Appendix 10

The linearly independent reaction parameters for various reactions and their relation to the helicity amplitudes

We give here the expressions for the fundamental CM reaction parameters (Section 5.6) in terms of the helicity amplitudes for various reactions. We also list the relation between the CM reaction parameters and those used in the Argonne convention.

A10.1 $0 + 1/2 \to 0 + 1/2$

An example is $\pi p \to \pi p$. We have

$$d\sigma/dt = |H_{++}|^2 + |H_{+-}|^2.$$

In Table A10.1 the expressions in the right-hand column correspond to the CM reaction parameters multiplied by $d\sigma/dt$. The order of the labels is (target|recoil).

The relation to Argonne Lab parameters is given in Table A10.2.

Table A10.1.

CM parameter	Shorthand notation	Expression in terms of helicity amplitudes
$(0Y\|00) = (00\|0Y)$	P	$2\ \mathrm{Im}\ (H^*_{+-}H_{++})$
$(0X\|0X)$	D_{XX}	$\|H_{++}\|^2 - \|H_{+-}\|^2$
$(0Z\|0X)$	D_{ZX}	$-2\ \mathrm{Re}\ (H_{++}H^*_{+-})$

https://doi.org/10.1017/9781009402040.024 Published online by Cambridge University Press

Table A10.2.

CM parameters	Argonne Lab parameters
P	$-P_{\mathrm{ARG}}$
D_{XX}	$-\cos\theta_{\mathrm{R}}\ D_{SS} + \sin\theta_{\mathrm{R}}\ D_{LS}$
D_{ZX}	$\sin\theta_{\mathrm{R}}\ D_{SS} + \cos\theta_{\mathrm{R}}\ D_{LS}$

In the old literature one finds $R = D_{SS}$, $A = D_{LS}$.

A10.2 $A(1/2) + B(1/2) \to 0 + 0$

An example is $\bar{p}p \to \pi\pi$. We have

$$\frac{d\sigma}{dt} = \frac{1}{2}\left(|H_{++}|^2 + |H_{+-}|^2\right).$$

In Table A10.3 the expressions in the right-hand column are the CM reaction parameters multiplied by $d\sigma/dt$.

A10.3 $A + B \to A + B$ all with spin 1/2

Because of their complexity we list in the second column of Table A10.4 a convenient set of 36 linearly independent Argonne Lab parameters. The cases $A \neq B$ and $A = B$ are both included.

Next, in Tables A10.5 and A10.6 we give expressions for the CM reaction parameters in terms of the helicity amplitudes. The results hold whether A is different from B or is identical to it. If $A = B$ then one should put $\phi_6 = -\phi_5$. Also, those parameters marked † are then no longer independent.

We have

$$\frac{d\sigma}{dt} = \frac{1}{2}\left(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2|\phi_5|^2 + 2|\phi_6|^2\right). \qquad \text{(A10.1)}$$

In Tables A10.5 and A10.6 the entries in the right hand column correspond to the CM reaction parameters multiplied by $d\sigma/dt$.

Table A10.3.

CM parameters	Shorthand notation	Expression in terms of helicity amplitudes
$(OY\|00)$	$A^{(B)}$	$2\ \mathrm{Im}\ (H_{++}H^*_{+-})$
$(Y0\|00)$	$A^{(A)} = -A^{(B)}$	$2\ \mathrm{Im}\ (H_{++}H^*_{-+})$
$(XX\|00)$	A_{XX}	$\|H_{++}\|^2 - \|H_{+-}\|^2$
$(XZ\|00)$	A_{XZ}	$-2\ \mathrm{Re}\ (H_{++}H^*_{+-})$

https://doi.org/10.1017/9781009402040.024 Published online by Cambridge University Press

Table A10.4. 36 linearly independent Argonne Lab parameters. As usual θ_L is the Lab scattering angle of A and θ_R is the recoil angle of B

Type of measurement	$A = B$	Additional parameters if $A \neq B$	Relation when $A = B$
No spin	$d\sigma/dt$		
One spin	$A^{(A)}_{ARG}$	$A^{(B)}_{ARG}$	$A^{(A)}_{ARG} = A^{(B)}_{ARG}$
Two spins	A_{SS}, A_{LL}, A_{NN}		
	A_{SL}	A_{LS}	$A_{LS} = A_{SL}$
	$D^{(B)}_{LL}, D^{(B)}_{NN}$		
	$D^{(B)}_{SS}$	$D^{(A)}_{SS}$	$D^{(A)}_{SS} = -\sin(\theta_R + \theta_L)D^{(B)}_{SL} - \cos(\theta_R + \theta_L)D^{(B)}_{SS}$
	$D^{(B)}_{SL}$	$D^{(A)}_{SL}$	$D^{(A)}_{SL} = -\cos(\theta_R + \theta_L)D^{(B)}_{SL} + \sin(\theta_R + \theta_L)D^{(B)}_{SS}$
	$K^{(A)}_{SS}, K^{(A)}_{LL}, K^{(A)}_{NN}$		
	$K^{(A)}_{SL}$	$K^{(A)}_{LS}$	$K^{(A)}_{LS} = -K^{(A)}_{SL} + \tan\theta_R \left[K^{(A)}_{SS} - K^{(A)}_{LL}\right]$
Three spins	$(SN\|0S)$	$(SN\|S0)$	$-(SN\|S0) = \cos(\theta_R + \theta_L)(NS\|0S) + \sin(\theta_R + \theta_L)(NS\|0L)$
	$(NS\|0S)$		
	$(SN\|0L)$	$(SN\|L0)$	$(SN\|L0) = \sin(\theta_R + \theta_L)(NS\|0S) - \cos(\theta_R + \theta_L)(NS\|0L)$
	$(LS\|0N)$		
	$(SL\|0N)$		
	$(LN\|0S)$	$(LN\|S0)$	$(LN\|S0) = (\cos\theta_L/\cos\theta_R)(NL\|0S) - \sin(\theta_L + \theta_R)[(NS\|0S) + \tan\theta_R(NS\|0L)]$
	$(NL\|0S)$		
	$(SS\|0N)$	$(SS\|N0)$	$(SS\|N0) = (SS\|0N)$
		$(LN\|0L)$	$(LN\|0L) = (SN\|0S)\tan\theta_R [(LN\|0S) + (SN\|0L)]$
Four spins	$(SS\|SS)$		
	$(SS\|LS)$	$(SS\|SL)$	$(SS\|SL) = (SS\|LS) + \sin(\theta_R + \theta_L)(A_{NN} - 1)$

https://doi.org/10.1017/9781009402040.024 Published online by Cambridge University Press

Table A10.5. Relation between CM reaction parameters and helicity amplitudes for one- and two-spin measurements

Shorthand	CM parameter	Formula
$A^{(B)}$	$(0Y\|00)$	$\mathrm{Im}\,[\phi_5^*(\phi_1+\phi_3)-\phi_6^*(\phi_2-\phi_4)]$
$A^{(A)}$	$\dagger(Y0\|00)$	$\mathrm{Im}\,[\phi_6^*(\phi_1+\phi_3)-\phi_5^*(\phi_2-\phi_4)]$
$D^{(B)}_{XX}$	$(0X\|0X)$	$\mathrm{Re}\,(\phi_1\phi_3^*+\phi_2\phi_4^*)-\|\phi_5\|^2+\|\phi_6\|^2$
$D^{(B)}_{ZZ}$	$(0Z\|0Z)$	$(1/2)(\|\phi_1\|^2-\|\phi_2\|^2+\|\phi_3\|^2$ $-\|\phi_4\|^2-2\|\phi_5\|^2+2\|\phi_6\|^2)$
$D^{(B)}_{ZX}$	$(0Z\|0X)$	$-\,\mathrm{Re}\,[(\phi_1+\phi_3)\phi_5^*+(\phi_2-\phi_4)\phi_6^*]$
$D^{(B)}_{YY}$	$(0Y\|0Y)$	$\mathrm{Re}\,[\phi_1\phi_3^*-\phi_2\phi_4^*]+\|\phi_5\|^2+\|\phi_6\|^2$
$D^{(A)}_{XX}$	$\dagger(X0\|X0)$	$\mathrm{Re}\,(\phi_1\phi_3^*+\phi_2\phi_4^*)+\|\phi_5\|^2-\|\phi_6\|^2$
$D^{(A)}_{XZ}$	$\dagger(X0\|Z0)$	$\mathrm{Re}\,[(\phi_1+\phi_3)\phi_6^*+(\phi_2-\phi_4)\phi_5^*]$
A_{XX}	$(XX\|00)$	$\mathrm{Re}\,(\phi_1\phi_2^*+\phi_3\phi_4^*)$
A_{YY}	$(YY\|00)$	$\mathrm{Re}\,(\phi_3\phi_4^*-\phi_1\phi_2^*+2\phi_5\phi_6^*)$
A_{ZZ}	$(ZZ\|00)$	$(1/2)(\|\phi_1\|^2+\|\phi_2\|^2-\|\phi_3\|^2-\|\phi_4\|^2)$
A_{XZ}	$(XZ\|00)$	$\mathrm{Re}\,[(\phi_1-\phi_3)\phi_6^*-(\phi_2+\phi_4)\phi_5^*]$
A_{ZX}	$\dagger(ZX\|00)$	$\mathrm{Re}\,[(\phi_1-\phi_3)\phi_5^*-(\phi_2+\phi_4)\phi_6^*]$
$K^{(A)}_{XX}$	$(X0\|0X)$	$\mathrm{Re}\,\left(\phi_1\phi_4^*+\phi_3\phi_2^*\right)$
$K^{(A)}_{XZ}$	$(X0\|0Z)$	$\mathrm{Re}\,\left[(\phi_1-\phi_3)\phi_6^*+(\phi_2+\phi_4)\phi_5^*\right]$
$K^{(A)}_{ZZ}$	$(Z0\|0Z)$	$(1/2)(\|\phi_1\|^2-\|\phi_2\|^2-\|\phi_3\|^2+\|\phi_4\|^2)$
$K^{(A)}_{ZX}$	$\dagger(Z0\|0X)$	$-\,\mathrm{Re}\,\left[(\phi_1-\phi_3)\phi_5^*+(\phi_2+\phi_4)\phi_6^*\right]$
$K^{(A)}_{YY}$	$(Y0\|0Y)$	$\mathrm{Re}\,\left(\phi_1\phi_4^*-\phi_3\phi_2^*+2\phi_5\phi_6^*\right)$

In Tables A10.7 and A10.8 we show the relationship between the CM reaction parameters and the Argonne Lab parameters. (Many of these are due to N.H. Buttimore, unpublished.) The tables are arranged so that pairs of relations can be used to solve for the Argonne Lab parameters in terms of the CM reaction parameters and then, via Tables A10.5 and A10.6, in terms of the helicity amplitudes. A few entries, marked $*$, involve parameters not listed in Table A10.4. α_C is, as usual, the Wick helicity rotation angle for particle $C = A$. The formulae (A10.2) enable one to eliminate them in favour of the Table A10.4 parameters, if so desired (for the meaning of the dagger see the text above (A10.1)):

Table A10.6. Relation between CM reaction parameters and helicity amplitudes for the three- and four-spin measurements

CM parameter	Formula
$(XX\|0Y)$	$-\,\mathrm{Im}\,\left[(\phi_1-\phi_3)\phi_6^*-(\phi_2+\phi_4)\phi_5^*\right]$
$\dagger(ZZ\|0Y)$	$\mathrm{Im}\,\left[(\phi_1-\phi_3)\phi_5^*-(\phi_2+\phi_4)\phi_6^*\right]$
$(XZ\|0Y)$	$\mathrm{Im}\,(\phi_2\phi_3^*-\phi_1\phi_4^*)$
$(ZX\|0Y)$	$\mathrm{Im}\,(\phi_2\phi_4^*-\phi_1\phi_3^*)$
$(ZY\|0X)$	$\mathrm{Im}\,(\phi_1\phi_3^*+\phi_2\phi_4^*)$
$\dagger(ZY\|0Z)$	$\mathrm{Im}\,\left[(\phi_1-\phi_3)\phi_5^*+(\phi_2+\phi_4)\phi_6^*\right]$
$(XY\|0X)$	$\mathrm{Im}\,\left[(\phi_1-\phi_3)\phi_6^*+(\phi_2+\phi_4)\phi_5^*\right]$
$(XY\|0Z)$	$\mathrm{Im}\,(\phi_1\phi_2^*+\phi_3\phi_4^*)$
$(YX\|0X)$	$\mathrm{Im}\,[(\phi_1+\phi_3)\phi_6^*+(\phi_2-\phi_4)\phi_5^*]$
$(YX\|0Z)$	$\mathrm{Im}\,\left(\phi_1\phi_2^*-\phi_3\phi_4^*+2\phi_5\phi_6^*\right)$
$\dagger(XY\|X0)$	$\mathrm{Im}\,\left[(\phi_1+\phi_3)\phi_5^*+(\phi_2-\phi_4)\phi_6^*\right]$
$\dagger(XY\|Z0)$	$\mathrm{Im}\,\left(\phi_1\phi_2^*-\phi_3\phi_4^*-2\phi_5\phi_6^*\right)$
$(YZ\|0X)$	$\mathrm{Im}\,\left(\phi_1\phi_4^*+\phi_2\phi_3^*-2\phi_5\phi_6^*\right)$
$\dagger(ZY\|X0)$	$\mathrm{Im}\,\left(\phi_1\phi_4^*+\phi_2\phi_3^*+2\phi_5\phi_6^*\right)$
$(XX\|XX)$	$(1/2)(\|\phi_1\|^2+\|\phi_2\|^2+\|\phi_3\|^2+\|\phi_4\|^2-2\|\phi_5\|^2-2\|\phi_6\|^2)$
$(XX\|XZ)$	$\mathrm{Re}\,\left[(\phi_1+\phi_3)\phi_5^*-(\phi_2-\phi_4)\phi_6^*\right]$
$\dagger(XX\|ZX)$	$\mathrm{Re}\,\left[(\phi_1+\phi_3)\phi_6^*-(\phi_2-\phi_4)\phi_5^*\right]$

$$
\begin{aligned}
D_{LS}^{(B)} &= -D_{SL}^{(B)}+\tan\theta_{\mathrm{R}}\left(D_{SS}^{(B)}-D_{LL}^{(B)}\right)\\
(NL|0L) &= (NS|0S)+\tan\theta_{\mathrm{R}}\left[(NS|0L)+(NL|0S)\right]\\
(LN|L0) &= (SN|S0)-\tan\alpha_C\left[(LN|S0)+(SN|L0)\right]\\
(SS|LL) &= -(SS|SS)+\tan(\alpha_C+\theta_{\mathrm{R}})\left[(SS|LS)-(SS|SL)\right]\\
&\quad +\sec(\alpha_C+\theta_{\mathrm{R}})[A_{NN}-1].
\end{aligned}
\tag{A10.2}
$$

A10.4 Photoproduction of pseudoscalar mesons

The simplest observables to measure are the *photon beam asymmetry* Σ, the *target asymmetry* T and the *polarizing power* P.

The asymmetry Σ is measured by comparing the differential cross-sections for the photon beam linearly polarized parallel to and perpendicular to the reaction plane, using an unpolarized proton target. One has

$$\Sigma=\frac{d\sigma_\perp-d\sigma_\parallel}{d\sigma_\perp+d\sigma_\parallel}.$$

https://doi.org/10.1017/9781009402040.024 Published online by Cambridge University Press

Table A10.7. Relation between CM and Argonne Lab parameters for one- and two-spin measurements. See the text for explanation

CM	Argonne
$(0Y\|00)$	$-A_{\mathrm{ARG}}^{(B)}$
$\dagger(Y0\|00)$	$A_{\mathrm{ARG}}^{(A)}$
$(0X\|0X)$	$-\cos\theta_{\mathrm{R}}\, D_{SS}^{(B)} - \sin\theta_{\mathrm{R}}\, D_{SL}^{(B)}$
$*(0Z\|0Z)$	$\sin\theta_{\mathrm{R}}\, D_{LS}^{(B)} - \cos\theta_{\mathrm{R}}\, D_{LL}^{(B)}$
$*(0Z\|0X)$	$\cos\theta_{\mathrm{R}}\, D_{LS}^{(B)} + \sin\theta_{\mathrm{R}}\, D_{LL}^{(B)}$
$(0Y\|0Y)$	$D_{NN}^{(B)}$
$\dagger(X0\|X0)$	$\cos\alpha_C\, D_{SS}^{(A)} - \sin\alpha_C\, D_{SL}^{(A)}$
$\dagger(X0\|Z0)$	$\sin\alpha_C\, D_{SS}^{(A)} + \cos\alpha_C\, D_{SL}^{(A)}$
$(XX\|00)$	A_{SS}
$(YY\|00)$	$-A_{NN}$
$(ZZ\|00)$	$-A_{LL}$
$(XZ\|00)$	$-A_{SL}$
$\dagger(ZX\|00)$	A_{LS}
$(X0\|0X)$	$-\cos\theta_{\mathrm{R}}\, K_{SS}^{(A)} - \sin\theta_{\mathrm{R}}\, K_{SL}^{(A)}$
$(X0\|0Z)$	$-\sin\theta_{\mathrm{R}}\, K_{SS}^{(A)} + \cos\theta_{\mathrm{R}}\, K_{SL}^{(A)}$
$(Z0\|0Z)$	$-\sin\theta_{\mathrm{R}}\, K_{LS}^{(A)} + \cos\theta_{\mathrm{R}}\, K_{LL}^{(A)}$
$\dagger(Z0\|0X)$	$-\cos\theta_{\mathrm{R}}\, K_{LS}^{(A)} - \sin\theta_{\mathrm{R}}\, K_{LL}^{(A)}$
$(Y0\|0Y)$	$-K_{NN}^{(A)}$

T is measured using an unpolarized photon beam incident on a transversely polarized proton target with spin polarization P_T along ($\uparrow$) or opposite ($\downarrow$) to the normal to the reaction plane. One has

$$T = \frac{1}{P_T}\frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow}.$$

Both Σ and T function as analysing powers of the reaction.

The polarizing power P is just the degree of polarization along the normal to the reaction plane of the recoil proton beam when an unpolarized photon beam is incident on an unpolarized proton target.

Table A10.8. Relation between CM and Argonne Lab parameters for the three- and four-spin measurements

CM	Argonne
$(XX\|0Y)$	$-(SS\|0N)$
$\dagger(ZZ\|0Y)$	$-(SS\|N0)$
$(XZ\|0Y)$	$(SL\|0N)$
$(ZX\|0Y)$	$-(LS\|0N)$
$(ZY\|0X)$	$\cos\theta_R\,(LN\|0S)+\sin\theta_R\,(LN\|0L)$
$\dagger(ZY\|0Z)$	$\sin\theta_R\,(LN\|0S)-\cos\theta_R\,(LN\|0L)$
$(XY\|0X)$	$\cos\theta_R\,(SN\|0S)+\sin\theta_R\,(SN\|0L)$
$(XY\|0Z)$	$\sin\theta_R\,(SN\|0S)-\cos\theta_R\,(SN\|0L)$
$(YX\|0X)$	$-\cos\theta_R\,(NS\|0S)-\sin\theta_R\,(NS\|0L)$
$(YX\|0Z)$	$-\sin\theta_R\,(NS\|0S)+\cos\theta_R\,(NS\|0L)$
$\dagger(XY\|X0)$	$\sin\alpha_C\,(SN\|L0)-\cos\alpha_C\,(SN\|S0)$
$\dagger(XY\|Z0)$	$-\cos\alpha_C\,(SN\|L0)-\sin\alpha_C\,(SN\|S0)$
$*(YZ\|0X)$	$\cos\theta_R\,(NL\|0S)+\sin\theta_R\,(NL\|0L)$
$\dagger*(ZY\|X0)$	$\sin\alpha_C\,(LN\|L0)-\cos\alpha_C\,(LN\|S0)$
$*(XX\|XX)$	$\sin\alpha_C\,[\cos\theta_R\,(SS\|LS)+\sin\theta_R\,(SS\|LL)]$
	$-\cos\alpha_C\,[\cos\theta_R\,(SS\|SS)+\sin\theta_R\,(SS\|SL)]$
$*(XX\|XZ)$	$\cos\alpha_C\,[\cos\theta_R\,(SS\|SL)+\sin\theta_R\,(SS\|SS)]$
	$+\sin\alpha_C\,[\sin\theta_R\,(SS\|LS)-\cos\theta_R\,(SS\|LL)]$
$\dagger*(XX\|ZX)$	$-\cos\alpha_C\,[\sin\theta_R\,(SS\|LL)+\cos\theta_R\,(SS\|LS)]$
	$-\sin\alpha_C\,[\cos\theta_R\,(SS\|SS)+\sin\theta_R\,(SS\|SL)]$

These observables are given in terms of the amplitudes defined in Appendix 5 by the following expressions:

$$\Sigma\frac{d\sigma}{dt}=2\ \mathrm{Re}\ (S_1S_2^*-ND^*)$$
$$T\frac{d\sigma}{dt}=2\ \mathrm{Im}\ (S_1N^*-S_2D^*)$$
$$P\frac{d\sigma}{dt}=2\ \mathrm{Im}\ (S_2N^*-S_1D^*)$$
$$\frac{d\sigma}{dt}=|N|^2+|S_1|^2+|S_2|^2+|D|^2.$$

For a very general discussion of the observables and possible measurements, see the review paper of Storrow (Storrow, 1978). Care should be taken regarding sign conventions for the various axes.

A10.5 Vector meson production in $0^-(1/2)^+ \to 1^-(1/2)^+$

The vector density matrix elements $\rho_{mm'}$ and transversely polarized target asymmetries T_0, T_+, T_-, which are commonly used observables, are given in terms of the vector meson production amplitudes P, defined above (Appendix 5); see also Field and Sidhu (1974) and Irving and Worden (1977):

$$\begin{aligned}
\sigma_0 \equiv \rho_{00}\sigma &= |P^0_{++}|^2 + |P^0_{+-}|^2 \equiv |P_0|^2 \\
\sigma_\pm \equiv (\rho_{11} \pm \rho_{1-1})\sigma &= |P^\pm_{++}|^2 + |P^\pm_{+-}|^2 \equiv |P_\pm|^2 \\
\sqrt{2}\ \mathrm{Re}\ (\rho_{10}\sigma) &= \ \mathrm{Re}\ (P^0_{++}P^{-*}_{++} + P^0_{+-}P^{-*}_{+-} \\
T_0\sigma_0 &= -2\ \mathrm{Im}\ (P^0_{++}P^{0*}_{+-}) \\
T_+\sigma_+ &= -2\ \mathrm{Im}\ (P^+_{++}P^{+*}_{+-}) \\
T_-\sigma_- &= -2\ \mathrm{Im}\ (P^-_{++}P^{-*}_{+-})
\end{aligned}$$

with

$$\sigma = d\sigma/dt = \sigma_0 + \sigma_+ + \sigma_-.$$

A10.6 Baryon resonance production in $0^-(1/2)^+ \to 0^-(3/2)^+$

The density matrix elements $\rho_{mm'}$ are given in terms of the M amplitudes defined above (Appendix 5):

$$\rho_{33}\sigma = \tfrac{1}{2}\left(|M_1|^2 + |M_2|^2\right) \qquad \rho_{11} + \rho_{33} = \tfrac{1}{2}$$

$$\left\{\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}\rho_{31}\sigma = \frac{1}{2}\left\{\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}(M_0M_1^* + M_1'M_2^*)$$

$$\left\{\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}\rho_{3-1}\sigma = \frac{1}{2}\left\{\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}(M_0M_2^* - M_1'M_1^*)$$

$$\mathrm{Im}\ \rho_{3-3}\sigma = \ \mathrm{Im}\ (M_1M_2^*) \qquad \mathrm{Im}\ \rho_{1-1}\sigma = \ \mathrm{Im}\ (M_0M_1^*)$$

with

$$\sigma = d\sigma/dt = |M_0|^2 + |M_1|^2 + |M_1'|^2 + |M_2|^2.$$