spaces are included in this scheme. Thus much fine work on Hankel operators on Bergman spaces and multivariable Hardy spaces is necessarily omitted. Indeed, this lengthy book contains a huge amount of interesting material, but even so there are other major topics which one is sorry not to see here: the reviewer's own personal choice would have included the very influential L^∞ 'sum of the tail' model reduction results of Glover, the explicit formulae for Hankel singular values of delay systems obtainable by solving two-point boundary-value problems, and the recent striking link between Bonsall's theorem on boundedness of Hankel operators and the admissibility of control and observation operators for semigroups. Further applications in non-commutative geometry and perturbation theory must also be sought elsewhere. Nonetheless, this is a very clear and well-written book, which will be a major source of reference for many years to come.

J. R. PARTINGTON

PELETIER, L. A. AND TROY, W. C. *Spatial patterns: higher order models in physics and mechanics* (Birkhäuser, 2001), 320 pp., 0 8176 4110 6 (hardback), £47.

One of the best-understood objects in the theory of nonlinear partial differential equations is the scalar reaction–diffusion equation in one space dimension:

$$u_t = du_{xx} + f(u), \tag{1.1}$$

where either $x \in I$, a bounded interval, in which case boundary conditions would be prescribed, or on the whole real axis $\mathbb{R}$.

Work of Kolmogorov, Fife, Henry, Hale, Temam and others (see, for example, [**1**, **2**] or, for a short overview, Chapter 1 of this book) has provided decisive answers to questions such as the asymptotic behaviour of solutions; dynamics of transients; structure of the set of equilibria and, remarkably, of stable and unstable manifolds of equilibria; existence and structure of compact attractors; existence of inertial manifolds, of travelling waves, etc.

In obtaining these results, one is helped by two facts. First of all, for stationary solutions of (1.1) phase-plane techniques are available, and, secondly, the maximum principle severely constrains the possibilities of dynamical behaviour and of static patterns.

While models of the type of (1.1) crop up in a number of applications (simplified models of combustion, population dynamics, etc.), one is often naturally led to parabolic equations more complicated than (1.1). Frequently, systems of reaction–diffusion equations are encountered. In addition, many different types of consideration, such as of viscous effects, lead to higher-order parabolic equations. The book under review is, to the present reviewer's knowledge, the first monograph dedicated to the analysis of fourth-order scalar parabolic equations in one space dimension.

From the above it should be clear that analysis of this class of equations, which include the Kuramoto–Sivashinsky, the Swift–Hohenberg and the elastic beam equations (the two latter ones are briefly covered in Chapters 9 and 10), as well as the Cahn–Hilliard equations and the thin-film equations, among many others (for a review, see Chapter 1), is much more difficult than that of (1.1), since the phase space of stationary solutions is now four dimensional and the maximum principle does not apply. Hence one should not expect universally applicable tools, and at this stage analysis has to be conducted on a case-to-case basis. How this can be done in practice is shown in an admirable way in this book.

The 'canonical equation' considered in detail in the present book is

$$u_{xxxx} + qu_{xx} + f(u) = 0, \quad x \in \mathbb{R}. \tag{1.2}$$

To see how (1.2) arises, consider, for example, the extended Fisher–Kolmogorov (EFK) equation:

$$u_t = -\gamma u_{xxxx} + u_{xx} + u - u^3.$$

Then after scaling $x \to \gamma^{1/4} y$, setting $q = -1/\sqrt{\gamma}$, the stationary solutions of the EFK satisfy (1.2) with $f(u) = u^3 - u$ (the symmetric bistable (SBS) equation). For other examples see Chapter 1.

The goal of the book is to give as complete an analysis as possible of the structure of bounded solutions of (1.2) for different source functions f, and $q \in \mathbb{R}$. In particular, one wants to understand the existence and multiplicity of kinks, pulses and of more spatially complex (periodic and chaotic) patterns.

The main tool used by the authors is a powerful topological shooting argument (introduced informally in § 1.3) that is developed in Chapter 3, and then used in Chapters 4–6 to prove existence of periodic solutions, kinks and pulses, and chaotic solutions (to see what is meant by that, the reader should have a look at Theorem 6.1.1). In Chapter 7 a variational approach to (1.2) is presented, with the emphasis being on exhibiting the connection between minimization properties of solutions and their symmetry.

The book is admirably clearly written. It abounds in numerically generated graphs of various types of solutions and bifurcation diagrams. Many proofs are left as exercises with sufficient information given so that a postgraduate student with a reasonable grasp of ordinary differential equation techniques can attempt them. Finally, since this is a rapidly developing area, the numerous open questions that require further work are clearly flagged.

This book is to be strongly recommended to anyone interested in infinite-dimensional dynamical systems, ordinary differential equations, topological methods, and applications of nonlinear partial differential equations in the natural sciences.

References

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