The translation of the text could also have been more precise: 'confrontation theorem' should be 'squeeze theorem', 'counterpositive sentence' should be 'contrapositive statement', and so on.

The book could be used for learning basic calculus but no more. It might possibly aid pre-university courses in countries where calculus is not met at school, but under systems such as that in the UK, where calculus techniques (without rigorous definitions) are often met at GCSE and are central to A-level, it has little to offer compared with standard textbooks. 'Mastering' in the title is an exaggeration; those students who want to master calculus in a more rigorous sense should look elsewhere.

10.1017/mag.2024.143 MARTIN LUKAREVSKI
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Calculus by Amber Habib, pp 391, £49.99 (paper), ISBN 978-1-00915-969-2, Cambridge University Press (2022)

The one-word title brings to mind books such as Spivak's *Calculus* and this indeed gives an accurate flavour of the style and content of the book under review. It is conceived as a first-year undergraduate bridging course between the calculus techniques met at school and either more advanced analysis courses or more sophisticated applications to science or economics. As well as supplying rigorous foundations for familiar techniques and an introduction to tools for future use, a related aim is to build confidence in writing out formal proofs. The author's style is impressively clear and crisp, with every topic fully motivated and proofs deftly handled with a minimum of fuss. As the detailed bibliography attests, full advantage is taken of recent thinking about such courses and there are some intriguing choices made of organisation and content: keep your eye on the order of chapters described below!

Chapter 1 deals with background material on functions and the real numbers. The completeness axiom used is Tarski's modification of Dedekind's: if A, B are non-empty subsets of $\mathbb{R}$ such that $a \leqslant b$ for all $a \in A$, $b \in B$ then there is a real number m such that $a \leqslant m \leqslant b$ for all $a \in A$, $b \in B$—notice that it is *not* assumed that $A \cup B = \mathbb{R}$. This axiom has the advantage of being well adapted both to handling lower and upper sums in integration and to using nested intervals to provide unified proofs that continuous functions on closed, bounded intervals are bounded, attain their bounds, have the intermediate value property (IVP), and are uniformly continuous.

Chapter 2 introduces Riemann integration via lower and upper step functions. The focus is on proving properties of the integral, including the integrability of monotone functions: this enables the algebraic properties of $\ln x$ (defined as $\int_1^x \frac{1}{t} dt$) and $\exp(x)$ (its inverse) to be readily established. Chapter 3 covers limits and continuity: as well as the usual results in this area, there is a quick review of trigonometric functions and a proof that continuous functions are integrable. The author's approach to differentiation in Chapter 4 emphasises 'local linearity' as a prelude to the standard results on combining derivatives: it was also good to see Darboux's theorem (that derivatives have the IVP) proved and later used and, in an excellent section on extrema and curve sketching, it was striking that the mean value theorem is not invoked.

Chapter 5 has a brisk résumé of techniques of integration squarely focused on anti-derivatives: two notable features here are a full account of the integration of partial fractions arising from general linear and quadratic denominators, and a treatment of first and second order differential equations that addresses the existence and uniqueness of solutions. In line with recent pedagogic thinking, the mean value theorem makes its late appearance in Chapter 6, with applications to the various versions of l'Hôpital's rule and the remainder formula for Taylor series approximations, as well as rigorous proofs (using tagged Riemann sums) of the familiar integration formulae for arc length and the volume and surface area of solids of revolution. The chapter ends with a look at the error formulae for the common methods of numerical integration.

Chapter 7 deals with sequences and series; leaving this topic late enables full advantage to be taken of the connections with limits of functions and l'Hôpital's rule. Finally, Chapter 8 starts by looking at power series in order to complete the work on Taylor expansions, before contrasting these with Fourier series (where the theory is taken far enough to show pointwise convergence for piecewise differentiable functions). The book ends with a quick account of complex numbers and how they clarify the foregoing results on power and Fourier series.

Throughout the text are 'tasks'—quick-fire checks on understanding—and there are some 400 exercises at the ends of sections of chapters. These are a carefully graded mixture of routine and more challenging examples, with full answers provided for the odd-numbered ones. (A nice touch was to include ten examples on l'Hôpital's rule from Euler's treatise on differential calculus!) There are also longer 'thematic exercises' dealing with extension topics such as cardinality, polynomial interpolation, Stirling's formula, the gamma and beta functions, and uniform convergence. I noticed just a handful of typos, including slips in the formulae for the first two derivatives of $(x - x^3)^{1/3}$ on p. 167, and in the integrals for the arc length of an ellipse on p. 244; there is also a missing minus sign in the error term for Simpson's rule on p. 257.

I thoroughly enjoyed reading this book. For students, it provides a review of familiar topics treated rigorously and an exposure to new and powerful ideas in real analysis. The material is very skilfully woven together with just the right amount of supporting detail and motivation and apposite examples (and counterexamples). And for lecturers, there is much food for thought in the author's innovative approach to what is ostensibly very standard fare, as well as some excellent and well thought-through collections of exercises.

10.1017/mag.2024.144 NICK LORD
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Visual differential geometry and forms: a mathematical drama in five acts by Tristan Needham, pp 501, £40 (paper), ISBN 978-0-69-120370-6, Princeton University Press (2021)

Visual differential geometry and forms is written in the same inimitable, highly visual style as Needham's widely acclaimed *Visual complex analysis* (VCA): indeed, there is some inevitable overlap of content, but also some differences. Among the 235 diagrams, relatively more are photos depicting the results of experiments with practical objects (often fruit and vegetables) and, in the text, fuller explicit use is made of the concept of "ultimate equality" applied geometrically in limiting arguments, as Newton did in the *Principia*. As in VCA, the author writes beautifully in a conversational manner which speaks directly to the reader. In the Prologue, he writes that, "I have made no attempt to write this book as a classroom textbook", but