III. MODELS

FOURIER ANALYSIS OF THE HYDRODYNAMIC LIMIT-CYCLE MODELS OF PULSATING STARS

YU. A. FADEYEV
Institute for Astronomy of the Russian Academy of Sciences
and
Institute for Mathematics of the Vienna University

Abstract. The pulsation motions of the limit-cycle model can be described as a superposition of the Fourier harmonics, in the adiabatic layers each harmonics being identified with the corresponding standing wave. Near the resonance $\Pi_0/\Pi_l = k$ the harmonics of order k is also identified with the overtone of order l. The spectra of the oscillatory moment of inertia obey to the power dependence on the Fourier harmonics order k. In cepheids with periods shorter than 9 days the bump is due to the wave packet generated by the second overtone, whereas at periods longer than 10 days the bump feature is due to the traveling pulse reflected off the stellar core.

1. Introduction

The Hertzsprung progression of Classical cepheids is one of the most conspicuous features which is unmistakably reproduced in hydrodynamic calculations on the theoretical light and radial velocity curves. At the same time the nature of the secondary bump is still unknown because the long competition between two alternative hypotheses on the nature of the secondary bump has not been ended yet. The first of these hypotheses considers the bump feature as the traveling pulse reflected off the stellar core (Christy, 1968; 1975), whereas the second one proposed by Simon and Schmidt (1976) assumes that the bump is due to the resonance between the second overtone and fundamental mode. The attempts to reconcile both these hypotheses also did not reach their logical completion (Whitney, 1983; Aikawa and Whitney, 1985). Below we try to shed the light onto the problem of the Hertzsprung progression using the fact of the strict repetition of the pulsation motions in Classical cepheids. This allows us to calculate the Fourier coefficients for the main hydrodynamic variables at each mass zone of the limit cycle model. Together with the problem of the Hertzsprung progression we briefly discuss also some other features of pulsating stars. In more detail some preliminary results of this study are given by Fadeyev and Muthsam (1992).

2. Radial Properties of Fourier Harmonics

The most conspicuous feature of the Fourier harmonics of hydrodynamic variables is that their radial dependence are very similar to the eigenfunctions of the linear equation of stellar pulsation. For example, shown on the upper panel of Fig. 1 are the normalized amplitudes of the three lowest

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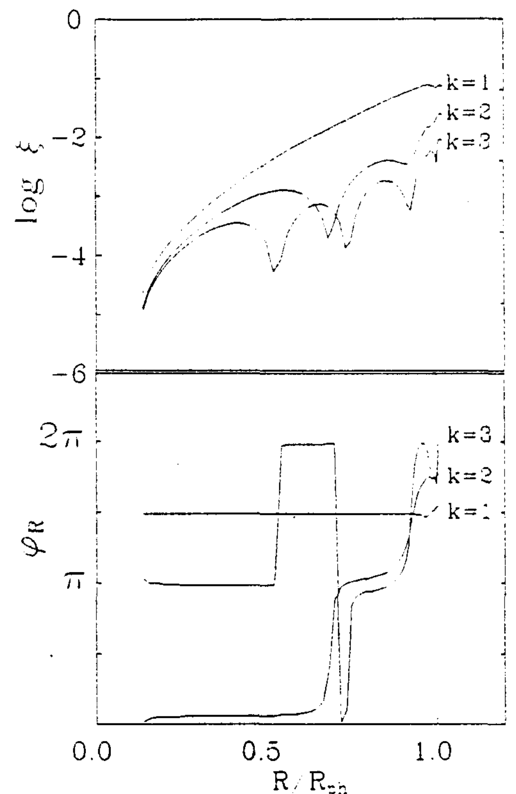

Fig. 1. The normalized amplitudes (upper panel) and the phases (lower panel) of the Fourier harmonics of order $1 \leq k \leq 3$ as a function of the radial distance.

Fourier harmonics of the radial displacement. This similarity is most prominent in the adiabatically pulsating layers where the difference between the adjacent maxima and minima is largest. Moreover, the phase of the Fourier harmonics is constant between two adjacent amplitude minima, whereas at the minimum the phase abruptly changes by π radian. Such a behaviour of the Fourier harmonics is recognized up to the order of $k = 8$. For higher Fourier harmonics this conclusion becomes uncertain due to the limited space resolution of the hydrodynamic models.

So, in the adiabatic layers the pulsation motions can be represented as a superposition of the standing waves. However in the radiative damping region as well in the outer layers where effects of nonadiabaticity become perceptible the amplitude minima and the corresponding phase changes become shallower and smoother, respectively. This implies that in the nonadiabatic layers the pulsation motions cannot be described in the terms of the pure standing waves due to the presence of the progressive wave component.

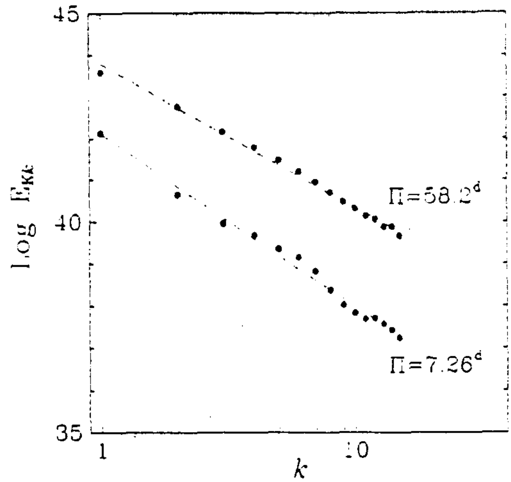

Fig. 2. The spectra of the kinetic energy E_{Kk} of the models of Classical cepheids with pulsation periods 7.26 and 58.2 days.

Comparison with the eigenfunctions of the linear equation of stellar pulsation shows that the Fourier harmonics of order k can be identified with the overtone of order l if the period ratio is $\Pi_0/\Pi_l = k$. When the period ratio is not integer and is in the range from k to $k+1$, the harmonics of order k reveals the features typical for both eigenfunctions of order l and $l-1$, respectively. When the period ratio becomes closer to one of these integer values, the properties of one of the overtones escape, whereas the properties of another overtone enhance.

Using the Fourier coefficients of radius and velocity we can calculate the oscillatory moment of inertia J_k and kinetic energy E_{Kk} for each Fourier harmonics of order k. Another conspicuous feature of the Fourier harmonics is that in all hydrodynamic limit-cycle models the pulsation spectra of J_k and E_{Kk} obey to the power law: $J_k = J_1 k^{-\nu_J}$ and $E_{Kk} = E_{K1} k^{-\nu_E}$ (see Fig. 2). Increase of nonadiabaticity in the envelope is accompanied by the decreasing slope of the spectrum, so that there is a correlation between the spectrum index ν_J (or ν_E) and the parameter related to nonadiabaticity (e.g. the growth rate of pulsation instability or mass to radius ratio). This implies that increase of nonadiabaticity is accompanied by the redistribution of the pulsation energy among higher harmonics.

3. Wave Packets

The compression wave propagating inwardly from the He^+ zone is easily traced using the temporal velocity dependence (see, e.g. Christy, 1975), whereas the outwardly propagating pulse reflected off the core is often lost

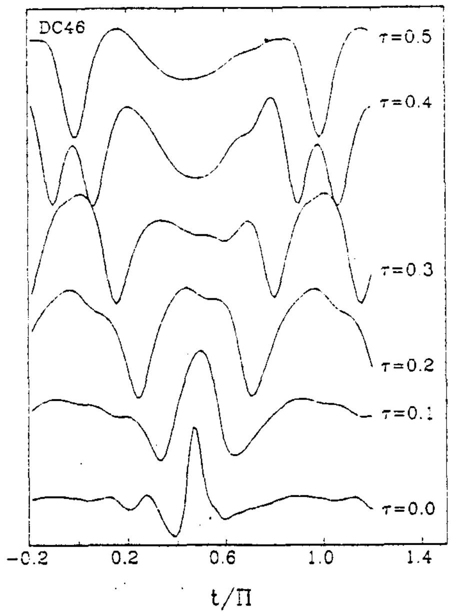

Fig. 3. The temporal dependence of the wave packets consisted of Fourier harmonics of order $2 \leq k \leq m = 15$. The dependencies are arbitrarily normalized, τ is acoustic coordinate of the layer.

in the helium ionizing region due to its relatively small amplitude. So, it is instructive to remove the influence of the high amplitude oscillations of the low-order harmonics and to consider the propagation of the wave packets consisted of Fourier harmonics of order $k_L \leq k \leq m$.

Shown in Fig. 3 are the temporal dependence of the velocity wave packets consisted of Fourier harmonics of order $k \geq k_L = 2$. For the sake of the graphic representation all these dependence are arbitrarily normalized. As is seen, the wave packets reveal the presence of the both inwardly and outwardly propagating pulses. However, though this method of the pulse tracing has a certain advantage, the procedure nevertheless remains rather cumbersome. In order to avoid such shortcoming, we considered the acoustic coordinates of most prominent maxima and minima of the wave packets. Fig. 4 shows the typical acoustic coordinate - time diagram for the cepheid model with the pulsation period of 8.5 day. On this diagram the minima and maxima of the velocity wave packets are shown as filled and open circles, respectively, the larger circles corresponding to the most prominent maxima and minima. So, the traveling pulse can be traced as a sequence of minima or maxima located along the characteristics ($|d\tau/dt| = 1$).

As is seen from Fig. 4, the inwardly propagating pulse is created in the instability excitation region at maximum compression (the phase $t/\Pi \simeq -0.1$). Below the instability excitation region the pulse propagates along the charac-

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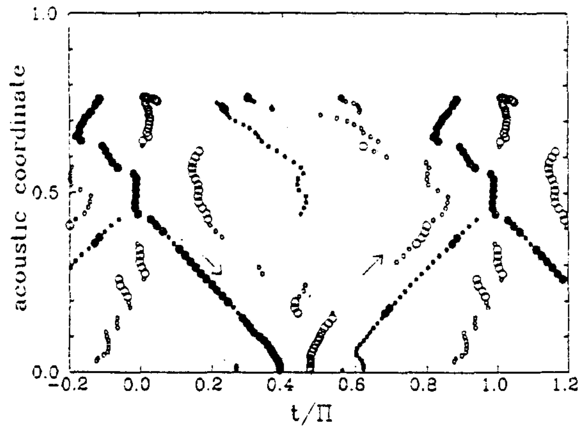

Fig. 4. The acoustic coordinate - phase diagram for the Classical cepheid model with period of 8.5 day. The filled and open circles show the minima and maxima of velocity, respectively. The largest circle corresponds to the most prominent maximum or minimum of the wave packet.

teristics and his trajectory is broken only at the acoustic midpoint ($\tau = 0.5$) due to interaction with the outwardly propagating pulse reflected off the stellar core. The reflected pulse also propagates along the characteristics but as is seen from Fig. 4, this pulse cannot be responsible for the secondary bump since his arrival at the photosphere nearly coincides with the main maximum.

The secondary bump appears due to the wave packet generated in the ionizing region at phases from 0.5 to 0.7 (see Fig. 4). The maximum of this wave packet coincides with the phase of the maximum of the second Fourier harmonics. Calculations show that the bump location changes in phase with the maximum of the second Fourier harmonics of the velocity. This implies that the secondary bump at periods shorter than 9 days is generated by the second Fourier harmonics and the nature of the Hertzsprung progression is tightly related to the second Fourier harmonics identified with the second overtone. Shown in Fig. 5 are the radial dependence of the phase difference between the fundamental mode and the second Fourier harmonics. This phase difference becomes close to $\pi/2$ radian near the resonance center. It is interesting to compare the change of the phase difference $\varphi_{U1} - \varphi_{U2}$ with the corresponding dependence of the secondary bump on the pulsation period. According to Fadeyev (1982), in the period range from 6 to 9 days the phase change of the secondary bump is $d\varphi/dlg\Pi = -1.66$. According to our models, the corresponding phase change is $d\varphi/dlg\Pi = -1.54$. There are hopes that the agreement will be improved when the more extended grid of the models is considered.

At periods longer than 10 days the secondary bump is due to the out-

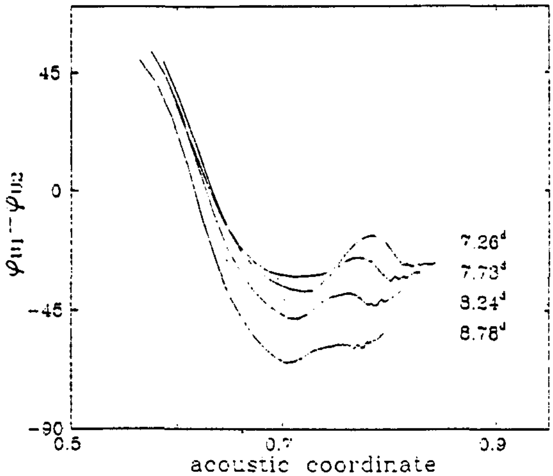

Fig. 5. The phase difference between the fundamental mode and second Fourier harmonics as a function of acoustic coordinate. The numbers near dependencies show the corresponding pulsation period.

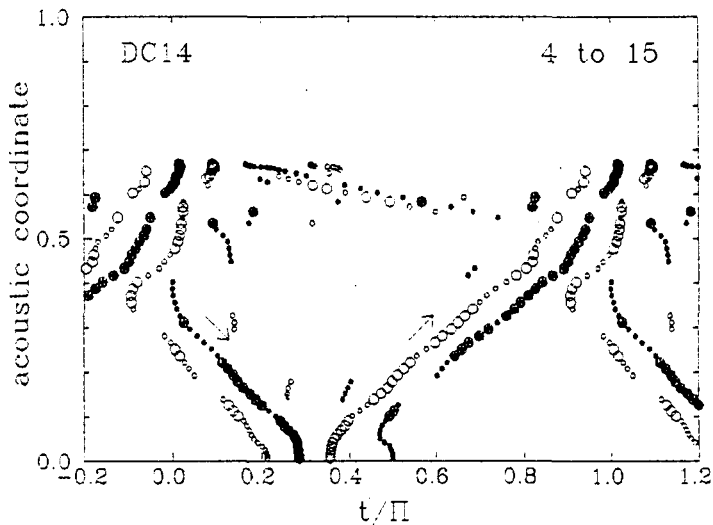

Fig. 6. The acoustic coordinate - time diagram for the Classical cepheid model with period of 17.4 day.

wardly propagating pulse reflected off the core (see Fig. 6). The energy of this pulse is distributed among the harmonics of order $k > 2$, i.e. the bump observed before the main maximum is not related to the second Fourier harmonics. As in observed light and radial velocity curves, the phase of the traveling pulse appearance does not change with the period.

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4. Concluding Remarks

Now we can certainly assert that transfer of the pulsation energy from the instability excitation region into the inner layers of the envelope is due to the traveling pulse created as a superposition of the standing waves. In population II cepheids the pulsation spectra are not so steep as in Classical cepheids so that the kinetic energy of the second Fourier harmonics identified with the first overtone is nearly a quarter of the kinetic energy of the fundamental mode. This implies that the second Fourier harmonics might be responsible for the alternating oscillations in RV Tau stars.

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