Student Problems

The solutions and successful solvers for the July Student Problem Corner are given below.

This will be the final appearance of the Student Problem Corner. There has been a steady decline of interest in this over the last few years. One reason for this is the late arrival of recent issues of *The Mathematical Gazette*, which has made it difficult for classroom teachers and their students to access the problems. As a result, the deadlines for submission (which are necessarily tight) have resulted in fewer solutions.

In addition, Dr Agnes Bokanyi-Toth has indicated that she cannot continue as SPC Editor, due to pressure of work. I am very grateful for her efforts since July 2024.

Solutions to 2025.1 and 2025.2

Both problems were solved by Chenyu Xie and Tejaswi Mishr. Problem 2025.1 was also solved by Dylan Davies and U. N. Owen. Problem 2025.2 was also solved by Harvey Gregg.

Problem 2025.1 (S. N. Maitra)

Given that $5a^2 + 5b^2 + 10c^2 + d^2 - 4ab - 4bc - 6cd = 4$. Determine the solutions where a, b, c, d are all distinct positive integers.

Solution (Dylan Davies and Tejaswi Mishr). Note that Chenyu Xie provided a similar solution to this problem. We begin with

$$5a^2 + 5b^2 + 10c^2 + d^2 - 4ab - 4bc - 6cd = 4.$$

The equation can be rewritten using some identities:

$$a^2 + (2a - b)^2 + (2b - c)^2 + (3c - d)^2 = 4.$$

As every term in the RHS is a squared number, it follows that $a^2 \leqslant 4$. So $a = 1$ and $a = 2$ are solutions.

If $a = 1$ then
$$\begin{aligned}(2a - b)^2 &= 1 \text{ and } 2a - b = \pm 1,\\ (2b - c)^2 &= 1 \text{ and } 2b - c = \pm 1,\\ (3c - d)^2 &= 1 \text{ and } 3c - d = \pm 1.\end{aligned}$$

- Now $a = 1$ and $b = 3$ gives $c = 5$ and $d = 14$ or $d = 16$.
- Also, $a = 1$ and $b = 3$ gives $c = 7$ and $d = 20$ or $d = 22$.

If $a^2 = 4$, $a = 2$ then $(2a - b)^2 = 0$, $(2b - c)^2 = 0$, $(3c - d)^2 = 0$. So $a = 2$ gives $b = 4$, $c = 8$ and $d = 24$.

Therefore, the solutions are

$$(1,3,5,14)\,(1,3,5,16),\ (1,3,7,20),\ (1,3,7,22) \text{ and } (2,4,8,24).$$

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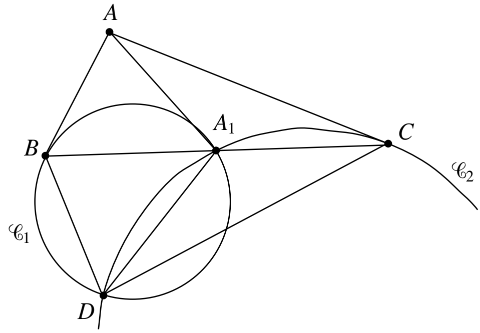
Problem 2025.2 (Gerry Leversha)

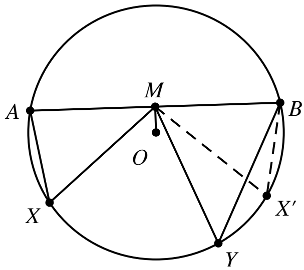
Let ΔABC be a triangle in which A_1 is the midpoint of BC.



Circle $\mathscr{C}_1$ through A_1 is tangential to AB at B, and circle $\mathscr{C}_2$ through A_1 is tangential to AC at C. Circles $\mathscr{C}_1$ and $\mathscr{C}_2$ meet again at D. Prove that

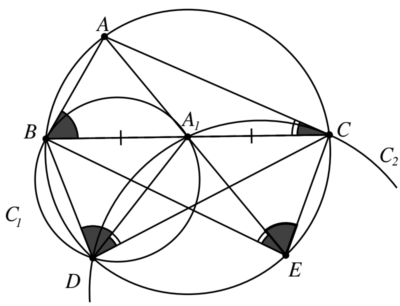
$$A_1A \times A_1D = A_1B \times A_1C.$$

Proof It is useful to begin with a lemma.



Proof: Reflect X in OM to X' and use the sine rule.

Let the chord AB of a circle have midpoint M and let X, Y be points on the circumference. Then $MX = MY$ if, and only if, $\angle AXM = \angle BYM$.



By alternate segments, $\angle ABC = \angle BDA$ and $\angle ACB_1 = \angle CDA_1$. Hence $\angle BAC + \angle BDC = 180°$ so $ABDC$ is cyclic.

Let AA_1 meet the circumcircle at E. By the lemma, $A_1E = A_1D$, and the result follows by intersecting chords.

Prize Winners

The first prize of £25 is awarded to Dylan Davies. The second prize of £20 is awarded to Harvey Gregg.

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