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# VANISHING COEFFICIENTS IN FOUR QUOTIENTS OF INFINITE PRODUCT EXPANSIONS

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**Abstract**

Motivated by Ramanujan's continued fraction and the work of Richmond and Szekeres ['The Taylor coefficients of certain infinite products', *Acta Sci. Math. (Szeged)* **40**(3–4) (1978), 347–369], we investigate vanishing coefficients along arithmetic progressions in four quotients of infinite product expansions and obtain similar results. For example, $a_1(5n+4)=0$, where $a_1(n)$ is defined by

$$\frac{(q,q^4;q^5)_\infty^3}{(q^2,q^3;q^5)_\infty^2}=\sum_{n=0}^{\infty}a_1(n)q^n.$$

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## 1. Introduction

In 1978, Richmond and Szekeres [9] proved that if

$$\sum_{n=0}^{\infty}\alpha(n)q^n=\frac{(q,q^7;q^8)_\infty}{(q^3,q^5;q^8)_\infty}\quad\text{and}\quad\sum_{n=0}^{\infty}\beta(n)q^n=\frac{(q^3,q^5;q^8)_\infty}{(q,q^7;q^8)_\infty},$$

then

$$\alpha(4n+2)=\beta(4n+3)=0. \tag{1.1}$$

Throughout the paper, we use the customary $q$-series notation: for $|q|<1$,

$$(a;q)_\infty:=\prod_{n=0}^{\infty}(1-aq^n),$$
$$(a_1,a_2,\ldots,a_m;q)_\infty:=(a_1;q)_\infty(a_2;q)_\infty\cdots(a_m;q)_\infty.$$

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Richmond and Szekeres [9] also conjectured that if

$$\sum_{n=0}^{\infty}\gamma(n)q^n=\frac{(q,q^{11};q^{12})_\infty}{(q^5,q^7;q^{12})_\infty}\quad\text{and}\quad\sum_{n=0}^{\infty}\delta(n)q^n=\frac{(q^5,q^7;q^{12})_\infty}{(q,q^{11};q^{12})_\infty},$$

then

$$\gamma(6n+3)=\delta(6n+5)=0. \tag{1.2}$$

Soon after, Andrews and Bressoud [3] not only confirmed the conjecture (1.2), but also proved a general result, which contains (1.1) and (1.2) as special cases. In 1994, Alladi and Gordon [1] further generalised the result of Andrews and Bressoud and obtained a companion result. McLaughlin [8] gave further extensions. Very recently, Hirschhorn [6] studied vanishing coefficients along arithmetic progressions in two $q$-series expansions. Soon after, the author [10] investigated some variants of these two $q$-series expansions and established comparable results.

The product representation of Ramanujan's continued fraction is given by

$$\sum_{n=0}^{\infty}u(n)q^n=R(q)=\frac{(q,q^4;q^5)_\infty}{(q^2,q^3;q^5)_\infty}$$

(see [7, Equation (16.2.1)]). In 1968 or early 1969, Szekeres noticed that the sign of $u(n)$ is periodic with period 5 except for some zeros near the beginning (specifically, $u(3)=u(8)=u(13)=u(23)=0$). In 1978, Richmond and Szekeres [9] obtained an asymptotic formula for $u(n)$ via the circle method, which implies that the sign of $u(n)$ is periodic with period 5 for sufficiently large $n$. A similar result was also obtained for the coefficients of $1/R(q)$. In 1981, Andrews [2] gave the first proof for Szekeres's observations from a combinatorial viewpoint. Later, Hirschhorn [5] provided an elementary proof.

We investigate vanishing coefficients in the quotients of four infinite product expansions similar to the ones considered above. Define the sequences $\{a_1(n)\}$, $\{a_2(n)\}$, $\{b_1(n)\}$ and $\{b_2(n)\}$ by the following Taylor series:

$$\sum_{n=0}^{\infty}a_1(n)q^n=\frac{(q,q^4;q^5)_\infty^3}{(q^2,q^3;q^5)_\infty^2}, \tag{1.3}$$

$$\sum_{n=0}^{\infty}a_2(n)q^n=\frac{(q^2,q^3;q^5)_\infty^3}{(q,q^4;q^5)_\infty^2}, \tag{1.4}$$

$$\sum_{n=0}^{\infty}b_1(n)q^n=\frac{(q,q^4;q^5)_\infty^3}{(-q^2,-q^3;q^5)_\infty^2}, \tag{1.5}$$

$$\sum_{n=0}^{\infty}b_2(n)q^n=\frac{(q^2,q^3;q^5)_\infty^3}{(-q,-q^4;q^5)_\infty^2}. \tag{1.6}$$

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THEOREM 1.1. *For any integer $n \geq 0$,*

$$a_1(5n+4) = 0, \tag{1.7}$$
$$a_2(5n+2) = 0, \tag{1.8}$$
$$b_1(5n+4) = 0, \tag{1.9}$$
$$b_2(5n+2) = 0. \tag{1.10}$$

## 2. Proofs

Ramanujan's theta function is given by

$$f(a,b) := \sum_{n=-\infty}^{\infty} a^{n(n+1)/2} b^{n(n-1)/2}, \quad |ab| < 1. \tag{2.1}$$

The function $f(a,b)$ satisfies the well-known Jacobi triple product identity [4, page 35, entry 19]:

$$f(a,b) = (-a,-b,ab;ab)_\infty. \tag{2.2}$$

Two important special cases of (2.1) are [7, Equations (1.5.4) and (1.5.5)]:

$$\varphi(q) := f(q,q) = \sum_{n=-\infty}^{\infty} q^{n^2} = \frac{(q^2;q^2)_\infty^5}{(q;q)_\infty^2 (q^4;q^4)_\infty^2}, \tag{2.3}$$

$$\psi(q) := f(q,q^3) = \sum_{n=0}^{\infty} q^{n(n+1)/2} = \frac{(q^2;q^2)_\infty^2}{(q;q)_\infty}. \tag{2.4}$$

We also require the quintuple product identity.

LEMMA 2.1 [7, Equation (10.1.3)]. *We have*

$$\frac{(a^{-2}, a^2 q, q; q)_\infty}{(a^{-1}, aq; q)_\infty} = \sum_{n=-\infty}^{\infty} (-1)^n a^{3n} q^{(3n^2+n)/2} + \sum_{n=-\infty}^{\infty} (-1)^n a^{3n-1} q^{(3n^2-n)/2}. \tag{2.5}$$

Let $k > 0$ and $l \geq 0$ be integers. If $G(q) = \sum_{n=0}^{\infty} g(n) q^n$ is a formal power series, define an operator $U_{k,l}$ by

$$U_{k,l}\Big(\sum_{n=0}^{\infty} g(n) q^n\Big) = \sum_{n=0}^{\infty} g(kn+l) q^{kn+l}.$$

PROOF OF (1.7). Replacing $q$ by $q^5$ in (2.5) and taking $a = q^{-2}$, and then replacing $n$ by $n-1$ in the second sum in the next line,

$$\begin{aligned}\frac{(q,q^4,q^5;q^5)_\infty^2}{(q^2,q^3;q^5)_\infty^2} &= \Big(\sum_{n=-\infty}^{\infty} (-1)^n q^{(15n^2-7n)/2} + q^2 \sum_{n=-\infty}^{\infty} (-1)^n q^{(15n^2-17n)/2}\Big)^2 \\ &= \Big(\sum_{n=-\infty}^{\infty} (-1)^n q^{(15n^2+7n)/2} - q \sum_{n=-\infty}^{\infty} (-1)^n q^{(15n^2+13n)/2}\Big)^2\end{aligned}$$

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$$
\begin{aligned}
&= \sum_{m,n=-\infty}^{\infty} (-1)^{m+n} q^{(15m^2+7m+15n^2+7n)/2} \\
&\quad + q^2 \sum_{m,n=-\infty}^{\infty} (-1)^{m+n} q^{(15m^2+13m+15n^2+13n)/2} \\
&\quad - 2q \sum_{m,n=-\infty}^{\infty} (-1)^{m+n} q^{(15m^2+7m+15n^2+13n)/2}.
\end{aligned}
$$

If $m$ and $n$ have the same parity, let $(m,n) = (r+s, r-s)$. If $m$ and $n$ have the opposite parity, take $(m,n) = (r+s-1, r-s)$, where $r$ and $s$ are integers. With the help of (2.2)–(2.4),

$$
\begin{aligned}
\frac{(q,q^4,q^5;q^5)_\infty^2}{(q^2,q^3;q^5)_\infty^2} &= \sum_{r,s=-\infty}^{\infty} (-1)^{(r+s)+(r-s)} q^{(15(r+s)^2+7(r+s)+15(r-s)^2+7(r-s))/2} \\
&\quad + \sum_{r,s=-\infty}^{\infty} (-1)^{(r+s-1)+(r-s)} q^{(15(r+s-1)^2+7(r+s-1)+15(r-s)^2+7(r-s))/2} \\
&\quad + q^2 \sum_{r,s=-\infty}^{\infty} (-1)^{(r+s)+(r-s)} q^{(15(r+s)^2+13(r+s)+15(r-s)^2+13(r-s))/2} \\
&\quad + q^2 \sum_{r,s=-\infty}^{\infty} (-1)^{(r+s-1)+(r-s)} q^{(15(r+s-1)^2+13(r+s-1)+15(r-s)^2+13(r-s))/2} \\
&\quad - 2q \sum_{m,n=-\infty}^{\infty} (-1)^{(r+s)+(r-s)} q^{(15(r+s)^2+7(r+s)+15(r-s)^2+13(r-s))/2} \\
&\quad - 2q \sum_{m,n=-\infty}^{\infty} (-1)^{(r+s-1)+(r-s)} q^{(15(r+s-1)^2+7(r+s-1)+15(r-s)^2+13(r-s))/2} \\
&= \varphi(q^{15}) \sum_{n=-\infty}^{\infty} q^{15n^2+7n} - 2q^4\psi(q^{30}) \sum_{n=-\infty}^{\infty} q^{15n^2+8n} \\
&\quad + q^2\varphi(q^{15}) \sum_{n=-\infty}^{\infty} q^{15n^2+13n} - 2q^3\psi(q^{30}) \sum_{n=-\infty}^{\infty} q^{15n^2+2n} \\
&\quad - 2q \sum_{m,n=-\infty}^{\infty} q^{15m^2+10m+15n^2+3n} + 2q^2 \sum_{m,n=-\infty}^{\infty} q^{15m^2+5m+15n^2+12n} \\
&= \varphi(q^{15}) \sum_{n=-\infty}^{\infty} q^{15n^2+7n} - 2q^4\psi(q^{30}) \sum_{n=-\infty}^{\infty} q^{15n^2+8n} \\
&\quad + q^2\varphi(q^{15}) \sum_{n=-\infty}^{\infty} q^{15n^2+13n} - 2q^3\psi(q^{30}) \sum_{n=-\infty}^{\infty} q^{15n^2+2n}
\end{aligned}
$$

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$$
\begin{aligned}
&- 2q(-q^5,-q^{25},q^{30};q^{30})_\infty \sum_{n=-\infty}^{\infty} q^{15n^2+3n} \\
&+ 2q^2(-q^{10},-q^{20},q^{30};q^{30})_\infty \sum_{n=-\infty}^{\infty} q^{15n^2+12n}. \qquad (2.6)
\end{aligned}
$$

In view of (2.2),

$$
\begin{aligned}
(q,q^4;q^5)_\infty &= \frac{1}{(q^5;q^5)_\infty} \sum_{n=-\infty}^{\infty} (-1)^n q^{(5n^2+3n)/2} \\
&= \frac{1}{(q^5;q^5)_\infty} \bigg( \sum_{n=-\infty}^{\infty} q^{10n^2+3n} - q \sum_{n=-\infty}^{\infty} q^{10n^2+7n} \bigg). \qquad (2.7)
\end{aligned}
$$

Combining (2.6) and (2.7),

$$
\begin{aligned}
\sum_{n=0}^{\infty} a_1(n) q^n = &\frac{\varphi(q^{15})}{(q^5;q^5)_\infty^3} \bigg( \sum_{m,n=-\infty}^{\infty} q^{15m^2+7m+10n^2+3n} - q^3 \sum_{m,n=-\infty}^{\infty} q^{15m^2+13m+10n^2+7n} \\
&+ q^2 \sum_{m,n=-\infty}^{\infty} q^{15m^2+13m+10n^2+3n} - q \sum_{m,n=-\infty}^{\infty} q^{15m^2+7m+10n^2+7n} \bigg) \\
&+ \frac{2\psi(q^{30})}{(q^5;q^5)_\infty^3} \bigg( q^4 \sum_{m,n=-\infty}^{\infty} q^{15m^2+2m+10n^2+7n} - q^4 \sum_{m,n=-\infty}^{\infty} q^{15m^2+8m+10n^2+3n} \\
&+ q^5 \sum_{m,n=-\infty}^{\infty} q^{15m^2+8m+10n^2+7n} - q^3 \sum_{m,n=-\infty}^{\infty} q^{15m^2+2m+10n^2+3n} \bigg) \\
&- \frac{2(-q^5,-q^{25},q^{30};q^{30})_\infty}{(q^5;q^5)_\infty^3} \\
&\quad \times \bigg( q \sum_{m,n=-\infty}^{\infty} q^{15m^2+3m+10n^2+3n} - q^2 \sum_{m,n=-\infty}^{\infty} q^{15m^2+3m+10n^2+7n} \bigg) \\
&+ \frac{2(-q^{10},-q^{20},q^{30};q^{30})_\infty}{(q^5;q^5)_\infty^3} \\
&\quad \times \bigg( q^2 \sum_{m,n=-\infty}^{\infty} q^{15m^2+12m+10n^2+3n} - q^3 \sum_{m,n=-\infty}^{\infty} q^{15m^2+12m+10n^2+7n} \bigg) \\
:= &\frac{\varphi(q^{15})}{(q^5;q^5)_\infty^3} (S_1 - S_2 + S_3 - S_4) + \frac{2\psi(q^{30})}{(q^5;q^5)_\infty^3} (S_5 - S_6 + S_7 - S_8) \\
&- \frac{2(-q^5,-q^{25},q^{30};q^{30})_\infty}{(q^5;q^5)_\infty^3} (S_9 - S_{10}) \\
&+ \frac{2(-q^{10},-q^{20},q^{30};q^{30})_\infty}{(q^5;q^5)_\infty^3} (S_{11} - S_{12}), \qquad (2.8)
\end{aligned}
$$

say, where

$$S_1 = \sum_{m,n=-\infty}^{\infty} q^{15m^2+7m+10n^2+3n}, \quad S_2 = q^3 \sum_{m,n=-\infty}^{\infty} q^{15m^2+13m+10n^2+7n},$$

$$S_3 = q^2 \sum_{m,n=-\infty}^{\infty} q^{15m^2+13m+10n^2+3n}, \quad S_4 = q \sum_{m,n=-\infty}^{\infty} q^{15m^2+7m+10n^2+7n},$$

$$S_5 = q^4 \sum_{m,n=-\infty}^{\infty} q^{15m^2+2m+10n^2+7n}, \quad S_6 = q^4 \sum_{m,n=-\infty}^{\infty} q^{15m^2+8m+10n^2+3n},$$

$$S_7 = q^5 \sum_{m,n=-\infty}^{\infty} q^{15m^2+8m+10n^2+7n}, \quad S_8 = q^3 \sum_{m,n=-\infty}^{\infty} q^{15m^2+2m+10n^2+3n},$$

$$S_9 = q \sum_{m,n=-\infty}^{\infty} q^{15m^2+3m+10n^2+3n}, \quad S_{10} = q^2 \sum_{m,n=-\infty}^{\infty} q^{15m^2+3m+10n^2+7n},$$

$$S_{11} = q^2 \sum_{m,n=-\infty}^{\infty} q^{15m^2+12m+10n^2+3n}, \quad S_{12} = q^3 \sum_{m,n=-\infty}^{\infty} q^{15m^2+12m+10n^2+7n}.$$

In $S_1$, we want $7m + 3n \equiv 4 \pmod 5$, that is, $m - n \equiv 2 \pmod 5$ and $4m + n \equiv 3 \pmod 5$. Writing $m - n = 5r + 2$ and $4m + n = 5s + 8$, it follows that $m = r + s + 2$ and $n = -4r + s$. Therefore,

$$U_{5,4}(S_1) = q^{74} \sum_{r,s=-\infty}^{\infty} q^{175r^2+55r-50rs+25s^2+70s}.$$

In $S_2$, if $13m + 7n \equiv 1 \pmod 5$, then $m - n \equiv 2 \pmod 5$ and $2m + 3n \equiv -1 \pmod 5$. Writing $m - n = 5r + 2$ and $2m + 3n = -5s - 11$, it follows in this case that $m = 3r - s - 1$ and $n = -2r - s - 3$. Therefore,

$$U_{5,4}(S_2) = q^{74} \sum_{r,s=-\infty}^{\infty} q^{175r^2+55r-50rs+25s^2+70s}.$$

Similarly,

$$U_{5,4}(S_{2i-1} - S_{2i}) = 0 \quad \text{for } 1 \le i \le 6. \tag{2.9}$$

This establishes (1.7). □

The proof of (1.8) is similar to that of (1.7).

Proof of (1.9). Replacing $q$ by $q^5$ and taking $a = -q^{-2}$ in (2.5), by manipulations similar to those used in deriving (2.8), we arrive at

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$$\frac{(q,q^4,q^5;q^5)_\infty^2}{(-q^2,-q^3;q^5)_\infty^2} = \Big(\sum_{n=-\infty}^{\infty} q^{(15n^2+7n)/2} - q\sum_{n=-\infty}^{\infty} q^{(15n^2+13n)/2}\Big)^2$$
$$= \sum_{m,n=-\infty}^{\infty} q^{(15m^2+7m+15n^2+7n)/2}$$
$$+ q^2 \sum_{m,n=-\infty}^{\infty} q^{(15m^2+13m+15n^2+13n)/2}$$
$$- 2q \sum_{m,n=-\infty}^{\infty} q^{(15m^2+7m+15n^2+13n)/2}$$
$$= \varphi(q^{15})\sum_{n=-\infty}^{\infty} q^{15n^2+7n} + 2q^4\psi(q^{30})\sum_{n=-\infty}^{\infty} q^{15n^2+8n}$$
$$+ q^2\varphi(q^{15})\sum_{n=-\infty}^{\infty} q^{15n^2+13n} + 2q^3\psi(q^{30})\sum_{n=-\infty}^{\infty} q^{15n^2+2n}$$
$$- 2q(-q^5,-q^{25},q^{30};q^{30})_\infty \sum_{n=-\infty}^{\infty} q^{15n^2+3n}$$
$$- 2q^2(-q^{10},-q^{20},q^{30};q^{30})_\infty \sum_{n=-\infty}^{\infty} q^{15n^2+12n}. \tag{2.10}$$

Combining (2.10) and (2.7) yields

$$\sum_{n=0}^{\infty} b_1(n)q^n = \frac{\varphi(q^{15})}{(q^5;q^5)_\infty^3}\Big(\sum_{m,n=-\infty}^{\infty} q^{15m^2+7m+10n^2+3n} - q^3\sum_{m,n=-\infty}^{\infty} q^{15m^2+13m+10n^2+7n}$$
$$+q^2\sum_{m,n=-\infty}^{\infty} q^{15m^2+13m+10n^2+3n} - q\sum_{m,n=-\infty}^{\infty} q^{15m^2+7m+10n^2+7n}\Big)$$
$$-\frac{2\psi(q^{30})}{(q^5;q^5)_\infty^3}\Big(q^4\sum_{m,n=-\infty}^{\infty} q^{15m^2+2m+10n^2+7n} - q^4\sum_{m,n=-\infty}^{\infty} q^{15m^2+8m+10n^2+3n}$$
$$+q^5\sum_{m,n=-\infty}^{\infty} q^{15m^2+8m+10n^2+7n} - q^3\sum_{m,n=-\infty}^{\infty} q^{15m^2+2m+10n^2+3n}\Big)$$
$$-\frac{2(-q^5,-q^{25},q^{30};q^{30})_\infty}{(q^5;q^5)_\infty^3}$$
$$\times\Big(q\sum_{m,n=-\infty}^{\infty} q^{15m^2+3m+10n^2+3n} - q^2\sum_{m,n=-\infty}^{\infty} q^{15m^2+3m+10n^2+7n}\Big)$$
$$-\frac{2(-q^{10},-q^{20},q^{30};q^{30})_\infty}{(q^5;q^5)_\infty^3}$$

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$$\times\Big(q^2\sum_{m,n=-\infty}^{\infty}q^{15m^2+12m+10n^2+3n}-q^3\sum_{m,n=-\infty}^{\infty}q^{15m^2+12m+10n^2+7n}\Big)$$
$$=\frac{\varphi(q^{15})}{(q^5;q^5)_\infty^3}(S_1-S_2+S_3-S_4)-\frac{2\psi(q^{30})}{(q^5;q^5)_\infty^3}(S_5-S_6+S_7-S_8)$$
$$-\frac{2(-q^5,-q^{25},q^{30};q^{30})_\infty}{(q^5;q^5)_\infty^3}(S_9-S_{10})$$
$$-\frac{2(-q^{10},-q^{20},q^{30};q^{30})_\infty}{(q^5;q^5)_\infty^3}(S_{11}-S_{12}),$$

where the $S_i$ for $1\le i\le 12$ are defined as before (following (2.8)). From (2.9),

$$b_1(5n+4)=0.$$

This proves (1.9). □

The proof of (1.10) is similar to that of (1.9).

## 3. Final remarks

We conclude with several remarks that merit further study.

Following the strategy used to prove (2.8) and applying the $U_{k,l}$ operator,

$$a_1(5n+2)=-a_2(5n+3),$$
$$a_1(5n+3)=a_2(5n+4),$$
$$b_1(5n+2)=-b_2(5n+3),$$
$$b_1(5n+3)=b_2(5n+4).$$

On the other hand, define

$$\frac{(q^r,q^{t-r};q^t)_\infty^3}{(q^s,q^{t-s};q^t)_\infty^2}=\sum_{n=0}^{\infty}a_{r,s,t}(n)q^n,$$
$$\frac{(q^r,q^{t-r};q^t)_\infty^3}{(-q^s,-q^{t-s};q^t)_\infty^2}=\sum_{n=0}^{\infty}b_{r,s,t}(n)q^n,$$

where $t\ge 5$ is a prime, $r,s$ are positive integers, $r\ne s$ and $r,s<t$. With the aid of a computer, there are no results similar to (1.7)–(1.10) in $a_{r,s,t}(n)$ and $b_{r,s,t}(n)$ for $t=7$, 11, 13, and 17. A natural question worth study is whether this is an isolated phenomenon or whether there are more such instances for another prime $t$.

Apart from some values near the beginning, the signs of the coefficients in the $q$-series expansions (1.3)–(1.6) appear to be periodic with period 5. For example, for any integer $n\ge 0$,

$$b_1(5n)>0\quad(n\ne 1),$$
$$b_1(5n+1)<0,$$

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$$b_1(5n+2) > 0,$$
$$b_1(5n+3) > 0.$$

We propose the following conjecture.

CONJECTURE 3.1. For given $r$, $s$ and $t$, the signs of $a_{r,s,t}(n)$ and $b_{r,s,t}(n)$ are periodic with period $t$ for sufficiently large $n$.

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