HIGHER ORDER AND ITERATIVE THEORIES TO COMPUTE ASTEROID MEAN ELEMENTS

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Mean orbital elements are obtained from their instantaneous, osculating counterparts by removal of the short periodic perturbations. They can be computed by means of different theories, analytical or numerical, depending on the problem and accuracy required. The most advanced contemporary analytical theory (Knežević 1988) accounts only for the perturbing effects due to Jupiter and Saturn, to the first order in their masses and to degree four in eccentricity and inclination. Nevertheless, the mean elements obtained by means of this theory are of satisfactory accuracy for majority of the asteroids in the main belt (Knežević et al. 1988), for the purpose of producing large catalogues of mean and proper elements, to identify asteroid families, to assess their age, to study the dynamical structure of the asteroid belt and chaotic phenomena of diffusion over very long time spans. In the vicinity of the main mean motion resonances, however, especially 2:1 mean motion resonance with Jupiter, these mean elements are of somewhat degraded accuracy.

We analysed a number of algorithms capable of producing mean elements of significantly better quality and much closer to even the strongest mean motion resonances, than it was the case so far (Milani and Knežević 1998). The methods we have tested belong to the two distinct classes: Breiter type (Breiter 1997a, 1997b) methods that take into account part of the effects of the second order in perturbing mass due to the combinations of the first order effects; and iterative methods which compute the convergent fixed frequency theory (Milani 1988). The former methods are based on application of some higher order numerical integration scheme, such as Runge-Kutta order 2, to the system of differential equations with determining function as Hamiltonian, whose solution is the transformation removing the fast angular variables; from this point of view, our current theory is equivalent to the solution of this auxiliary differential equation by means of the classical, first order numerical integration scheme of Euler type. The latter procedures involve fixed point iterative schemes, with current first order theory, or some second order Breiter type theory, as initial iteration step; at convergence they compute the inverse map, i.e. the transformation from mean to osculating elements; a similar method has already been employed in the computation of proper elements from the mean ones (Milani and Knežević 1990).

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Ten different algorithms were tested on a sample of asteroid orbits taken from the Themis family, covering a range in semimajor axis up to the very edge of the 2:1 resonance. The dispersion and the maximum excursion of the instantaneous mean values with respect to an average value of the mean semimajor axis, computed over 100 000 yr, was used as an accuracy control. The results of these tests led us to a number of interesting conclusions, the most important of which can be summarized as follows.

(i) The iterative methods turned out to be superior to the Breiter-type methods, in accuracy and in reliability; this can be understood if there are cancellations among second order terms, and some of these terms become unbalanced when the second order effects are only partially accounted for. In other words, unless we take into account a full second order transformation (very difficult in the asteroidal case, due to the complexity of the required second order determining function), the improvement of the results might be compromised to the point that some first order solutions have better performance.

(ii) As all the iterative methods at convergence supply practically identical results, the simplest Euler type starter (that is the current theory) was the most efficient; using a more sophisticated starter reduces somewhat the number of iterations, but the computational cost involved in deriving a more accurate initial step in practice often exceeds the gain due to the faster convergence.

(iii) A remarkable improvement of the accuracy of the resulting mean elements has been achieved mainly in the vicinity of the strongest mean motion resonances, in particular close to the 2:1, where the accuracy reached that common for the main belt objects even for a semimajor axis up to 3.20 AU. The conjecture proposed by Milani and Knežević (1994), that the further improvements of the existing analytical theory to compute mean and proper asteroid elements can only be of a local character, has thus been confirmed.

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