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ARTICLE

Three liquid assets

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Abstract

We examine a theoretical model of liquidity with three assets—money, government bonds, and equity—that are used for transaction purposes. Money and bonds complement each other in the payment system. The liquidity of equity is derived as an equilibrium outcome. Liquidity cycles arise from the loss of confidence of the traders in the liquidity of the system. Both open market operations and credit easing play a beneficial role for different purposes.

Keywords: Money; bonds; equity; liquidity; credit easing

1. Introduction

The events associated with the 2008 financial crisis have revived interest in the theoretical macroeconomic literature with financial constraints. In such a literature, the liquidity of assets plays a key role in the onset, propagation, and amplification of business cycles.[1]

Kiyotaki and Moore (2019) have presented a model economy in which equity is assumed less than fully liquid for exogenous reasons. In such a world, money compensates for the partial illiquidity of equity and business cycles arise from exogenous shocks to the asset liquidity. The model matches well several business cycle facts, except for the counterfactual prediction that equity prices should be higher in bad than in good times. Shi (2015) has pointed out that the problem may lie in the exogeneity of asset liquidity.[2]

This paper suggests a way to endogenize the liquidity of equity in a model that places money center stage as the payment instrument and attributes changes in the liquidity of equity to the self-fulfilling beliefs of the traders, in the tradition of the sunspot literature.[3] Three assets, namely money, government bonds, and equity, are available to lubricate transactions when enforcement is limited due to lack of commitment and anonymity.[4] Capital needs to be transferred from investors to entrepreneurs. Money serves as payment instrument to trade capital, but ends up being partly misallocated relative to best use due to trading uncertainty. Government bonds, that cannot be used as payment instruments due to a legal restriction as in Kocherlakota (2003), help reallocate money toward the traders who need it most and reward with interest those who do not. Depending on the traders' choice, equity may be used as payment instrument or not.

To determine whether equity is liquid or not, we assume, following Lester et al. (2012), that equity may be counterfeited but a costly technology is available that allows those who adopt it to distinguish the counterfeits. The individual incentive to adopt the technology depends on how many other traders decide to adopt it, giving rise to a strategic complementarity that is responsible for the presence of two Pareto-ordered pure strategy equilibria in which equity is either liquid or illiquid, representing a liquidity boom and a liquidity crisis, respectively. The key difference between the two situations is that capital carries a premium over and above its fundamental value when equity is liquid while it reflects only its fundamental value when illiquidity prevails. The

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liquidity of equity sets off a pecuniary externality with beneficial knock-on effects for output and welfare. Consistent with the evidence, asset prices and leverage are higher when the economy is in a boom than in a liquidity crisis.[5]

As is customary in economies in which liquidity plays a role in the allocation of resources, for example in Diamond and Dybvig (1983), liquidity dry-ups may arise endogenously as the result of the coordination of the agents' expectations on different scenarios, through the realization of sunspot uncertainty in the sense of Cass and Shell (1983). According to Lucas and Stokey (2011), this element is bound to be at the heart of any sound explanation of liquidity crises. In this paper, a crisis arises from the loss of confidence of the traders in the genuine value of asset-backed securities.[6]

Several papers have extended the framework of Kiyotaki and Moore (2019), endogenizing liquidity by adding imperfections such as asymmetric information, for example Kurlat (2013) and Bigio (2015), nominal rigidities, for example Ajello (2016), search frictions, for example Venkateswaran and Wright (2014), and costly participation in over-the-counter markets, for example Cui and Radde (2020). Our model keeps the market structure perfectly competitive without informational asymmetries, nominal rigidities, search or participation costs, in the spirit of Kiyotaki and Moore (2019). Crucially, our paper explores a different notion of endogenous liquidity as an equilibrium-dependent feature arising from a strategic complementarity that generates multiple equilibria over which the traders coordinate through sunspot events.[7]

The paper exploits the presence of misallocated money when the trading prospects are uncertain to introduce, alongside money and equity, also government bonds as in Kocherlakota (2003).[8] Money, bonds, and equity play mutually complementary roles. Bonds serve to reallocate money towards those traders who need it most and reward with interest those who would otherwise hold it idle. When liquid, equity helps money in its role as payment instrument and, at the same time, creates elbow room for bonds to shift money around. The presence of both bonds and equity is key to be able to distinguish between different types of monetary intervention.

In the model, public authorities have access to three policy instruments, namely lump-sum taxation, the acquisition or sale of bonds for money, and the acquisition of equity with money. As usual in the monetary literature, the best policy is the Friedman rule. However, due to the enforcement limitations, the amount of taxation needed to support such a policy is not always feasible in this environment. When this is the case, the other two policies may be helpful. Acquiring bonds through an expansionary open market operation has a liquidity effect on the interest rate, as in Lucas (1990), and is beneficial in terms of output and welfare at a given equilibrium, but cannot help the economy move across equilibria.[9] Acquiring equity may, instead, help restore the traders' confidence in the liquidity of the system, thus rescuing the economy from a liquidity crisis. The acquisition of public bonds turns out to be appropriate in normal times, while the acquisition of private securities during liquidity crises.

During the 2008 financial crisis, the Fed has injected liquidity into the economy acquiring temporarily asset-backed securities from the private sector, an unconventional policy that was dubbed *credit easing* by the then Fed chairman Ben Bernanke.[10] Due to well-known irrelevance results à la Wallace (1981), open market operations and credit easing cannot matter in theoretical models in which money and other assets are perfect substitutes.[11] This is not the case, however, when the perfect substitutability across assets is severed by the presence of liquidity or credit constraints, as in Kiyotaki and Moore (2019). In our model, money and government bonds are not perfect substitutes due to the presence of the legal restriction, and money and equity are not perfect substitutes when the economy is plunged into a liquidity crisis by the miscoordination of the traders' expectations on the equilibrium in which equity is illiquid. In this context, the temporary purchase of private equity backed by real assets with fresh liquidity by the public authorities helps set off the pecuniary externality that allows to achieve an equilibrium with larger output and welfare.[12]

The rest of the paper proceeds as follows. Section 2 presents the model and Section 3 the monetary equilibria. Section 4 examines the alternative arrangements and Section 5 policy. Section 6 concludes. The proofs are in the Appendix.

2. Model

2.1 Fundamentals

Time is discrete and continues forever. The economy is inhabited by a unit mass of anonymous ex-ante identical agents, who cannot commit to future actions. Two different goods are available in the economy. One good is durable and can be either consumed or accumulated as capital, k. All the agents can produce the durable good with labor used as input into a linear production function. Labor generates linear disutility for the agents. All the agents wish to consume the durable good from whose consumption they derive linear utility. The other good is perishable, and its production requires capital as an input. The perishable good is produced with a differentiable production function $f(k)$ that satisfies $f(0)=0, f'(0)=\infty$, and $f'(\infty)=0$, with $f'>0>f''$. At the beginning of each period, half of the agents are randomly selected by an i.i.d. process to become producers of the perishable good. We call these agents entrepreneurs and the rest investors. Capital depreciates after production at rate $\delta<1$. We will use the notation $F(k)\equiv f(k)+(1-\delta)k$. If not used in production, capital can be stored without cost. Only the producers of the perishable good wish to consume it, obtaining linear utility from its consumption. The perishable good disappears if not consumed immediately after production. All the agents discount future payoffs at rate $\beta<1$.

2.2 Trade

In each period, the agents can interact in three different markets, a liquidity market (LM), a secondary market (SM) for previously accumulated capital, and a primary market (PM) for new capital, opening sequentially. In the LM, the agents may choose to trade liquidity at a competitive price p.[13] There is neither production nor consumption at this stage. Capital accumulated from the past can be traded in the SM. Trade in this market is bilateral, with each entrepreneur being matched to an investor, and the price is determined by an increasing function, $q(\kappa)$, of the amount traded in the SM, κ, which we assume linear in order to stay as close as possible to the competitive framework of Kiyotaki and Moore (2019).[14] Production and consumption of the perishable good occur after capital is traded in the SM but before the PM opens. The durable good is the numeraire and can be traded in the competitive PM. Production and consumption of the durable good, as well as the accumulation of new capital, all occur before the end of the current period. The economy begins in the first period in the PM. Three assets are available for trade, money, m, public bonds, b, and capital, k. Money is an intrinsically worthless durable object whose supply, M, is controlled by the monetary authorities. Its price in numeraire units is v. Bonds, whose supply is denoted by B, are sold by the fiscal authority for money during the PM of each period and reimbursed in money during the following PM with a competitively determined gross interest rate i. The bonds cannot be traded in the SM due to a legal restriction that prevents them from serving as direct payment instruments, but they can be traded in the LM for money.[15] The output produced with capital is perishable and specific for the use of entrepreneurs; hence, it cannot be pledged to investors to acquire capital in the SM, but capital, whose aggregate stock is denoted with K, is identifiable and can be seized by outsiders; hence, the agents can issue equity on its undepreciated value. Equity, however, can be counterfeited on the spot without cost by the entrepreneurs.[16] The investors are unable to recognize counterfeits unless they incur a cost c at the beginning of the period. If the investors incur the cost, in the SM, they can use a technology that distinguishes counterfeited from genuine equity. Equity issued in any given period, whether genuine or counterfeited, disappears before the beginning of the following period. Equity may also be interpreted as a one-period security or loan backed by a real asset.

2.3 Government

The monetary and fiscal authority are consolidated in a single agency called the government. The government has a limited ability to tax the agents due to their anonymity. However, since

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capital is identifiable by outsiders, the government may be able to tax the agents up to the value of their undepreciated capital holdings. The government fulfills its budget constraint, $M_{+1} = M + B - i^{-1}B_{+1} - T$.[17] Lump-sum taxation is represented by $T \equiv \tau(M+B)$ as a fraction of the outstanding stock of liquid government instruments, with a negative τ representing a subsidy. We define the bonds to money ratio $x \equiv \frac{B}{M}$ and assume that the government keeps it constant over time. With this notation, the budget constraint of the government can be written as

$$\frac{M_{+1}}{M} = \frac{(1+x)(1-\tau)}{1+xi^{-1}}. \tag{1}$$

Hence, we will use x as the monetary policy parameter, capturing open market operations, namely relative shift in the long run proportion of the stock of money and bonds available in the economy, and indirectly controlling the growth rate of the money stock. The fiscal policy parameter τ also partially controls the growth rate of the money stock. The policy pair (τ, x) is decided and implemented once and forever at the initial date. We restrict attention to the empirically relevant case with $x \geq 1$ and $\tau \in [0, 1-\beta]$. To save on notation, we will sometimes use $z \equiv 1 - \beta - \tau \geq 0$.

2.4 Efficiency

Every period, a randomly selected half of the agents turns out to have an amount of capital for which they have no immediate better use than storing it, while the rest can use it in production. The efficient level of capital accumulation solves the recursive problem,

$$V\left(k\right) = \max\ \frac{1}{2}F\left(2k\right) + k - k_{+1} + \beta V\left(k_{+1}\right),$$

where $2k = K$, as efficiency requires that capital should be allocated entirely to the entrepreneurs for the production of the perishable good if the marginal productivity is larger than depreciation. This is because the best alternative for the investors consists in storing the capital, which gives a zero net return. Optimization with respect to capital accumulation leads to the Euler condition

$$1 = \beta F'(K). \tag{2}$$

By continuity and the Inada conditions, a positive solution of (2) exists and by concavity of the production function is unique. Since the marginal productivity of undepreciated capital is $\beta^{-1} > 1$, which is the return of storage, the allocation of the entire amount of available capital to the entrepreneurs is vindicated. The efficient amount of capital is

$$K^* = f'^{-1}\left(\beta^{-1} - 1 + \delta\right). \tag{3}$$

The efficient amount of capital accumulation, (3), could be decentralized as a competitive equilibrium for all parameter values if there was full commitment or for β sufficiently large if the traders were not anonymous.

3. Monetary equilibrium

Since every period some of the agents have an amount of capital for which they have no immediate use, while the rest can use it in production, there is a motive to trade capital in a secondary market before production of the perishable good occurs. However, trade in such a market is impeded by the inability of the agents to commit themselves to future actions and by anonymity. There are two imperfections. First, the output generated with capital is perishable and specific for the use of the entrepreneurs; hence, it cannot be pledged to the investors to acquire capital in the secondary market. Second, the agents cannot credibly commit to pay for transactions that occur in the LM or SM with work done in the PM or in future periods. This is because the agents are unable to commit to deliver on promises of future payments, and anonymous; hence, neither bilateral

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nor multilateral credit deals can be enforced threatening to punish defectors. In this situation, physical assets play a useful role as payment instruments. Capital may be either liquid or illiquid, depending whether the investors decide to pay the cost c to find out whether equity has been counterfeited or not by their trading partner in the SM. Either way, counterfeiting does not occur in equilibrium, since counterfeits are not accepted if the cost is incurred by the investors, and equity is not accepted if the cost is not incurred. Equity can be used as payment only in the SM, since the technology to distinguish the counterfeits is available there. Under the assumption that the pricing function in the SM is linear, we treat such a market as competitive.

3.1 Liquid equilibrium

Consider the case in which the investors, at the beginning of the period, decide to incur the cost c to distinguish genuine from counterfeited equity. In this scenario, bonds are used to trade money in the LM, and then, money and equity are traded in the SM for second hand capital. An entrepreneur chooses how much money to acquire in the liquidity market, m^d, and how much capital to trade in the secondary market, κ, to maximize

$$V^E\left(m, b, k\right) = \max\ f\left(k+\kappa\right) + (1-\delta)\left(k+\kappa\right) - q\kappa + vm^d - pm^d + W\left(m, b\right)$$

subject to the constraints

$$pm^d \leq vb. \tag{4}$$

in the LM and the constraint

$$q\kappa \leq vm + vm^d + (1-\delta)\,k, \tag{5}$$

in the SM. These two constraints reflect the purchase of money in the liquidity market using bonds and, then, the purchase of additional capital in the secondary market using all the money held by the entrepreneur, including the cash just acquired in the liquidity market, and equity on undepreciated capital. An investor chooses how much money to sell in the liquidity market, m^s, and trade in the secondary market, to maximize

$$V^I\left(m, b, k\right) = \max\ k - \kappa + q\kappa + pm^s - vm^s + W\left(m, b\right)$$

subject to the constraint

$$vm^s \leq vm, \tag{6}$$

and

$$\kappa \leq k. \tag{7}$$

These constraints reflect the limited amount of cash and capital currently in the hands of the investor. In the primary market, an agent chooses money, m_{+1}, bonds, b_{+1}, and capital, k_{+1}, to solve

$$W\left(m, b\right) = \max\ vm + vb - t - vm_{+1} - i^{-1}vb_{+1} - k_{+1} + \beta V\left(m_{+1}, b_{+1}, k_{+1}\right),$$

where $t \equiv \tau vM(1+x)$ and the value function satisfies

$$V\left(m, b, k\right) = \frac{1}{2}\left[V^E\left(m, b, k\right) + V^I\left(m, b, k\right)\right].$$

The distribution of assets is degenerate at equilibrium by virtue of the linearity of the payoffs, as in Lagos and Wright (2005). The market clearing conditions are $m^s = m^d$ for trade in the LM; $m = M$ and $b = B$ for money and bonds, and the market clearing condition for the durable good. The amount traded in the SM match since meetings are bilateral. A competitive equilibrium requires the agents to optimize taking prices as given and the prices to clear the competitive markets. We

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consider stationary equilibria, namely equilibria in which real variables are time invariant. The equilibrium conditions are fully derived in the Appendix. Since money and bonds are traded for each other in the liquidity market, by arbitrage, their returns are equated, that is $\frac{p}{v}=i$. Define the gross return of capital as $r\equiv\frac{F'(K)}{q}$. All the constraints are binding at equilibrium, except possibly (4). The stationary equilibrium conditions are the Euler condition for money

$$1=\frac{\beta}{2}\frac{(i+x)(r+i)}{(1-\tau)\,i\,(1+x)}, \tag{8}$$

since money is used by the entrepreneurs to acquire capital in the secondary market and sold for bonds by the investors in the liquidity market earning the corresponding interest, with the return of money being the inverse of (1); the Euler condition for capital accumulation

$$1=\frac{\beta}{2}\left[q\,(r+1)+(r-1)\,(1-\delta)\right], \tag{9}$$

where the first term in brackets on the RHS is the fundamental value of capital, while the second corresponds to the liquidity premium, reflecting a pecuniary externality that is at work in the accumulation of capital; and the complementary slackness condition for the liquidity constraint (4)

$$(r-i)\,(x-i)=0. \tag{10}$$

To guarantee that the investors have the incentive to accept bonds in the LM and sell capital in the SM, it has to be that $i\geq 1$, $r\geq 1$, and $q\geq 1$. The technology to distinguish genuine from counterfeited equity is adopted by all investors. Next, we define the equilibrium with liquid equity.

Definition 1. *A stationary equilibrium with liquid equity is a time invariant triple (i, r, q) such that (8), (9), and (10) hold with $i\geq 1$, $r\geq 1$ and $q\geq 1$.*

The next Proposition establishes existence and uniqueness of such an equilibrium. An equilibrium is constrained if the liquidity constraint is binding and unconstrained otherwise.

Proposition 1. *There exist a $\widetilde{c}>0$ and $\widetilde{\tau}>0$, such that, if $c<\widetilde{c}$ and $\tau\geq\widetilde{\tau}$, a stationary equilibrium with liquid equity exists and is unique. If $x<\frac{1-\tau}{2\beta-1+\tau}$, the equilibrium is constrained; if $x\geq\frac{1-\tau}{2\beta-1+\tau}$, the equilibrium is unconstrained.*

The restriction for τ guarantees $q\geq 1$. The restriction for c guarantees that each investor has the incentive to adopt the technology that distinguishes genuine from counterfeited equity when everybody else does. There are two possibilities, as the liquidity constraint may be binding or not. When the liquidity constraint is binding, the equilibrium is constrained and the interest rate is $\widetilde{i}=x\geq 1$; when the constraint is slack, the equilibrium is unconstrained and the interest rate is $\widetilde{i}=\widetilde{r}\geq 1$. In both cases, the return of capital is

$$\widetilde{r}=\frac{\beta+z\,(1+x)}{\beta}, \tag{11}$$

and the price of second hand capital is

$$\widetilde{q}=\frac{2-z\,(1+x)\,(1-\delta)}{2\beta+z\,(1+x)}. \tag{12}$$

This is an equilibrium in which the liquidity market is open, debt serves to reshuffle and reward with interest money holdings that would otherwise remain idle, and equity is used as payment instrument together with money to trade capital in the secondary market. In this sense, equity is liquid.

3.2 Illiquid equilibrium

Suppose now that the investors decide not to pay the cost c, and, hence, payment with equity is refused in the SM since the investors are worried that equity may be counterfeited. The only difference with the previous case is that the constraint (5) becomes

$$q\kappa \leq vm + vm^d, \tag{13}$$

since capital is illiquid. The equilibrium conditions are the same except for (9) that becomes

$$1 = \frac{\beta}{2} q\,(r+1)\,, \tag{14}$$

since there is no liquidity premium for capital anymore, and, hence, capital has only fundamental value. To guarantee that the investors have the incentive to accept bonds in the LM and sell capital in the SM, it has to be that $i \geq 1$, $r \geq 1$, and $q \geq 1$. The technology to distinguish genuine from counterfeited equity is not adopted by any investor. Next, we define the equilibrium with illiquid capital.

Definition 2. *A stationary equilibrium with illiquid capital is a time invariant triple* (i, r, q) *such that (8), (14), and (10) hold with* $i \geq 1$, $r \geq 1$ *and* $q \geq 1$.

The next Proposition establishes existence and uniqueness of such an equilibrium. An equilibrium is constrained if the liquidity constraint is binding and unconstrained otherwise.

Proposition 2. *There exist values* $\widehat{c}$ *and* $\widehat{\tau}$, *such that, if* $c > \widehat{c}$ *and* $\tau \geq \widehat{\tau}$, *a stationary equilibrium with illiquid capital exists and is unique. If* $x < \frac{1-\tau}{2\beta-1+\tau}$, *the equilibrium is constrained; if* $x \geq \frac{1-\tau}{2\beta-1+\tau}$, *the equilibrium is unconstrained.*

The restriction for τ guarantees $q \geq 1$. The restriction for c guarantees that each investor has no incentive to adopt the technology that distinguishes genuine from counterfeited equity when nobody else does. The equilibrium values of i and r are the same as in the previous case, with $\widehat{i} = x$, when the equilibrium is constrained, $\widehat{i} = \widehat{r}$, when unconstrained; and

$$\widehat{r} = \frac{\beta + z\,(1+x)}{\beta}. \tag{15}$$

The price of second hand capital, instead, is

$$\widehat{q} = \frac{2}{2\beta + z\,(1+x)}, \tag{16}$$

which is larger than at the liquid equilibrium for $\tau < 1-\beta$ and the same for $\tau = 1-\beta$. In this equilibrium, capital is illiquid. When this occurs, the equity premium disappears, which makes the price of second hand capital higher than with liquid equity.

3.3 Implications

The model has implications for the so-called Tobin's Q, leverage, capital, and output, as we document next.

Tobin's Q The Tobin's Q is the ratio between the market value of a company and the replacement cost of its capital. In the current model, the market value of the entrepreneur's company is $(1-\delta)K$ and the replacement cost of capital is $qK/2$. Hence, the Tobin's Q is $Q \equiv 2(1-\delta)q^{-1}$. Inserting into it the equilibrium value of q, at the two equilibria, (12) and (16), we obtain, respectively,

$$\widetilde{Q} = \frac{2\,(1-\delta)\,[2\beta + z\,(1+x)]}{2 - z\,(1+x)\,(1-\delta)}, \tag{17}$$

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at the liquid equilibrium, and

$$\widehat{Q} = (1-\delta)\left[2\beta + z(1+x)\right], \tag{18}$$

at the illiquid equilibrium, with (17) strictly larger than (18) for $\tau < 1-\beta$ and equal to it for $\tau = 1-\beta$. We conclude that the Tobin's Q is higher when equity is liquid. This is consistent with the available evidence and stands in contrast to the model of Kiyotaki and Moore (2019), that gives rise to the counterfactual prediction that the equity price is higher when equity is less liquid, as pointed out by Shi (2015).[18]

Leverage Interpreting equity as an asset-backed security, the model determines also leverage in the sense of Geanakoplos (2010). Leverage is the ratio of the asset value to the cash needed to purchase it. In our model, this is given by $L \equiv qK(4vM)^{-1}$. In this model, leverage depends on the equilibrium that realizes. At the liquid equilibrium, $4vM = (q-1+\delta)K$, by the binding payment constraint. Inserting into it the value of q at the liquid equilibrium, (12), we obtain

$$\widetilde{L} = \frac{2 - z(1+x)(1-\delta)}{2\left[1-\beta(1-\delta) - z(1+x)(1-\delta)\right]}, \tag{19}$$

which is larger than 1. At the illiquid equilibrium, we have that $4vM = qK$, by the binding payment constraint. Hence, at the illiquid equilibrium, there is no leverage,

$$\widehat{L} = 1. \tag{20}$$

This is because the asset-backed securities are not traded. This is broadly consistent with the evidence for the US, where the leverage for asset-backed securities was largely above 1 for the entire period between 2000 and the 2008 financial crisis, and was down to 1 during the crisis.[19]

Capital and Output The stock of capital is $K = f'^{-1}(rq - 1 + \delta)$, which is a decreasing function of its argument, since the production function is strictly concave. At the liquid equilibrium, inserting the equilibrium values (11) and (12), we obtain the capital stock

$$\widetilde{K} = f'^{-1}\left(\frac{2\left[1-\beta(1-\delta)\right]\left[\beta + z(1+x)\right] - z^2(1+x)^2(1-\delta)}{\beta\left[2\beta + z(1+x)\right]}\right). \tag{21}$$

At the illiquid equilibrium, inserting the equilibrium values (15) and (16), we obtain the capital stock

$$\widehat{K} = f'^{-1}\left(\frac{2\left[1-\beta(1-\delta)\right]\left[\beta + z(1+x)\right] + z(1+x)\beta(1-\delta)}{\beta\left[2\beta + z(1+x)\right]}\right), \tag{22}$$

which is strictly smaller than (21) for $\tau < 1-\beta$ and equal to it for $\tau = 1-\beta$. Since output is an increasing function of the capital stock, $f(K)$, and the capital stock is larger at the liquid than the illiquid equilibrium, it follows that GDP is higher when the economy is liquid than illiquid. This is consistent with the evidence on the great recession for the US that saw a drop of 8% of GDP in the last quarter of 2008.[20]

3.4 Welfare

Due to the linearity of the payoffs, the ex-ante welfare of the individuals at any stationary equilibrium of the model is given by

$$V(K) = \frac{\beta F(K) - K}{2(1-\beta)}. \tag{23}$$

Definition 3. *An equilibrium dominates another if it gives rise to a value of (23) which is at least as large and sometimes strictly larger than the alternative.*

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Since $rq \geq \beta^{-1}$, the equilibrium capital stock is never larger than K^*. It follows that $\beta F'(K) \geq 1$. The function (23) is strictly concave in K. Hence, any increase in capital accumulation turns directly into a welfare improvement. Since (21) is never smaller than (22) and strictly larger for $\tau < 1 - \beta$, it follows immediately that the equilibrium with liquid equity dominates the illiquid one in terms of welfare. We need only to establish that both equilibria exist for the same parameter values, in particular, that there exists a non-empty region of the cost of adopting the technology that allows traders to distinguish genuine from counterfeited equity such that both the liquid and illiquid equilibrium exist. This is what the next Proposition does. Define $\alpha = \alpha(K) \equiv -\frac{f''(K)K}{f'(K)}$, as the curvature of the production function. Assume that $\alpha'(K) \geq 0$, taht is the curvature of the production function is non-decreasing in the capital stock.

Proposition 3. *There exist values $\overline{\tau} > 0$ and $\overline{\alpha} > 0$, such that, for $\tau > \overline{\tau}$ and $\alpha < \overline{\alpha}$, the cutoffs for the cost at the liquid and illiquid equilibria are $\widetilde{c} > \widehat{c}$; if $c \in (\widehat{c}, \widetilde{c})$ and $\tau \geq \max\{\widetilde{\tau}, \widehat{\tau}, \overline{\tau}\}$, both equilibria exist and the former dominates the latter.*

When all investors adopt the technology that allows to distinguish counterfeits from genuine private securities, these securities are accepted as payment, setting off the pecuniary externality that affects the price and amount of capital traded in the SM; when no investor adopts the technology, instead, the pecuniary externality generates opposite effects. Hence, the strategic complementarity arises from this general equilibrium effect, but one still needs to check that there is indeed a multiplicity of equilibria within the same region of parameters. Since the benefit of adopting the technology for an investor depends both on the price and the amount traded of second hand capital, and these move in opposite directions across the two equilibria, it is key to find out whether the price or quantity effect dominates, that is the elasticity of the benefit, which is controlled by the curvature of the production function. The bound on the curvature of the production function serves precisely to make sure that both equilibria above, with and without liquid securities, exist for the same parameters values. When both equilibria exist, they are immediately Pareto-ordered as capital accumulation is higher at the liquid than illiquid equilibrium. Since capital accumulation, output, welfare, the Tobin's Q, and leverage are all larger when the equilibrium is liquid than illiquid, it seems appropriate to interpret the former as a boom and the latter as a liquidity crisis.

3.5 Endogenous liquidity cycles

In this model, whether the economy ends up liquid or illiquid and, hence, in a liquidity boom or crisis, depends on how the traders coordinate their expectations over different outcomes. To illustrate this point, we introduce sunspot events in the spirit of Cass and Shell (1983), namely payoff irrelevant events that can affect the fundamentals only through the agents' expectations. For concreteness, we limit attention to stationary sunspot events of order two that alternate randomly over time, with the probability that next period the event $s' = 1, 2$ will occur, given that today the event $s = 1, 2$ has occurred, represented by $\pi_{ss'}$. Suppose that, if the first event occurs, all the investors pay the cost c to activate the verification technology that helps them distinguish between genuine and counterfeit equity, while, if the second event occurs, they do not pay the cost and the technology is not activated. Debt is never used as a payment instrument, due to the legal restriction advocated by Kocherlakota (2003), but still serves to reallocate money. Define $\mu \equiv (1 - \tau)(1 + x)$ and the indicator function $I_s \in \{0, 1\}$ with $I_1 = 1$ and $I_2 = 0$. The equilibrium conditions are the Euler condition for money,

$$v_s = \frac{\beta}{\mu} \sum_{s'=1,2} \pi_{ss'} v_{s'} \left(r_{s'} + x\right), \tag{24}$$

and the Euler condition for capital, that can be rewritten so as to obtain

$$q_s = \frac{2 - \beta\,(1-\delta)\,I_s\,(r_s - 1)}{\beta\,(r_s + 1)} \equiv g_s\,(r_s)\,, \tag{25}$$

for each s. The complementary slackness condition for the liquidity constraint in the LM determines the interest rate, which is either $i_s = x$ or $i_s = r_s$, for both s. We limit attention to equilibria that are always either constrained or unconstrained in both states. From the binding constraint in the SM, obtain the value of money, $v_s = G_s(r_s)(4M)^{-1}$, where $G_s(r_s) \equiv f'^{-1}(r_s g_s(r_s) - 1 + \delta)[g_s(r_s) - (1-\delta)I_s]$, for each s. Using this into (24), we obtain the two equilibrium conditions

$$G_s\,(r_s) = \frac{\beta}{\mu} \sum_{s'=1,2} \pi_{ss'}\,(r_{s'} + x)\,G_{s'}\,(r_{s'})\,, \tag{26}$$

for each $s = 1, 2$ in the two unknowns, that is the returns in the two sunspot states, r_s for $s = 1, 2$. The returns need to be different in the two states and larger than or equal to one, to guarantee the incentive to lend money in each state. Next, we define stationary sunspot equilibria.

Definition 4. *A two state stationary sunspot equilibrium is a pair* (r_1, r_2) *that satisfies (26) with* $r_1 \neq r_2$ *and* $r_s \geq 1$ *for* $s = 1, 2$.

The following Proposition proves the existence of a sunspot equilibrium.

Proposition 4. *There exist values* $\bar{\delta} < 1$ *and* $\bar{\pi} > 0$*, such that, for* $\delta \in (\bar{\delta}, 1)$ *and* $\pi_{ss'} \in [0, \bar{\pi})$ *with* $s' = s$ *for each s, a two state stationary sunspot equilibrium exists.*

Since the case with $\pi_{ss'} = 0$ with $s' = s$ for each s is included, this also establishes the existence of a deterministic cycle of period two. Due to the self-fulfilling expectations of the traders, the economy ends up oscillating randomly between good times in which equity is liquid and the economy is booming and bad times in which capital is illiquid and the economy is in the dumps. These situations arise endogenously from the coordination of the agents' expectations on different scenarios in which equity is perceived as either trustworthy, hence, liquid, or not. Notice that this is not the conventional situation in monetary theory where there is a no-trade equilibrium in which money has no value and another equilibrium with trade and valued money.[21] Here, in both equilibria, money is valued and there is some trade of capital in the SM, but equity may be liquid or not. In turn, this determines whether capital has a liquidity premium, which alters the price of capital with knock-on effects on the capital stock, output, and welfare.

4. Alternative arrangements

The reader may wonder whether there are other arrangements that are feasible and may give rise to higher welfare than those examined so far. We examine two classes of alternative arrangements. First, we consider the alternative arrangements that are available without dropping any of the restrictions adopted so far, namely the legal restriction on the use of bonds in the SM, and the technological restriction on the use of equity in the LM. Second, we consider the alternative arrangements that are available when these two restrictions are lifted.

4.1 Subset of instruments

Given the imperfections of the environment, the legal restriction on the use of government bonds in the SM, and the technological restriction on the use of equity in the LM, there are three feasible alternative arrangements, in which either trade in the SM is conducted using only equity, only money, or a combination of money and equity but no bonds. In all three cases, the agents skip

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trade in the LM and show up directly for trade in the SM. In the first case, the entrepreneurs issue equity on the value of the undepreciated capital stock after production to pay for the acquisition of second hand capital in the SM, without using any other payment instrument. Trade by the entrepreneurs in the SM is subject to the constraint $q\kappa \leq (1-\delta)k$. We assume that the cost c is not too large to impede the use of equity as the sole means of payment. In the second case, the entrepreneurs pay for the acquisition of second hand capital in the SM with money, without using any other payment instrument. Trade by the entrepreneurs in the SM is subject to the constraint $q\kappa \leq vm$. In the third case, the entrepreneurs pay for the acquisition of second hand capital in the SM with money and equity. Trade by the entrepreneurs in the SM is subject to the constraint $q\kappa \leq vm + (1-\delta)k$. We assume that the cost c is not too large to impede the use of equity as means of payment. We compare the equilibria of these trading arrangements with those examined above according to the ex-ante welfare criterion.

Definition 5. *An arrangement dominates another one if it gives rise to a value of equilibrium welfare which is at least as large and sometimes strictly larger than the alternative.*

The following result compares the feasible arrangements in terms of ex-ante welfare.

Proposition 5. *a. There exists a value $\underline{\tau}$ such that, for $\tau \geq \underline{\tau}$, the liquid equilibrium of the arrangement with money, bonds, and equity dominates all the feasible alternative arrangements; b. The illiquid equilibrium of the arrangement with money, bonds, and equity is dominated by the arrangement with only money.*

The equilibrium with only equity limits the amount of capital that can be reallocated trading in the SM. This is because the equilibrium price of second hand capital cannot be smaller than 1, since the investors can store capital one-for-one. A price larger than 1, however, is incompatible with the constraint $q\kappa \leq (1-\delta)k$, since depreciation is non-negative and the amount of capital that is available to be reallocated is k. Therefore, at equilibrium, it has to be that $q=1$ and $\kappa = (1-\delta)k$. Thus, not all the capital in the hands of the investors is acquired by the entrepreneurs in the SM. This is a distortion relative to efficiency, that requires all the capital to end up in the hands of the entrepreneurs. The traders compensate this distortion accumulating capital above the efficient amount over time. The liquid equilibrium with money, government bonds, and equity outperforms this arrangement at least for τ not too small, since it reallocates capital properly in the SM, thus, avoiding the overaccumulation of capital over time. As regards the arrangement with money and equity, without the use of bonds in the LM, the liquid equilibrium with money, government bonds, and equity outperforms this arrangement for τ not too small since it reallocates money according to trading needs and rewards idle money balances with interest using government bonds. The equilibrium of the arrangement with only money being used as payment instrument is dominated by the one with money and equity. This shows that the equilibrium we have identified as a liquidity boom is indeed the best arrangement among those that are feasible given the imperfections of the environment and the existing legal and technical restrictions. Thus, the results do not depend on having selected a dominated arrangement. On the other hand, the illiquid equilibrium at best replicates the arrangement in which only money is used as payment instrument, which shows that this equilibrium can indeed be identified as a liquidity crisis.

4.2 No legal or technical restrictions

Next, we ask what would happen if we were to lift the legal restriction on bonds for their use in the SM and the technological restriction on equity for its use in the LM. To address this question, in the Appendix, we set up a general scheme that captures all the alternative possibilities and compare the ensuing equilibrium allocations and the associated equilibrium welfare. It turns out that the technological restriction on the use of equity in the LM is not crucial for our results, since equity would have a premium even when used as a liquid instrument in the LM. The only difference

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would be that the premium would be smaller, being discounted by the interest rate. Hence, the price of second hand capital would be larger and the equilibrium allocation worse than the one obtained at the liquid equilibrium above. The question whether the legal restriction imposed on government bonds in the SM is socially beneficial as in Kocherlakota (2003) is a bit more subtle, since there are four different situations to be compared, when bonds are used in the LM with liquid and illiquid equity and when bonds are used in the SM with liquid and illiquid equity. There exist values of taxation such that the liquid equilibrium when bonds are used in the LM dominates the liquid equilibrium when bonds are used in the SM. The same holds for the illiquid equilibria of the arrangements with bonds in the LM and SM, respectively, provided the bonds to money ratio is not too large. However, the illiquid equilibrium when bonds are used in the LM is dominated by the liquid equilibrium when bonds are used in the SM for all parameter values. Hence, the payment of interest on otherwise idle money balances alone is not enough to generate a welfare improvement in this economy. The presence of a liquidity premium for capital, with the ensuing pecuniary externality generated through the price of second hand capital, is required to obtain a welfare improvement relative to an economy in which the legal restriction on the use of bonds as payment instruments is absent. Overall, the combination of these results confirms that the liquid equilibrium of the arrangement considered above with money, government bonds used in the LM, and equity used in the SM is the best scenario, given the informational imperfections of the environment, while the illiquid equilibrium of the same arrangement may be pretty bad.

5. Optimal policy

Next, we discuss optimal fiscal and monetary policy. The public authorities can influence market outcomes through the policy instruments, fiscal, and monetary. Optimal policy maximizes equilibrium welfare. In what follows, we show that the Friedman rule is optimal, but may not be feasible. When this is the case, conventional monetary policy may be able to improve the situation at a particular equilibrium but cannot help the economy move across equilibria, while unconventional policy can help the economy move across equilibria. By (23), since the equilibrium capital stock is always inefficiently small, any change induced in the accumulation of capital by policy intervention has a direct effect on welfare. In turn, the effects of fiscal and monetary policy on the capital stock at the two equilibria are immediate from (21) and (22). The effects of fiscal and monetary policy on the interest rate and prices at the two equilibria are also immediate from (11), (15), (12), and (16).

5.1 Lump-sum taxation

The fiscal policy parameter, τ, which represents lump-sum, that is, non-distortionary, taxation, has a positive effect on capital accumulation at both equilibria. Thus, the optimal policy consists in setting $\tau = 1 - \beta$, which induces the efficient allocation, K^*. This corresponds to the so-called Friedman rule. However, the Friedman rule may not be feasible in this environment due to the limitation of enforcement. The ability to tax the traders is limited by the available undepreciated capital stock. The following Proposition determines when the Friedman rule can be achieved or not.

Proposition 6. *The Friedman rule is optimal. a. At the equilibrium with liquid equity, there exists* $\widetilde{\delta} < 1$ *such that the Friedman rule can be achieved if and only if* $\delta \leq \widetilde{\delta}$*; b. At the equilibrium with illiquid capital, there exists* $\widehat{\delta} < 1$ *such that the Friedman rule can be achieved if and only if* $\delta \leq \widehat{\delta}$*.*

When depreciation is sufficiently close to 1, the undepreciated capital stock is small and lump-sum taxation is severely limited, so that the Friedman rule cannot be achieved. The optimal fiscal policy consists in any case in setting τ as close as possible to the bound.

5.2 Open market operations

When the ability to tax the traders is limited and, hence, the efficient allocation cannot be achieved through fiscal policy alone, monetary policy may be helpful. One type of monetary policy is captured in the model by changes in the bonds to money ratio, x, which represents open market operations. In both equilibria, with liquid and illiquid equity, the interest rates, and the Tobin's Q are increasing in x. Hence, there is a liquidity effect of open market operations, as an expansionary open market operation—corresponding to a smaller value of x—lowers the interest rate. Notice that, when the equilibrium is constrained, that is when the bonds to money ratio is relatively small, open market operations affect asymmetrically the interest rate of bonds and capital, while, when the equilibrium is unconstrained, the effect is symmetric. This is broadly consistent with the US evidence over the last thirty years, as the returns of equity, the Bond Bill rate and the Tobin's Q, all correlate positively with the bond to money ratio.[22] Leverage, on the other hand, is increasing in x at the liquid equilibrium but insensitive to conventional policy at the illiquid equilibrium. In both equilibria and independently whether the equilibrium is constrained or not, capital accumulation is decreasing in the bonds to money ratio, implying that an expansionary open market operation increases capital accumulation with beneficial effects for output and welfare. This is also broadly consistent with the evidence for the US over the last three decades, as gross physical capital formation in the US has been inversely related to the bond to money ratio since the mid-1990s.[23] We summarize the discussion in the next Proposition.

Proposition 7. *At both equilibria and independently whether the equilibrium is constrained or not, the interest rate and Tobin's Q are increasing and the capital stock is decreasing in x. Leverage is increasing in x at the liquid equilibrium and insensitive to x at the illiquid equilibrium.*

Thus, at any given equilibrium, expansionary open market operations have a liquidity effect and are socially beneficial. This policy is, instead, ineffective in moving the economy from an equilibrium with illiquid capital, that is from a liquidity crisis, to an equilibrium with liquid equity, that is a boom, as reflected in its inability to affect leverage when the economy is in a liquidity crisis. When the economy is stuck at the illiquid equilibrium, the government may consider lifting the legal restriction on the use of bonds as payment instrument. The traders would then be able to replace equity which does not function as payment instrument with a publicly issued instrument, such as government bonds. As can be seen from the analysis in the Appendix, the new equilibrium would give rise to a better allocation of resources relative to the illiquid equilibrium, although not as good as the liquid equilibrium. If used as payment instrument, government bonds would become perfect substitutes of money and, as a consequence, the nominal interest rate would vanish, hitting the zero lower bound. This equilibrium would resemble a liquidity trap, in which, depending on the bonds to money ratio, output, and welfare may be slightly higher than during the crisis but still lower than in normal times, with a zero nominal interest rate and ineffective open market operations.[24] The economy may end up being stuck at a second-rate equilibrium as a consequence of this policy move.[25] A better alternative is examined next.

5.3 Credit easing

When the economy is mired in a liquidity crisis, a type of unconventional monetary policy known as credit easing may help rescue the situation. Consider the equilibrium in which the investors do not pay the cost and fail to adopt the technology that allows them to distinguish genuine from counterfeited equity. Suppose that not only the private sector but also the government has access to this technology and its cost is $c \in (\widehat{c}, \widetilde{c})$. The government internalizes the benefits for all traders, that is both entrepreneurs and investors, and decides to pay the cost and adopt the technology. At the beginning of every period, the government issues an amount of money, $\overline{m}$, which is exchanged at current market price, v, for equity issued by the traders against a fraction γ of

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their undepreciated capital holdings. Such an amount is withdrawn from circulation before the end of the period when the capital is bought back by the agents in the PM; hence, this policy does not alter the stock of money over time. In other words, the public authorities acquire some private assets with a temporary injection of liquidity, in a move that resembles what has come to be known as credit easing. The constraint (5) becomes $q\kappa \le vm + vm^d + v\overline{m}$, with $v\overline{m} = \gamma(1-\delta)k$. The equilibrium is the same as when equity is liquid, with the price of capital in the SM given by

$$q = \frac{2 - \gamma z (1+x)(1-\delta)}{2\beta + z(1+x)}, \tag{27}$$

while the return, r, is still given by (11). This policy makes capital at least partially liquid without generating extra inflationary pressures since the stock of money is not altered further over time. This, in turn, helps the economy move from the inferior equilibrium with illiquid capital to the superior equilibrium with liquid equity. With the right injection of liquidity, the allocation of the dominating equilibrium can be attained. Since (27) is decreasing in γ, the rate of return r is unaffected, and the capital stock is decreasing in q, the equilibrium capital stock is increasing in γ. Welfare is increasing in the equilibrium capital stock; therefore, it is optimal to set γ as high as possible. The next Proposition follows immediately.

Proposition 8. *Credit easing is welfare improving, and it is optimal to set $\gamma = 1$. Tobin's Q, leverage, and the capital stock are all increasing in γ.*

In other words, in a liquidity crisis, this unconventional policy reinstates the liquidity premium of capital, giving rise to a pecuniary externality that lowers the price of capital with socially beneficial effects. By restoring the confidence in asset-backed securities, unconventional policy also brings back leverage. This provides a rationale for the purchase of asset-backed private securities by the US Federal Reserve during the 2008 financial crisis. Between the end of 2008 and the beginning of 2009, the balance sheet of the Fed doubled, going from about 1 trillion to 2 trillion dollars.[26] Over that period of time, the Fed exchanged government liquidity for private assets through various facilities, including the Term Auction Facility, the Primary Dealer Credit Facility, and the Term Securities Lending Facility.[27] The consensus seems to be that unconventional policy had a positive impact on the US economy. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) claim that "the economy may have suffered a second Great Depression in the absence of interventions."[28]

6. Conclusion

We have presented a model in which money, bonds, and equity may all be used for transaction purposes. We have shown that there are two Pareto-ordered equilibria. In the first, bonds help reshuffle misallocated money and money and equity serve as a means of payment. In the second equilibrium, capital is illiquid. We have interpreted the first equilibrium as a boom and the second as a liquidity crisis. The multiplicity of equilibria is driven by a strategic complementarity reinforced by a pecuniary externality. Different types of monetary policy intervention can be effective for different purposes and at different times. Expansionary open market operations are socially beneficial once the economy settles on one of the equilibria, but are ineffective in moving the economy across equilibria. This policy is ineffective once the economy settles in a situation known as a liquidity trap. An unconventional policy resembling credit easing may instead be effective in moving the economy away from a liquidity crisis and trap.

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Notes

1 As in Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and, more recently, Brunnermeier and Sannikov (2014).
2 See also Bigio and Schneider (2017). The Kiyotaki and Moore (2019) paper has circulated in unpublished form for almost two decades. This is why the related literature often has an earlier publication date than the original paper itself.
3 Del Negro et al. (2017) suggest this possibility in a footnote.
4 These are the key imperfections that open up a role for money, see Kocherlakota (1998).
5 For the role of leverage in the 2008 crisis, the key reference is Geanakoplos (2010).
6 Allegedly, this occurred during the 2008 financial crisis, see Gorton and Metrick (2012).
7 Allen and Gale (1994) generate multiple Pareto-ordered equilibria through limited participation in asset markets. Our paper also generates multiple equilibria through strategic complementarities. See also the related paper by Cui and Radde (2016).
8 Several papers have exploited this feature in the monetary literature, including Berentsen et al. (2007), Ferraris and Watanabe (2008), Geromichalos and Herrenbrueck (2017), Ferraris and Mattesini (2020), and Araujo and Ferraris (2020). See Lagos et al. (2017) for a survey of liquidity in this class of models.
9 The literature on open market operations is vast. A recent paper that examines their role in a new monetarist framework is Rocheteau, et al. (2018).
10 According to Bernanke, this policy should be distinguished from quantitative easing, which focuses on the quantity of reserves. See Bernanke's 2009 public lecture at the LSE, quoted by Del Negro et al. (2017).
11 One such argument is given by Eggertsson and Woodford (2003). Hence, Bernanke's tongue-in-cheek remarks that "unconventional policy works in practice but not in theory." For this reason, in Williamson (2012) unconventional policy is never useful.
12 The literature on quantitative easing is also vast. Joyce et al. (2012) offer a survey of the literature on quantitative easing. Gertler and Karadi (2011) model quantitative easing in a DSGE environment; Williamson (2014) in a new monetarist framework.
13 As it will become clear, the presence of a pecuniary externality is key for our results. Any trading arrangement that preserves such a feature would do. Market arrangements with imperfect competition, however, would make it harder to disentangle the inefficiencies arising because of strategic complementarity from those arising because of market power.
14 This is a version of the pricing mechanism adopted in Gu et al. (2016). See Rocheteau and Wright (2005) for a discussion of bilateral trade with competitive price mechanisms.
15 This is the same legal restriction on government bonds that was adopted by Kocherlakota (2003).
16 This is reminiscent of Lester et al. (2012).
17 For our purposes, it is immaterial whether taxes are denominated in cash or goods. We omit public expenditures as they are not directly relevant for our purposes, that relate to monetary policy issues.
18 For the US data on Q, see Tobin's Q for non-financial corporate business in the FRED dataset.
19 According to Geanakoplos (2010), average leverage on mortgage securities was as high as 16–1 in 2006, for instance, but a meager 1.2–1 in the second quarter of 2009.
20 See the FRED dataset.
21 See Ferraris and Watanabe (2011) for this type of deterministic cycles and sunspot equilibria.
22 We checked this with yearly data for the period 1990–2020. For the equity returns, we used the inverse of the price/earning ratio computed dividing the earnings by the price per share, using the dataset at *http://people.stern.nyu.edu/adamodar* which also provides the Bond bill rate; for the money stock, we used M3 national currency, annualized, from FRED; for the outstanding government debt, we used the dataset at *https://fiscaldata.treasury.gov;* for the Tobin's Q, we used the corporate equities and liabilities to net worth ratio, for non-financial corporate business from FRED.
23 See Penn World table data for the gross physical capital formation at current PPPs.
24 Allegedly, this phenomenon has been observed in the US after the 2008 financial crisis. See Eggertsson (2017).
25 One may wonder whether some policy instrument, such as the bonds to money ratio, may be used to eliminate the illiquid equilibrium altogether, collapsing the region of the cost in which it exists. In the Appendix, we show for δ sufficiently close to 1, the existence region is independent of the policy parameters.
26 See Del Negro et al. (2017), figure 1.
27 See, for instance, Adrian et al. (2009).
28 Notice that this policy also eliminates inflationary equilibria for a reason similar to the one that applies to the lender of last resort lending at a zero real interest rate in Antinolfi et al. (2001).

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A. APPENDIX

In this Appendix, we derive explicitly the equilibrium conditions and prove the Propositions that appear in the text.

A.1 LIQUID EQUILIBRIUM

Consider the case in which equity is liquid. The multipliers are $\lambda^E \geq 0$ and $\theta^E \geq 0$ for (4) and (5), respectively. The optimality condition for κ is

$$f'\left(k+\kappa\right)+1-\delta-q-\theta^E q=0, \tag{A1}$$

and for m^d

$$v-p-\lambda^E p+\theta^E v=0. \tag{A2}$$

The multipliers are $\lambda^I \geq 0$ and $\theta^I \geq 0$ for (6) and (7), respectively. The optimality condition for κ is

$$q-1-\theta^I=0, \tag{A3}$$

and for m^s

$$p-v-\lambda^I v=0. \tag{A4}$$

Denoting with $V_n(\cdot)$ the partial derivative wrt $n=m,b,k$, the optimality conditions for the assets holdings are:

$$v=\beta V_m\left(m_{+1},b_{+1},k_{+1}\right), \tag{A5}$$

for money;

$$vi^{-1}=\beta V_b\left(m_{+1},b_{+1},k_{+1}\right), \tag{A6}$$

for bonds; and

$$1=\beta V_k\left(m_{+1},b_{+1},k_{+1}\right), \tag{A7}$$

for capital. The envelope conditions are

$$V_m\left(m,b,k\right)=v\left(1+\frac{\theta^E+\lambda^I}{2}\right), \tag{A8}$$

for money;

$$V_b\left(m,b,k\right)=v\left(1+\frac{\lambda^E}{2}\right), \tag{A9}$$

for bonds; and

$$V_k\left(m,b,k\right)=\frac{1}{2}\left[F'\left(k+\kappa\right)+\lambda^E\left(1-\delta\right)+\theta^I+1\right], \tag{A10}$$

for capital. Insert the multipliers λ^E, θ^E, λ^I, and θ^I obtained from (A1), (A2), (A3), and (A4) into (A8), (A9), and (A10), delay them one period and combine them with (A5), (A6), and (A7), obtaining the following optimality conditions: the Euler conditions for money holdings

$$1=\frac{\beta}{2}\frac{v_{+1}}{v}\left(\frac{F'\left(k_{+1}+\kappa_{+1}\right)}{q_{+1}}+\frac{p_{+1}}{v_{+1}}\right), \tag{A11}$$

for government bonds

$$1=\frac{\beta}{2}\frac{v_{+1}}{v}i_{+1}\frac{v_{+1}}{p_{+1}}\left(\frac{F'\left(k_{+1}+\kappa_{+1}\right)}{q_{+1}}+\frac{p_{+1}}{v_{+1}}\right), \tag{A12}$$

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for capital accumulation

$$1 = \frac{\beta}{2}\left[F'\left(k_{+1} + \kappa_{+1}\right) + q_{+1} + \left(\frac{F'\left(k_{+1} + \kappa_{+1}\right)}{q_{+1}} - 1\right)(1-\delta)\right], \tag{A13}$$

and the complementary slackness conditions for the constraint (4)

$$\left(F'\left(k+\kappa\right) - qi\right)\left(vb - pm^d\right) = 0, \tag{A14}$$

for the constraint (5)

$$\left[F'\left(k+\kappa\right) - q\right]\left[vm + vm^d + (1-\delta)\,k - q\kappa\right] = 0, \tag{A15}$$

for the constraint (6)

$$(p - v)\left(vm - vm^s\right) = 0, \tag{A16}$$

and for the constraint (7), which is

$$(q-1)\left(k - \kappa\right) = 0. \tag{A17}$$

The first result follows from arbitrage and equates the price of bonds to the relative price of money in the liquidity and primary markets.

Lemma 1. *At an optimum,* $i_{+1} = \frac{p_{+1}}{v_{+1}}$.

Proof. By arbitrage, $i_{+1} = \frac{p_{+1}}{v_{+1}}$ follows immediately from (A11) and (A12). □

By this Lemma, among the Euler conditions we only need to check (A11) and (A13). We look for equilibria in which $m^s = m$ even when $p = v$ and $\kappa = k$ even when $q = 1$. The next Lemma simplifies the equilibrium system, under this assumption.

Lemma 2. *Constraint (4) implies (5).*

Proof. Since $i \geq 1$, $F'(k+\kappa) \geq qi \geq q$, hence, by (A15), (5) is implied by (4). □

Therefore, among the complementary slackness conditions we only need to check (A14), which rewrites as

$$\left(F'\left(k+\kappa\right) - qi\right)(x - i) = 0. \tag{A18}$$

The market clearing conditions are $m^s = m^d$, $m = M$, $b = B$, and the market clearing condition for the durable good, which holds by Walras Law whenever the other markets are in equilibrium. At a stationary equilibrium, the return of money is determined by

$$\frac{v_{+1}}{v} = \frac{i+x}{(1-\tau)\,i\,(1+x)}, \tag{A19}$$

that is the inverse of (1). Using the binding constraints and the market clearing conditions, we obtain the equilibrium conditions (8), (9), and (10) in the text. Next, we prove Proposition 1.

Proof of Proposition 1. Let every investor adopt the technology. From (8), obtain

$$r = \frac{2\,(1+x)\,(1-\tau) - \beta\,(i+x)}{\beta\,(i+x)}\,i. \tag{A20}$$

Insert (A20) into (10), obtaining

$$\left[z\,(1+x) + \beta - \beta i\right](x - i) = 0. \tag{A21}$$

Since both terms in brackets are linear in i, a unique $\tilde{i}$ exists that satisfies (A21), whether the constraint is binding or not, with $\tilde{i} \geq 1$ since $x \geq 1$. Once $\tilde{i}$ is determined, (A20) gives r uniquely and,

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then, (9) gives q uniquely. With $\tau \geq (1-\beta)\frac{(2-\delta)x-\delta}{(2-\delta)(1+x)} \equiv \widetilde{\tau}$, $q \geq 1$. Substitute $i = \frac{z(1+x)+\beta}{\beta}$ into $x \geq i$, obtaining that the constraint is slack iff $x \geq \frac{1-\tau}{2\beta-(1-\tau)}$. The benefit of trade for an investor is $(q-1)k + (r-1)vM$, where $vM = \frac{(q-1+\delta)k}{2}$. Hence, there is an incentive to adopt the technology when everybody else does if

$$\frac{(r+1)\left(q-1\right)+(r-1)\,\delta}{2}k \geq c.$$

Insert the equilibrium values for q, k, r, taking the unconstrained case, obtaining the cutoff for the cost

$$\widetilde{c} \equiv \frac{1-\beta-z\,(1+x)\,(1-\delta)}{\beta}\widetilde{K}, \tag{A22}$$

where $\widetilde{K}$ is given by (21). To insure that $\widetilde{c} > 0$, z should be sufficiently small, that is τ sufficiently large. Analogously, for the constrained case. □

A.2 ILLIQUID EQUILIBRIUM

The equilibrium conditions are the same as in the previous case, except for the Euler equation of capital, which is (14).

Proof of Proposition 2. The proof is the same as the one for Proposition 1. To guarantee $q \geq 1$, assume $\tau \geq (1-\beta)\frac{x-1}{x+1} \equiv \widehat{\tau}$. In this case, nobody adopts the technology. The benefit for an investor is $qk + (r-1)vM$, where $vM = \frac{qk}{2}$. Hence, there is no incentive to adopt the technology when nobody else does if

$$\frac{(r+1)\left(q-1\right)+r-1}{2}k < c.$$

Insert the equilibrium values for q, k, r, taking the unconstrained case. Obtaining the cutoff for the cost

$$\widehat{c} \equiv \frac{1-\beta}{\beta}\widehat{K}, \tag{A23}$$

where $\widehat{K}$ is given by (22). Analogously, for the constrained case. □

A.3 WELFARE

Proof of Proposition 3. Compare (A22) and (A23) in the previous Propositions and notice that $\widehat{c} = \widetilde{c}$ for $z = 0$. Compute the derivatives of (A22) and (A23) wrt z, obtaining

$$\frac{d\widetilde{c}}{dz} = -\frac{(1+x)\,(1-\delta)}{\beta}\widetilde{K} + \widetilde{c}\frac{d\widetilde{K}}{dz}\frac{1}{\widetilde{K}}, \tag{A24}$$

$$\frac{d\widehat{c}}{dz} = \widehat{c}\frac{d\widehat{K}}{dz}\frac{1}{\widehat{K}}. \tag{A25}$$

Let $A\,(K) \equiv \frac{1+x}{2\beta+z(1+x)}\frac{1}{\alpha(K)}$. The derivatives of the capital stock are

$$\frac{d\widetilde{K}}{dz} = \frac{-\left\{2\beta-(1-\delta)\left[2\beta^2+4z\beta\,(1+x)+z^2\,(1+x)^2\right]\right\}A\left(\widetilde{K}\right)\widetilde{K}}{[1-\beta\,(1-\delta)]\,[2\beta+z\,(1+x)]+z\,(1+x)\,(1-\delta)\,[1-\beta-z\,(1+x)]}, \tag{A26}$$

$$\frac{d\widehat{K}}{dz} = \frac{-2\beta A\left(\widehat{K}\right)\widehat{K}}{[1-\beta\,(1-\delta)]\,[2\beta+z\,(1+x)]+z\,(1+x)\,(1-\delta)}. \tag{A27}$$

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Insert (A26) and (A27) into (A24) and (A25) and evaluate them at $z=0$. Notice that (A26) is less negative than (A27) at such a value. Since the curvature is a non-decreasing function of K, we can find an upper bound on the curvature, $\overline{\alpha}>0$, such that, for $\alpha<\overline{\alpha}$, (A24) is larger than (A25). By continuity, there exists a positive cutoff for z; hence, for τ, $\overline{\tau}$, such that, for $\tau>\overline{\tau}$, we still have $\widehat{c}<\widetilde{c}$. When $c\in(\widehat{c},\widetilde{c})$ and $\tau\geq\max\{\widetilde{\tau},\widehat{\tau},\overline{\tau}\}$, the existence of both equilibria follows from Propositions 1 and 2. The equilibrium with liquid equity dominates the illiquid one, since (21) is never smaller than (22) and strictly larger for $\tau<1-\beta$. □

Lemma 3. *There exists a lower bound $\underline{\delta}<1$, such that if $\delta>\underline{\delta}$, $\widehat{c}<\widetilde{c}$ for any feasible value of x and τ.*

Proof. From (21) and (22), we have $\widetilde{K}>\widehat{K}$ for all feasible values of δ, x, and τ. Therefore, from (A22) and (A23), when $\delta=1$, $\widehat{c}<\widetilde{c}$. By continuity, there exists a $\underline{\delta}<1$, such that if $\delta>\underline{\delta}$, we still have $\widehat{c}<\widetilde{c}$ for all feasible values of x and τ. □

A.4 ENDOGENOUS LIQUIDITY CYCLES

Proof of Proposition 4. Consider first the case in which the liquidity constraint is binding in both states, that is $i_s=x$ for both s. By Proposition 1 and 2, this requires $x<\frac{1-\tau}{2\beta-1+\tau}$. Since the equilibrium is constrained in each state, by assumption, the returns are $r_s\geq i_s=x\geq 1$ for each $s=1,2$. Take the limit with $\pi_{ss'}=0$ for each s with $s'=s$. Define $\rho\equiv\frac{\mu}{\beta}$. From (26) obtain

$$r_2=\frac{\rho^2-(r_1+x)\,x}{r_1+x}, \tag{A28}$$

plug it back into one of the equilibrium equations and obtain the condition

$$\Phi\left(r_1\right)\equiv\left(r_1+x\right)G_1\left(r_1\right)-\rho G_2\left(\frac{\rho^2-(r_1+x)\,x}{r_1+x}\right)=0. \tag{A29}$$

Define $\omega\equiv\frac{\rho^2-2x^2}{2x}$. The interest rate $r_1\in[x,\omega]$, where the upper bound serves to guarantee that $r_2\geq x$, from equation (A28). Since the liquidity constraint is binding, we have $x<\frac{1-\tau}{2\beta-1+\tau}$, which implies that $\omega>x$. The function $\Phi(r_1)$ is continuous in r_1 and the product of the function evaluated at the boundary of the interval for the feasible values of the interest rate is

$$\Phi\left(x\right)\Phi\left(\omega\right)=-\left[2xG_1\left(x\right)-\rho G_2\left(\omega\right)\right]\left[2xG_2\left(x\right)-\rho G_1\left(\omega\right)\right]. \tag{A30}$$

For $\delta\to 1$, $G_1(\cdot)$ and $G_2(\cdot)$ are the same function, $G(\cdot)$, and, thus, (A30) becomes $-[2xG(x)-\rho G(\omega)]^2<0$. By continuity, we can find values of δ close to but different from 1, so that (A30) remains negative. Hence, for such values of δ, by the intermediate value theorem $\Phi(r_1)=0$ admits a solution with $r_1\geq x\geq 1$. Once r_1 is determined by (A29), equation (A28) gives uniquely r_2, with $r_1\neq r_2\geq x\geq 1$. By continuity of the equilibrium system (26) in the switching probabilities, there is an equilibrium with $\pi_{ss'}\approx 0$ but $\pi_{ss'}\neq 0$ for each s with $s'=s$. Consider the non-binding case with $x>\frac{1-\tau}{2\beta-1+\tau}$. This is treated in the same way, since, as seen above, the real return and price of capital are insensitive to whether the equilibrium is constrained or not. However, the upper and lower bounds for the rates of return need to be swapped, as $\omega<x$ in this case. The rest of the argument remains the same. When $x=\frac{1-\tau}{2\beta-1+\tau}$, $x=\omega=\rho/2$ and $r_1=r_2$. This is inconsistent with a negative (A30). □

A.5 FEASIBLE ARRANGEMENTS

Here, we set up a general scheme to include all the feasible arrangements, with or without the legal restriction on the use of government bonds. Let $\iota_j\in\{0,1\}$ be indicator functions, with

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$j = m, b, k, d, e$, where $\iota_m, \iota_b, \iota_k, \iota_d, \iota_e$ denote whether money (ι_m) and government bonds (ι_b) are available for trade (1) or not (0), capital (ι_k) is liquid (1) or not (0) and bonds (ι_d) and equity (ι_e) are used as a payment instrument in the SM (1) or to exchange liquidity in the LM (0). Moreover, $\chi \in \{0, 1\}$ denotes whether the LM is active (1) or not (0). We consider only situations in which $\iota_m = \iota_b$. An entrepreneur chooses how much capital to buy in the secondary market, κ, and, possibly, how much money and bonds to acquire in the liquidity market, m^d and b^d, to maximize $V^E(\iota_m m, \iota_b b, k) =$

$$F\left(k + \kappa\right) - q\kappa + \chi\left[\iota_m\left(vm^d - pm^d\right) + \iota_b\iota_d\left(vb^d - pb^d\right)\right] + W\left(\iota_m m, \iota_b b\right),$$

subject to the constraints

$$\chi\left[\iota_k\left(1 - \iota_e\right)\left(1 - \delta\right)k + \iota_b\left(1 - \iota_d\right)vb - \iota_b\iota_d pb^d - pm^d\right] \geq 0, \tag{A31}$$

in the LM and the constraint

$$\iota_m\left(vm + \chi vm^d\right) + \iota_b\iota_d\left(vb + \chi vb^d\right) + \iota_k\iota_e\left(1 - \delta\right)k \geq q\kappa, \tag{A32}$$

in the SM. An investor sells capital in the secondary market, κ, and, possibly, money and bonds in the liquidity market, m^s and b^s, to maximize $V^I(\iota_m m, \iota_b b, k) =$

$$k - \kappa + q\kappa + \chi\left[\iota_m\left(pm^s - vm^s\right) + \iota_b\iota_d\left(pb^s - pb^s\right)\right] + W\left(\iota_m m, \iota_b b\right),$$

subject to the constraint for money and bonds

$$\chi\iota_m\left(vm - vm^s\right) \geq 0, \tag{A33}$$

$$\chi\iota_b\iota_d\left(vb^s - vb^s\right) \geq 0, \tag{A34}$$

in the LM and

$$k \geq \kappa, \tag{A35}$$

in the SM. The value function when the CM opens is

$$W\left(\iota_m m, \iota_b b\right) = \iota_m\left(vm - vm_{+1}\right) + \iota_b\left(vb - vi^{-1}b_{+1}\right) - t - k_{+1} + \beta V\left(\iota_m m_{+1}, \iota_b b_{+1}, k_{+1}\right),$$

where $t \equiv \iota_m\tau(1 + x)vM$, and the value function when the period begins is

$$V\left(\iota_m m, \iota_b b, k\right) = \frac{1}{2}\left[V^E\left(\iota_m m, \iota_b b, k\right) + V^I\left(\iota_m m, \iota_b b, k\right)\right].$$

The choice for the future concerns m_{+1}, b_{+1}, and capital, k_{+1}. The market clearing conditions are $m^s = m^d$ and $b^s = b^d$ for trade in the LM, which applies when it is active; $m = M$ and $b = B$ for the public instruments; and the market clearing condition for the durable good. A competitive equilibrium requires the agents to optimize taking prices as given and the prices to clear all the active markets. We consider stationary equilibria, in which real variables are time invariant. Define $\rho \equiv \frac{p}{v}$. The equilibrium condition for the accumulation of capital is the Euler equation for capital

$$1 = \frac{\beta}{2}\left\{q\left(r + 1\right) + \iota_k\left[\iota_e\left(r - 1\right) + \chi\left(1 - \iota_e\right)\left(\frac{r}{\rho} - 1\right)\right]\left(1 - \delta\right)\right\}, \tag{A36}$$

where the first term in (A36) reflects the fundamental value of capital, the term in square brackets reflects the premium for capital that arises for acting as either payment or liquid instrument. The equilibrium condition for the accumulation of money holdings, when $\iota_m = 1$, is the Euler equation for money

$$1 = \frac{\beta}{2}\frac{\left(1 + \iota_b x i^{-1}\right)\left[r + 1 + \chi\left(\rho - 1\right)\right]}{\left(1 + \iota_b x\right)\left(1 - \tau\right)}. \tag{A37}$$

When money is traded in the LM, it pays off an interest for the investors; otherwise, only the entrepreneurs benefit from its use in the SM, obtaining extra capital whose return is r. The return of money is given by the inverse of (1). By arbitrage, the interest rate of the bond is $i=\rho$, when the bonds are available and subject to the legal restriction, that is $\iota_b=1$ and $\iota_d=0$, while $i=1$ when available but used as payment instruments in the SM, that is $\iota_d=1$. Hence, a stationary equilibrium is a four-tuple (i, r, ρ, q) such that (A36) and (A37) hold for the different possibilities, with the appropriate complementary slackness conditions for the relevant constraints. To guarantee that the investors have the incentive to trade the instruments, it has to be that $i \geq 1$, $r \geq 1$, $\rho \geq 1$, and $q \geq 1$. Once the equilibrium interest rates and price of second hand capital are determined, the capital stock is given by $K=f'^{-1}(rq-1+\delta)$. The arrangement with liquid equity in the text has $\iota_m=\iota_b=\iota_k=\iota_e=\chi=1$; the arrangement with illiquid capital has $\iota_m=\iota_b=\chi=1$ and $\iota_k=0$. Next, we consider all the relevant alternative arrangements that are feasible given the imperfections of the environment.

A.5.1 SUBSET OF INSTRUMENTS

Equity Only This arrangement has $\iota_m=\iota_b=\chi=\iota_d=0$ and $\iota_k=\iota_e=1$. The entrepreneurs issue equity on the value of the undepreciated capital stock after production to pay for the acquisition of second hand capital in the SM, without using any other payment instrument. In this case, all agents skip trade in the LM and show up directly for trade in the SM. The Euler condition for the accumulation of capital is

$$1=\frac{\beta}{2}\left[q\left(F'\left(k+\kappa\right)+1\right)+\left(\frac{F'\left(k+\kappa\right)}{q}-1\right)(1-\delta)\right]. \tag{A38}$$

At equilibrium, the price of second hand capital cannot be smaller than 1, since the investors can store capital one-for-one. Should the price be larger than 1, the investors would want to sell it all, setting $\kappa=k$. By market clearing, this situation would be incompatible with (A32), since $1-\delta \leq 1$. Therefore, at equilibrium, it has to be that $q=1$ and $\kappa=\kappa=(1-\delta)k$. Thus, not all the capital in the hands of the investors is acquired by the entrepreneurs in the secondary market. Substituting these conditions into (A38), with $2k=K$, we obtain the equilibrium condition for this case, namely

$$1=\frac{\beta}{2}\left[F'\left(\frac{2-\delta}{2}K\right)(2-\delta)+\delta\right]. \tag{A39}$$

A stationary equilibrium with secured credit is a time invariant K such that (A39) holds. By the properties of the production function, it follows immediately that an equilibrium exists and is unique. Notice that not all the second hand capital is transferred to the entrepreneurs, and there is sometimes overaccumulation of capital relative to the efficient allocation arising from the excessive use of capital as trading instrument to compensate for the lack of other liquid instruments.

Money Only This arrangement has $\iota_m=\iota_b=1$ and $\iota_k=\iota_e=\iota_d=\chi=0$, that is the case with only money used as payment instrument and the bonds are not used in the LM. The return of capital is $\underline{r}=\frac{\beta+2z(1+x)}{\beta}$, and the price of second hand capital $\underline{q}=\frac{1}{z(1+x)+\beta}$. To guarantee that the price of second hand capital does not fall below 1, it has to be that $\tau \geq 0$. The capital stock is

$$\underline{K}=f'^{-1}\left(\frac{[1-\beta(1-\delta)][\beta+2z(1+x)]+z(1+x)\beta(1-\delta)}{\beta[\beta+z(1+x)]}\right). \tag{A40}$$

Money and Equity, No Bonds in the LM This arrangement has $\iota_m=\iota_b=\iota_k=\iota_e=1$ and $\chi=\iota_d=0$, that is the case with only money and equity as payment instruments, while bonds are not used in the LM. The return of capital is $\overline{r}=\frac{\beta+2z(1+x)}{\beta}$, and the price of second hand capital $\overline{q}=$

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$\frac{1-z(1+x)(1-\delta)}{z(1+x)+\beta}$. To guarantee that the price of second hand capital does not fall below 1, it has to be that $\tau \geq \frac{(1-\beta)(1-\delta+2x-x\delta)}{(2-\delta)(1+x)}$. The capital stock in this case is

$$\overline{K} = f'^{-1}\left(\frac{[1-\beta(1-\delta)][\beta+2z(1+x)]-2z^2(1+x)^2(1-\delta)}{\beta[\beta+z(1+x)]}\right). \tag{A41}$$

Proof of Proposition 5. a. Consider the liquid equilibrium of the arrangement with money, bonds, and equity and compare it with the arrangement with only equity. Using (A39), we have that welfare, (23), at an equilibrium of the latter arrangement is always lower than at K^* for any positive δ. Since the allocation is independent of τ, a cutoff value for τ can be found so that the liquid equilibrium with money, bonds, and liquid equity dominates the arrangement with only equity in terms of welfare when τ is above the cutoff. Compare it now with the arrangement with only money. Compare it now with the arrangement with money and equity and no bonds. Check that $\overline{K} \leq \widetilde{K}$ with a strict inequality for $\tau < 1-\beta$, provided $z \leq \widetilde{z}$, where $\widetilde{z}$ is the positive value that satisfies

$$\beta[1-\beta(1-\delta)]-3\beta(1-\delta)z(1+x)-(1-\delta)(1+x)^2z^2=0.$$

The upper bound on z implies a lower bound for τ, so that the equilibrium with money, bonds, and liquid equity dominates this last arrangement when τ is larger than the cutoff. Take $\underline{\tau}$ as the maximum between the two cutoffs for τ identified here and above. From (A40) and (A41), we have that $\overline{K} \geq \underline{K}$ with a strict inequality for $z > 0$. b. Consider the illiquid equilibrium of the arrangement with money, bonds, and equity and compare it with the arrangement with only money, obtaining, from (22) and (A40), $\widehat{K} \leq \underline{K}$ for all parameter values, with a strict inequality iff $\tau < 1-\beta$. □

A.5.2 No legal or technical restrictions

Equity in the LM This arrangement has $\iota_m = \iota_b = \iota_k = \iota_d = \chi = 1$ and $\iota_e = 0$, that is the case with only money as payment instrument, equity as a liquidity instrument in the LM, while bonds are not used in the LM. We consider only the case in which the liquidity constraint (A31) is not binding, which is the best possible scenario for this case. The return of capital is $\overline{\overline{r}} = \frac{z(1+x)+\beta}{\beta}$, and the price of second hand capital $\overline{\overline{q}} = \frac{2}{z(1+x)+2\beta}$. The price of second hand capital does not fall below 1, for $\tau \geq 0$. The capital stock in this case is

$$\overline{\overline{K}} = f'^{-1}\left(\frac{2[1-\beta(1-\delta)][\beta+z(1+x)]+z(1+x)\beta(1-\delta)}{\beta[2\beta+z(1+x)]}\right). \tag{A42}$$

Comparing (21) and (A42), we obtain $\overline{\overline{K}} \leq \widetilde{K}$ for any τ, with a strict inequality iff $\tau < 1-\beta$. Hence, this arrangement is dominated by the liquid equilibrium of the arrangement with money, bonds, and equity.

Bonds in the SM This arrangement has $\iota_m = \iota_b = \iota_k = \iota_e = \iota_d = 1$ and $\chi = 0$, that is the case in which bonds are not legally restricted from being used as payment instruments in the SM together with money and equity. The equilibrium of this case has $i = 1$, since the bonds are perfect substitutes of money. The return of capital is $r' = \frac{\beta+2z}{\beta}$, and the price of second hand capital $q' = \frac{1-z(1-\delta)}{z+\beta}$. The capital stock in this case is

$$K' = f'^{-1}\left(\frac{[1-\beta(1-\delta)](\beta+2z)-2z^2(1-\delta)}{\beta(\beta+z)}\right). \tag{A43}$$

Comparing (21) and (A43), we obtain a bound on z such that $K' \leq \widetilde{K}$, implying that the equilibrium with money, bonds, and liquid equity dominates this arrangement. On the other hand, comparing (22) and (A43), we obtain $\widehat{K} \leq K'$ for all parameter values. This shows that the use

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of bonds to reward idle cash with interest is not enough to generate a welfare improvement. The presence of a liquidity premium for capital is key. Finally, consider the arrangement in which money and bonds are used as payment instruments in the SM but equity is illiquid, that is with $\iota_m = \iota_b = \iota_d = 1$ and $\chi = \iota_k = \iota_e = 0$. The return of capital is $r' = \frac{\beta+2z}{\beta}$, and the price of second hand capital $q' = \frac{1}{z+\beta}$. The capital stock in this case is

$$K'' = f'^{-1}\left(\frac{[1-\beta(1-\delta)](\beta+2z)+z\beta(1-\delta)}{\beta(\beta+z)}\right). \tag{A44}$$

Comparing (22) and (A44), we obtain $\widehat{K} \geq K''$ iff $x \leq \frac{1}{1-\beta(1-\delta)}$. Hence, when capital is illiquid, the legal restriction on the bonds is beneficial if the stock of bonds relative to money is not too large.

A.6 OPTIMAL POLICY

Proof of Proposition 6. By (21) and (22), τ should be pushed as close as possible to $1-\beta$. Taxation is limited by $\tau(1+x)vM \leq (1-\delta)K/2$. a. Consider the equilibrium with liquid equity. In this case, the binding constraint in the SM gives $(1+x)vM = (q-1+\delta)K/4$, which can be inserted into the constraint on taxation, obtaining $\tau \leq 2(1-\delta)(q-1+\delta)^{-1}$. Substituting (12) into this condition, it follows that $\tau = 1-\beta$ can be achieved iff $\delta \leq (4\beta-1-\beta^2)/(3\beta-\beta^2) \equiv \widetilde{\delta}$. b. Next, consider the equilibrium with illiquid capital. In this case, the binding constraint in the SM gives $(1+x)vM = qK/4$, which can be inserted into the constraint on taxation, obtaining $\tau \leq 2(1-\delta)q^{-1}$. Substituting (16) into this condition, it follows that $\tau = 1-\beta$ can be achieved iff $\delta \leq (3\beta-1)/2\beta \equiv \widehat{\delta}$. □

Proof of Proposition 7. The comparative statics results follow by direct computation from (21), (22), (11), (15), (17), and (18). At the liquid equilibrium, leverage is given by (19), and the effect of x follows from direct computation. At the illiquid equilibrium, leverage is (20). □

Proof of Proposition 8. The results follow by direct computation from (27), (21), computed with the value (27) and (23). Leverage is $L = \frac{2-z(1+x)(1-\delta)\gamma}{2[1-\beta(1-\delta)-z(1+x)(1-\delta)\gamma]}$, the effect of a change in γ follows by direct computation. □

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