$ contains some $h'$ with $xh' \neq x$. Then $g' \in F_1^{\perp\perp} \vee g^{\perp\perp}$ and $h' \in F_2^{\perp\perp} \vee h^{\perp\perp}$, for some finite subsets $F_1$, $F_2$ of $H$, so $(g')^{\perp\perp} \cap (h')^{\perp\perp} \subseteq H$. Then there is a $k \in G$ with $x \in \operatorname{supp}(k) \subseteq \operatorname{supp}(g) \cap \operatorname{supp}(h)$ so $k \in (g')^{\perp\perp} \cap (h')^{\perp\perp} \subseteq H \subseteq G_x$–a contradiction.

Thus, $H$ is a *prime* $\pi$-subgroup of $G$ and $\bigcap\{k^{-1}Hk\colon k \in G\} \subseteq \bigcap\{k^{-1}Hk\colon k \in G\} \subseteq \bigcap\{k^{-1}G_xk\colon k \in G\} = 1$.

The motivating class of groups for the study of (finite) Boolean orthogonalities is the class of lattice-ordered groups: a lattice ordered group $G$ has a finite Boolean orthogonality $\perp$ defined by $g \perp h$ in $G$ if and only if $|g| \wedge |h| = 1$ (where $|g| = g \vee g^{-1}$).

Our representation theorem for groups with finite Boolean orthogonalities was modelled on Holland's (1963) representation of lattice-ordered groups as groups of order-preserving permutations of chains.

Another class of examples is provided by the (not necessarily commutative) reduced rings $R$ where we define $g \perp h$ if and only if $gh = 0$. Then $g^{\perp}$ is just the left annihilator of $g$ in $R$ which, since $R$ is reduced, is a two-sided ideal. It follows then that $g^{\perp} \subseteq (gh)^{\perp}$ for all $g, h \in R$, so *$\pi$-subgroups* of $R$ are in fact two-sided ideals of $R$. Our representation theorem can then be converted to a sheaf-theoretic representation for $R$. This has been carried out by Davis (1971b) and Stewart (1975).

Another–by Theorem 8 fairly typical–example is $\Gamma(\mathbf{R}^2)$, the homeomorphism group of $\mathbf{R}^2$. Indeed, suppose $g, h \in \Gamma(\mathbf{R}^2)$ and $x \in \operatorname{supp}(g) \cap \operatorname{supp}(h) = U$. Find inside $U$ an isometric copy $C$ of the unit open disc $O$, with $x \in C$. Let $\varphi$: $O \to C$ be a homeomorphism from $O$ onto $C$. Define a map $\psi$: $\overline{O} \to \overline{O}$ as follows:

$z\psi = e^{2\pi i d(z, \partial O)z}$, where $d$ is the Euclidean metric and $\partial O$ is the boundary of $O$.

Now define $k$: $\mathbf{R}^2 \to \mathbf{R}^2$ by

$$zk = \begin{cases} z & \text{if } z \notin C, \\ z\phi^{-1}\psi\varphi & \text{if } z \in C. \end{cases}$$

Then $k$ is a homeomorphism of $\mathbf{R}^2$ and $x \in \operatorname{supp}(k) \subseteq U$, so $\Gamma(\mathbf{R}^2)$ is a $d^*$-group and $\perp$, defined by $g \perp h$ if $\operatorname{supp}(g) \cap \operatorname{supp}(h) = \varnothing$, is a Boolean orthogonality.

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