

THE
JOURNAL
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SMITH COLLEGE

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UNIVERSITY OF MICHIGAN

UNIVERSITY OF PENNSYLVANIA

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ERRATA

VOLUME 1

Page 116, line 9 from the bottom. For $CpCpq$, read $CqCpq$.

VOLUME 14

Page 79, line 31. For « turns of », read « turns on ».

VOLUME 15

Page 85, line 35. Replace the second occurrence of C_α by $A_{\alpha\alpha}$.

Page 90, line 30. After the word « theorem », insert the numeral « 3 ».

Page 159, line 2. For « Dinnik », read « Dynnik »; and for « Ánovskij », read « Ánovskaá ».

Page 200, line 6. For t , read i .

Page 204, line 2. For « XIV 159 », read « paper in this JOURNAL, vol. 14, pp. 159–166 ».

Page 204, line 15. For « II 131 », read « in this JOURNAL, vol. 2, p. 131 ».

Page 274, line 3 from the bottom (of text). In the formula at the beginning of the line, replace the first Cxy by Bxy .

Page 276, line 22. For the first S , read \bar{S} .

Page 302, line 2 from the bottom. The cross-reference to « Self-contradictory propositions (or sentences) significance of » may be deleted, since the entry in question does not appear in this index.

Page 309, line 30. For « Modes », read « modes ».

Page 312, line 19 from the bottom. For « Comfirmation », read « Confirmation ».

VOLUME 16

Page 11, line 10. Correct the latter part of the line, after the word « formulae », to read: « logically equivalent in L_m if they are logically ».

Page 18, line 28. For « consistent », read « constituent ».

Page 22, line 2. Replace the second occurrence of $>$ by \geq . Also insert a space after the first s .

Page 31, line 15. For « All », read « All ».

Page 33, line 7 from the bottom. Replace $=$ by \equiv .

Page 36, line 14. For $m < n \leq k$, read $n < m$.

Page 46, line 2. For the semicolon, read $($.

Page 53, line 6 from the bottom. For « 1949 », read « 1948 ».

Page 58, line 21. For the first « ” », read « “ ».

Page 62, line 3. After « vol. 74 », insert « no. 1 ».

Page 65, line 16. At the beginning of the line, for « ng », read « ing ».

Page 70, line 18. After « assez », insert « ample ».

Page 79, line 6 from the bottom. For « Bryne », read « Byrne ».

Page 133, line 1. For « 14.1–1.11 », read « 14.1–11 ».

Page 133, line 4. S' should be lightface (rather than bold).

Page 138, line 17. For « unipue », read « unique ».

Page 142, line 1. For I , read 1 .

Page 160, line 5 from the bottom. For « 27 », read « 28 ».

Page 222, line 14 from the bottom. At the beginning of the line, delete the inverted comma after BS.

Page 222, line 2 from the bottom. For « Pradikatenkalküls », read « Prädikatenkalküls ».

Page 238, line 21 from the bottom. For « deductive », read « deduction ».