Acknowledgement

The author is grateful to the referee for the reference to Takács (1979).

References

Csáki, E. and Mohanty, S. (1981) Excursion and meander in random walk. *Canad. J. Statist.* **9**, 57–70.

Csáki, E. and Mohanty, S. (1986) Some joint distributions for conditional random walks. *Canad. J. Statist.* **14**, 19–28.

Smith, L. and Diaconis, P. (1988) Honest Bernoulli excursions. *J. Appl. Prob.* **25**, 464–477.

Takács, L. (1979) Fluctuation problems for Bernoulli trials. *SIAM Survey* **21**, 222–228.

Vervaat, V. (1979) A relation between Brownian bridge and Brownian excursion. *Ann. Prob.* **7**, 143–149.

Graduate School of Management,
Rutgers University,
Newark, NJ 07102–1895,
USA.

Yours sincerely,
Arie Harel

Dear Editor,

A note on the stability of interior ESSs in a diploid population

An outstanding problem in evolutionarily stable strategy (ESS) theory has been the consistency of the underlying genetic structure of a population with the method of finding stable equilibria by the ESS criteria of comparing phenotypic (i.e. strategy) fitnesses. Even the most basic problem of analyzing the stability properties of an interior ESS, S^*, in a randomly-mating diploid population where strategy is determined at a single multi-allelic locus has produced misleading and/or incorrect statements in the literature.

For instance, Cressman and Hines (1984) asserted in their Theorem 4.1(d) that, for semi-dominant inheritance patterns (i.e. incompletely dominant in the terminology of Cressman (1988)), the mean strategy evolves either towards the boundary or towards S^*. The proof relied on the exponentiation of a matrix product and, to be correct, needed the matrices to commute — an unwarranted assumption in the circumstances. The inappropriateness of this approach was pointed out to one of us in private communications with Josef Hofbauer. As a consequence, the corollary at the end of this paper (asserting that S^* is globally stable if there are three or less alleles) is incorrect as can be seen by a counter-example in the last exercise of Section 28.4 of Hofbauer and Sigmund (1988).

On the other hand, the other main conclusions of Cressman and Hines (1984), that is Theorems 4.1 and 4.2, remain valid. In particular, S^* is locally stable for semi-dominant

inheritance patterns (see Cressman (1988), Theorem 6.4 and Hofbauer and Sigmund (1988), Section 28 for alternate proofs). However, this local stability cannot be claimed, or assumed, to hold for all inheritance patterns. For example, the apparently general discussion by Hines and Bishop (1984) is restricted to cases in which the covariance matrix is 'effectively constant', thereby excluding a somewhat vaguely specified class of cases. A paper in preparation by one of us re-examines, in more depth, the situation addressed by Hines and Bishop and establishes the existence of non-generic counter-examples to the general claim — that is, counter-examples in which the covariance matrix cannot be considered effectively constant.

References

CRESSMAN, R. (1988) Frequency-dependent viability selection (a single-locus, multi-phenotype model). *J. Theor. Biol.* **130**, 147–165.

CRESSMAN, R. AND HINES, W. G. S. (1984) Evolutionarily stable strategies of diploid populations with semi-dominant inheritance patterns. *J. Appl. Prob.* **21**, 1–9.

HINES, W. G. S. AND BISHOP, D. T. (1984) On the local stability of an evolutionarily stable strategy in a diploid population. *J. Appl. Prob.* **21**, 215–224.

HOFBAUER, J. AND SIGMUND, K. (1988) *The Theory of Evolution and Dynamical Systems*. Cambridge University Press.

Department of Mathematics,
Wilfrid Laurier University,
Waterloo, Ontario,
Canada N2L 3C5.

Yours sincerely,
R. CRESSMAN

Department of Mathematics and Statistics,
University of Guelph,
Guelph, Ontario,
Canada N1G 2W1.

W. G. S. HINES