Correspondence

DEAR EDITOR,

In Note 81.26 (July 1997) there appears a version of an often-repeated incorrect statement about Pythagorean triples, namely that for integers $m > n > 0$ the formulae

$$x = m^2 - n^2, \qquad y = 2mn, \qquad z = m^2 + n^2 \tag{1}$$

give all the positive-integer solutions of $x^2 + y^2 = z^2$.

The correct result is, of course, that these formulae with *coprime* integers $m > n > 0$ *of opposite parity* give all the **primitive** Pythagorean triples (x, y, z), i.e. those positive-integer solutions x, y, z having no common divisor (apart from 1); and then all Pythagorean triples are given by

$$x = \left(m^2 - n^2\right)k, \qquad y = 2mnk, \qquad z = \left(m^2 + n^2\right)k \tag{2}$$

for any positive integer k.

If m and n have greatest common divisor d, say $m = m'd$ and $n = n'd$, then (1) becomes

$$x = \left(m'^2 - n'^2\right)d^2, \qquad y = 2m'n'd, \qquad z = \left(m'^2 + n'^2\right)d^2, \tag{3}$$

where $m' > n' > 0$ and m', n' are coprime; but (3) fails to yield *all* the Pythagorean triples because, even when m', n' have opposite parity, k in (2) need not be a perfect square.

If we employ (2) instead of (1), we find that the argument in Note 81.26 gives $b = \frac{1}{2}\left(m^2 + n^2\right)h$ and $ac = \frac{1}{4}mn\left(m^2 - n^2\right)h^2$, where $m > n > 0$ are coprime integers with opposite parity and h is now any *even* positive integer, say $h = 2k$. So all the desired monic quadratics (i.e. with $a = 1$) are

$$x^2 + \left(m^2 + n^2\right)kx + mn\left(m^2 - n^2\right)k^2 = 0.$$

They can be listed systematically in families (for given m, n and variable k), starting with $m = 2, n = 1$. For example, the (4, 1) family is

$$\left\{x^2 + 17kx + 60k^2 = 0 : k \in \mathbb{N}\right\}.$$

Yours sincerely,

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DEAR EDITOR,

I would like to comment on two recent notes.

1. Note 81.1 *A Pascal-like triangle for* $\alpha^n + \beta^n$.

Since α and β are roots of $ax^2 + bx + c = 0$ then

$$\alpha^{n+2} - l\alpha^{n+1} + m\alpha^n = 0,$$

where $\alpha + \beta = l$ and $\alpha\beta = m$ as defined in Note 81.1. An equivalent result holds for β.