Substituting $a'^{\frac{2}{2}}=a'-\frac{1}{4}$, (1) becomes

$$\pi(a'-\tfrac{1}{4})=pa' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2);$$

thus we get

$$\pi=\frac{pa'}{a'-\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Substituting $a'=1+a$, (3) becomes

$$\pi=p\,.\,\frac{1+a}{1+a-\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\pi=p\left(1+\frac{0{\cdot}25}{0{\cdot}75+a}\right) \quad . \quad . \quad . \quad . \quad (5).$$

This is Laundy's expression.

To find the annual premium on the payment of $\frac{1}{m}$ yearly premiums, we start from the approximate expression

$$a'^{\frac{m}{m}}=a'-\frac{m-1}{2m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Thus we have

$$\pi a'^{\frac{m}{m}}=pa',$$

$$\pi=p\,.\,\frac{a'}{a'-\frac{m-1}{2m}}$$

$$=\frac{1+a}{1+a-\frac{m-1}{2m}};$$

or, after a simple operation,

$$\pi=p\cdot\left(1+\frac{\frac{m-1}{2m}}{\frac{m+1}{2m}+a}\right) \quad . \quad . \quad . \quad . \quad . \quad (7).$$

This is Laundy's general expression.

I am, Sir,

Yours most obediently,

DR. AUGUST WIEGAND,

Director of Life Assurance Society, Iduna.

Halle, Germany, Prussia,
May 14, 1864.

ON THE SAME SUBJECT.

To the Editor of the Assurance Magazine.

SIR,—Having been allowed the privilege of perusing the preceding letter of Dr. August Wiegand, I can but express my gratification that a subject of such minor importance should have attracted the attention of your distinguished correspondent: and beg to thank him, through the medium of your *Journal*, for supplementing, as he has done, the demonstration I gave upon this subject in my letter which appeared in vol. xi., p. 232. The value of the Doctor's remarks will be duly appreciated by your readers.

It is quite true that I might have deduced the expression (5) in the Doctor's letter by the "nearer and more direct" method which he lays down; but had I done so I should have failed to show that which I intended and expressed—Mr. Orchard's quantity c_x in terms of the annual premium (π_x), viz.:—

$$\pi_x \cdot \frac{\cdot 25}{\cdot 75 + a_x} = c_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

that is, the increment to be made to the annual premium when paid half-yearly.

Having, then, had in view the two objects, first, of deducing, as above stated, an expression for c_x in terms of π_x, and secondly, another for the value of $\frac{1}{2}(\pi_x + c_x)$, I obtained at once the following obvious equation:—

$$\cdot \frac{1}{2}\left(\pi_x + \pi_x \cdot \frac{\cdot 25}{\cdot 75 + a_x}\right) = \frac{\pi_x}{2}\left(1 + \frac{\cdot 25}{\cdot 75 + a_x}\right) \quad . \quad . \quad (b)$$

Hence it will appear that my process furnishes, as I proposed, both the means of obtaining the increment (a) to be made to the annual premium when payable half-yearly, and by (b) of finding, by a method as simple in the one case as the other, the half-yearly premium itself by direct calculation. I submit, therefore, that the expression I gave could not, under the circumstances, well be derived in a much nearer or more direct way.

Allow me to add, that when I before addressed you upon this subject, I intended only to submit what appeared to me to be a convenient and simple *arithmetical* process for forming a table for passing from the yearly premium to the equivalent premium when paid half-yearly, quarterly, or otherwise; and I think that the table appended to my letter, computed by that process, was produced with, perhaps, about as small an amount of labour as possible.

Another method of arriving at the same results, which I now beg to submit, may not be deemed unworthy of being added to my previous communication.

By eliminating the value a_x and solving in terms of π_x, more symmetry will be imparted to the expression, and we further obtain a constant for each rate of interest as one of the terms of the denominator. I am indebted to Mr. Samuel Younger, one of your talented contributors, for the suggestion, and avail myself of an extract from a communication from that gentleman upon this subject, in which he thus treats the case:—

"The half-yearly premium being H_x, we have

$$H_x = \frac{1 - d(1 + a_x)}{2(a_x + \cdot 75)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

"Now, from $\pi_x = \dfrac{1}{1 + a_x} - d$, we find

$$1 + a_x = \frac{1}{\pi_x + d} \text{ and } \cdot 75 + a_x = \frac{1}{\pi_x + d} - \cdot 25.$$

"Substituting these values of $1+a_x$ and $\cdot 75+a_x$ in (1), we get

$$\text{H}_x=\frac{1-\dfrac{d}{\pi_x+d}}{2\left(\dfrac{1}{\pi_x+d}-\cdot 25\right)}=\frac{1}{2}\,\frac{\pi_x}{1-\cdot 25(\pi_x+d)}."$$

This latter equation may be again reduced to

$$\frac{\pi_x}{2-\frac{1}{2}d-\frac{1}{2}\pi_x},$$

which, when the rate of interest is 3 per cent., becomes

$$\text{H}_x=\frac{\pi_x}{1\cdot 9854-\frac{1}{2}\pi_x},$$

which is extremely simple.

The example worked in my last letter will, by this formula, be as follows:—

$$\pi_x=\cdot 0410 \;\therefore\; 1\cdot 9854-\cdot 0205=1\cdot 9649,$$
and $\cdot 0410\div 1\cdot 9649=\cdot 02087$, as before.

The general formula will be found readily from the foregoing to be

$$\frac{1}{m}\cdot\frac{\pi_x}{1-\dfrac{m-1}{2m}(\pi_x+d)}.$$

Apologizing for the length of this communication,

I am, Sir,

Your obedient servant,

SAMUEL L. LAUNDY.

Eagle Insurance Company,
6th September, 1864.

ON A PARTICULAR ARRANGEMENT OF ELEMENTARY VALUES.

To the Editor of the Assurance Magazine.

SIR,—1. The values of annuities and assurances of all kinds consist of certain elements variously combined. These elements are not usually exhibited in detail, their combinations being otherwise attainable.

2. But an intimate knowledge of details will enable its possessor to surmount such difficulties as occur in the treatment of complex questions involving many lives. Something may also be gained by particular arrangements of elementary values. For these reasons the following brief exposition is offered.

3. Our elementary table contains the logarithms, for the Carlisle life table, from age 90 upwards, of the following quantities:—

l_x =number that complete age x;
d_x=number that die in the $(x+1)$th year of age;
v =an unit of money, discounted for one year (3 per cent.);

and combinations of these, as shown in the headings.

4. The present value of £1, to be received in event of a life now aged x completing the $(x+1)$th year of age, is $\frac{vl_{x+1}}{l_x}=vp_x$. This is an endowment.

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