COMPLEX THEORY OF GRAVITATIONAL LENSES PART II

Cluster-Lensing: Statistics of the Arclet Distribution

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Applying the complex formalism of Part I we investigate the transformation of the ellipticity and orientation distribution of elliptical background sources by local lensing. One important result is that we can apply the techniques from Part I directly in most parts of the cluster field from which we can also reconstruct the distribution of the background sources, without further information. Knowing the distribution we can apply the techniques from Part I everywhere.

Observed arclets are in general not images of circular sources. Additionally, the sources are distributed over a redshift range. Although the redshift distribution can be accounted for in the Beltrami equation we here assume for simplicity that all sources are at the same redshift.

However, we can also describe the situation including intrinsic ellipticities using the Beltrami equation introduced in Part I if a composed mapping $h(z) = \omega(\mathrm{w}(z))$ is introduced. h is a composition of two mappings w and ω. The mapping w maps the observed elliptical arclets to the unknown *real* elliptical source and ω maps this source to a circle. The corresponding Beltrami parameters are μ_h, μ_{w} and μ_ω, respectively.

For simplicity we use now indices in parentheses to denote derivatives. For the Beltrami parameter of the composed mapping we find after some calculation

$$\mu_h = \frac{\mathrm{w}_{(\bar{z})} + \mu_\omega \overline{\mathrm{w}}_{(\bar{z})}}{\mathrm{w}_{(z)} + \mu_\omega \overline{\mathrm{w}}_{(z)}} \quad . \tag{1}$$

If we suppose now that we are in the linear regime of the mapping, we can always write

$$\omega = \mathrm{w} + \mu_\omega \overline{\mathrm{w}}, \quad 0 \leq |\mu_\omega| \leq 1 \quad \textit{and} \quad \mathrm{w} = (1-\kappa)z + \gamma\bar{z} \quad . \tag{2}$$

Note that we do not have to restrict the lens equation to the case of real γ. However, the only difference would be a constant phase in the appropriate Beltrami parameter, which could also be attached afterwards.

M. Kafatos and Y. Kondo (eds.), Examining the Big Bang and Diffuse Background Radiations, 535–536.

Applying Eq.(1) we find

$$\mu_{\mathrm{w}} = \frac{\gamma}{1-\kappa}, \quad \text{real !} \tag{3}$$

$$\mu_h = \frac{\gamma + \mu_\omega(1-\kappa)}{1-\kappa+\gamma\mu_\omega} = \frac{\mu_{\mathrm{w}}+\mu_\omega}{1+\mu_{\mathrm{w}}\mu_\omega} \quad \text{and the inverse} \tag{4}$$

$$\mu_\omega = \frac{\mu_{\mathrm{w}}-\mu_h}{\mu_{\mathrm{w}}\mu_h - 1} \quad . \tag{5}$$

These equations describe the transformation of the Beltrami parameter under local lensing, given by μ_{w}. We are now interested to see how the mean of a local distribution Φ of μ_ω transforms under local lensing. This means how the statistical properties of a field of elliptical sources alter if mapped by a lens locally described by constant density and shear. For isotropical distributions we find

$$\langle\mu_h\rangle = \frac{1}{2\pi}\int_0^{\mu_\omega^{\max}}\left[\int_0^{2\pi}\frac{\mu_{\mathrm{w}}+\mu_\omega}{\mu_{\mathrm{w}}\mu_\omega+1}\mathrm{d}\varphi_\omega\right]\Phi(|\mu_\omega|)|\mu_\omega|\,\mathrm{d}|\mu_\omega|. \tag{6}$$

We therefore find for $\langle\mu_h\rangle$ using residue calculus for the integral in square brackets

$$\langle\mu_h\rangle = \mu_{\mathrm{w}}\int_0^{\frac{1}{\mu_{\mathrm{w}}}}|\mu_\omega|\Phi(|\mu_\omega|)\,\mathrm{d}|\mu_\omega| + \tag{7}$$

$$\frac{1}{\mu_{\mathrm{w}}}\int_{\frac{1}{\mu_{\mathrm{w}}}}^{\infty}|\mu_\omega|\Phi(|\mu_\omega|)\,\mathrm{d}|\mu_\omega| \quad , \tag{8}$$

$$= \left(\mu_{\mathrm{w}} - \frac{1}{\mu_{\mathrm{w}}}\right)\int_0^{\frac{1}{\mu_{\mathrm{w}}}}|\mu_\omega|\Phi(|\mu_\omega|)\,\mathrm{d}|\mu_\omega| + \frac{1}{\mu_{\mathrm{w}}} \quad , \tag{9}$$

$$= \mu_{\mathrm{w}} \quad \text{if } \Phi(|\mu_\omega|) = 0 \text{ for } |\mu_\omega| > \tfrac{1}{\mu_{\mathrm{w}}} \quad . \tag{10}$$

Since $|\mu_\omega| \leq 1$ the last condition is fulfilled for $|\mu_{\mathrm{w}}| \leq 1$, which is valid for a region where the lens mapping is orientation preserving (quasi conformal). This important results states that outside the critical curves (subcritical regions) the mean of the measured Beltrami parameter is only determined by the local lens. **The bad news is that if we know that $|\mu_{\mathrm{w}}| > 1$ from other (global) information, the mean axail ratio of the observed arclets is influenced by the distribution of the intrinsic axial ratios. The good news is that we can conclude the intrinsic distribution from regions where the mapping is surely quasi conformal.** For details see Schramm & Kayser 1994c in the reference list of Part I.