of inexpressibility, but it is also useful in establishing normal forms for logical formulas. There are generally two forms of locality: (i') if two structures $\mathfrak{A}$ and $\mathfrak{B}$ realize the same multiset of types of neighborhoods of radius d, then they agree on a given sentence Φ. Here d depends only on Φ; (ii') if the d-neighborhoods of two tuples $\vec{a}_1$ and $\vec{a}_2$ in a structure $\mathfrak{A}$ are isomorphic, then $\mathfrak{A} \models \Phi(\vec{a}_1) \Leftrightarrow \Phi(\vec{a}_2)$. Again, d depends on Φ, and not on $\mathfrak{A}$. Form (i') originated from Hanf's works. Form (ii') came from Gaifman's theorem. There is no doubt about the usefulness of the notion of locality, which as seen applies to a huge number of situations. However, there is a deficiency in such a notion: all versions of the notion of locality refer to isomorphism of neighborhoods, which is a fairly strong property. For example, where structures simply do not have sufficient isomorphic neighborhoods, versions of the notion of locality obviously cannot be applied. So the question that immediately arises is: would it be possible to weaken such a condition and maintain Hanf/Gaifman-localities? Arenas, Barceló, and Libkin establish a new condition for the notions of locality, weakening the requirement that neighborhoods should be isomorphic, establishing only the condition that they must be indistinguishable in a given logic. That is, instead of requiring $N_d(\vec{a}) \cong N_d(\vec{b})$, you should only require $N_d(\vec{a}) \equiv_k N_d(\vec{b})$, for some $k \geq 0$. Using the fact that logical equivalence is often captured by Ehrenfeucht–Fraïssé games, the authors formulate a game-based framework in which logical equivalence-based locality can be defined. Thus, the notion defined by the authors is that of *game-based locality*. Although quite promising as well as easy to apply, the game-based framework (used to define locality under logical equivalence) has the following problem: if a logic $\mathcal{L}$ is local (Hanf-, or Gaifman-, or weakly) under isomorphisms, and $\mathcal{L}'$ is a sub-logic of $\mathcal{L}$, then $\mathcal{L}'$ is local as well. The same, however, is not true for game-based locality: properties of games guaranteeing locality need not be preserved if one passes to weaker games. The question that immediately arises is: is it possible to define the notion of locality under logical equivalence without resorting to game-based frameworks? In this thesis, I present a homotopic variation for locality under logical equivalence, namely a Quillen model category-based framework for locality under k-logical equivalence, for every primitive-positive sentence of quantifier-rank k.

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Will Stafford, ***Something Valid This Way Comes: A Study of Neologicism and Proof-Theoretic Validity***, University of California, Irvine, USA, 2021. Supervised by Kai Wehmeier and Sean Walsh. MSC: 00A30, 03B55, 03F99. Keywords: Proof-theoretic semantics, neologicism, inquisitive logic.

Abstract

The interplay of philosophical ambitions and technical reality have given birth to rich and interesting approaches to explain the oft-claimed special character of mathematical and logical knowledge. Two projects stand out both for their audacity and their innovativeness. These are logicism and proof-theoretic semantics. This dissertation contains three chapters exploring the limits of these two projects. In both cases I find the formal results offer a mixed blessing to the philosophical projects.

Chapter 1. Is a logicist bound to the claim that as a matter of analytic truth there is an actual infinity of objects? If Hume's Principle is analytic then in the standard setting the answer appears to be yes. Hodes's work pointed to a way out by offering a modal picture in which only a potential infinity was posited. However, this project was abandoned due to apparent failures of cross-world predication. I re-explore this idea and discover that in the setting of the potential infinite one can interpret first-order Peano arithmetic, but not second-order Peano arithmetic. I conclude that in order for the logicist to weaken the metaphysically loaded claim of necessary actual infinities, they must also weaken the mathematics they recover.

Chapter 2. There have been several recent results bringing into focus the super-intuitionistic nature of most notions of proof-theoretic validity. But there has been very little work

evaluating the consequences of these results. In this chapter, I explore the question of whether these results undermine the claim that proof-theoretic validity shows us which inferences follow from the meaning of the connectives when defined by their introduction rules. It is argued that the super-intuitionistic inferences are valid due to the correspondence between the treatment of the atomic formulas and more complex formulas. And so the goals of proof-theoretic validity are not undermined.

Chapter 3. Prawitz (1971) conjectured that proof-theoretic validity offers a semantics for intuitionistic logic. This conjecture has recently been proven false by Piecha and Schroeder-Heister (2019). This chapter resolves one of the questions left open by this recent result by showing the extensional alignment of proof-theoretic validity and general inquisitive logic. General inquisitive logic is a generalisation of inquisitive semantics, a uniform semantics for questions and assertions. The chapter further defines a notion of quasi-proof-theoretic validity by restricting proof-theoretic validity to allow double negation elimination for atomic formulas and proves the extensional alignment of quasi-proof-theoretic validity and inquisitive logic.

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Mark Kamsma, ***Independence Relations in Abstract Elementary Categories***, University of East Anglia, UK, 2022. Supervised by Jonathan Kirby. MSC: Primary 03C45, 18C35, Secondary 03C48, 03C10. Keywords: independence relation, dividing, Kim-dividing, Kim-independence, accessible category, abstract elementary category, positive logic, stable theory, simple theory, NSOP1 theory.

Abstract

In model theory, a branch of mathematical logic, we can classify mathematical structures based on their logical complexity. This yields the so-called stability hierarchy. Independence relations play an important role in this stability hierarchy. An independence relation tells us which subsets of a structure contain information about each other, for example, linear independence in vector spaces yields such a relation.

Some important classes in the stability hierarchy are stable, simple, and NSOP_1, each being contained in the next. For each of these classes there exists a so-called Kim-Pillay style theorem. Such a theorem describes the interaction between independence relations and the stability hierarchy. For example, simplicity is equivalent to admitting a certain independence relation, which must then be unique.

All of the above classically takes place in full first-order logic. Parts of it have already been generalised to other frameworks, such as continuous logic, positive logic, and even a very general category-theoretic framework. In this thesis we continue this work.

We introduce the framework of AECats, which are a specific kind of accessible category. We prove that there can be at most one stable, simple, or NSOP_1-like independence relation in an AECat. We thus recover (part of) the original stability hierarchy. For this we introduce the notions of long dividing, isi-dividing, and long Kim-dividing, which are based on the classical notions of dividing and Kim-dividing but are such that they work well without compactness.

Switching frameworks, we generalise Kim-dividing in NSOP_1 theories to positive logic. We prove that Kim-dividing over existentially closed models has all the nice properties that it is known to have in full first-order logic. We also provide a full Kim-Pillay style theorem: a positive theory is NSOP_1 if and only if there is a nice enough independence relation, which then must be given by Kim-dividing.

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