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Luka Mikec, ***On Logics and Semantics for Interpretability***, University of Barcelona, Spain, and University of Zagreb, Croatia (cotutelle), 2021. Supervised by Joost J. Joosten and Mladen Vuković. MSC: 03B45, 03B60, 03F45. Keywords: modal logic, metamathematics, formalised interpretability, interpretability logics, generalised Veltman semantics.

Abstract

We study various properties of formalised relativised interpretability. In the central part of this thesis we study for different interpretability logics the following aspects: completeness for modal semantics, decidability and algorithmic complexity.

In particular, we study two basic types of relational semantics for interpretability logics. One is the Veltman semantics, which we shall refer to as the regular or ordinary semantics; the other is called generalised Veltman semantics. In the recent years and especially during the writing of this thesis, generalised Veltman semantics was shown to be particularly well-suited as a relational semantics for interpretability logics. In particular, modal completeness results are easier to obtain in some cases; and decidability can be proven via filtration in all known cases. We prove various new and reprove some old completeness results with respect to the generalised semantics. We use the method of filtration to obtain the finite model property for various logics.

Apart from results concerning semantics in its own right, we also apply methods from semantics to determine decidability (implied by the finite model property) and complexity of provability (and consistency) problems for certain interpretability logics.

From the arithmetical standpoint, we explore three different series of interpretability principles. For two of them, for which arithmetical and modal soundness was already known, we give a new proof of arithmetical soundness. The third series results from our modal considerations. We prove it arithmetically sound and also characterise frame conditions w.r.t. ordinary Veltman semantics. We also prove results concerning the new series and generalised Veltman semantics.

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Pedro Pinto, ***Proof Mining with the Bounded Functional Interpretation***, Faculdade de Ciências, Universidade de Lisboa, Lisboa, Portugal, 2019. Supervised by Fernando Ferreira. MSC: 03F10, 47H09, 47H10, 47J25. Keywords: bounded functional interpretation, majorants, metastability, weak compactness, fixed points.

Abstract

In this doctoral thesis, we show how the bounded functional interpretation of F. Ferreira and P. Oliva can be used and contribute to the Proof Mining program, a program which aims to extract computational information from mathematical theorems using proof-theoretic techniques. We present a method for the elimination of sequential weak compactness arguments from the quantitative analysis of certain mathematical results. This method works as a "macro" and allowed us to obtain quantitative versions of important results of F. E. Browder, R. Wittmann, and H. H. Bauschke in fixed point theory in Hilbert spaces. Although the theorems of Browder and Wittmann were previously analyzed by U. Kohlenbach using the monotone functional interpretation, it was not clear why such analyses did not require the use of functionals defined by bar recursion. This phenomenon is now fully understood by a theoretical justification for the elimination of sequential weak compactness in the context

of the bounded functional interpretation. Bauschke's theorem is an important generalization of Wittmann's theorem and its original proof is also analyzed here. The analyses of these results also require a quantitative version of a projection argument which turned out to be simpler when guided by the bounded functional interpretation than when using the monotone functional interpretation. In the context of the theory of monotone operators, results due to Boikanyo/Moroşanu and Xu for the strong convergence of variants of the proximal point algorithm are analyzed and bounds on the metastablility property of these iterations are obtained. These results are the first applications of the bounded functional interpretation to the proof mining of concrete mathematical results.

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MANLIO VALENTI, ***A journey through computability, topology and analysis***, Università degli studi di Udine, Udine, Italy, 2021. Supervised by Alberto Marcone. MSC: 03D78, 03D30, 03D55, 03E15, 28A75, 28A78. *Key words and phrases*: computable analysis, descriptive theory, Weihrauch reducibility, Hausdorff dimension, Fourier dimension.

Abstract

This thesis is devoted to the exploration of the complexity of some mathematical problems using the framework of computable analysis and (effective) descriptive set theory. We will especially focus on Weihrauch reducibility as a means to compare the uniform computational strength of problems. After a short introduction of the relevant background notions, we investigate the uniform computational content of problems arising from theorems that lie at the higher levels of the reverse mathematics hierarchy.

We first analyze the strength of the open and clopen Ramsey theorems. Since there is not a canonical way to phrase these theorems as multi-valued functions, we identify eight different multi-valued functions (five corresponding to the open Ramsey theorem and three corresponding to the clopen Ramsey theorem) and study their degree from the point of view of Weihrauch, strong Weihrauch, and arithmetic Weihrauch reducibility.

We then discuss some new operators on multi-valued functions and study their algebraic properties and the relations with other previously studied operators on problems. In particular, we study the *first-order part* and the *deterministic part* of a problem f, capturing the Weihrauch degree of the strongest multi-valued problem that is reducible to f and that, respectively, has codomain $\mathbb{N}$ or is single-valued.

These notions proved to be extremely useful when exploring the Weihrauch degree of the problem DS of computing descending sequences in ill-founded linear orders. They allow us to show that DS, and the Weihrauch equivalent problem BS of finding bad sequences through non-well quasi-orders, while being very "hard" to solve, are rather weak in terms of uniform computational strength. We then generalize DS and BS by considering $\mathbf{\Gamma}$-presented orders, where $\mathbf{\Gamma}$ is a Borel pointclass or $\mathbf{\Delta}^1_1$, $\mathbf{\Sigma}^1_1$, $\mathbf{\Pi}^1_1$. We study the obtained DS-hierarchy and BS-hierarchy of problems in comparison with the (effective) Baire hierarchy and show that they do not collapse at any finite level.

Finally, we work in the context of geometric measure theory and we focus on the characterization, from the point of view of descriptive set theory, of some conditions involving the notions of Hausdorff/Fourier dimension and Salem sets. We first work in the hyperspace $\mathbf{K}([0,1])$ of compact subsets of $[0,1]$ and show that the closed Salem sets form a $\mathbf{\Pi}^0_3$-complete family. This is done by characterizing the complexity of the family of sets having sufficiently large Hausdorff or Fourier dimension. We also show that the complexity does not change if we increase the dimension of the ambient space and work in $\mathbf{K}([0,1]^d)$. We also generalize the results by relaxing the compactness of the ambient space and show that the closed Salem sets are still $\mathbf{\Pi}^0_3$-complete when we endow $\mathbf{F}(\mathbb{R}^d)$ with the Fell topology. A similar result holds also for the Vietoris topology.