If it is assumed that the distribution of error follows a normal distribution in practice then 95 per cent of readings will be within approximately two standard deviations of the true value. In this example therefore 95 per cent of errors are less than 12′. Taking $\delta\theta$ to be 12′ it is thus possible to construct error contours for specified values of the displacement (e). These are known as the 95 per cent error contours. In practical work they are often the most useful, since for points along the contour only one out of twenty observations may be expected to give arcs of position lines which are displaced from the true position line by more than the nominal displacement error (e) of the contour.

It is worth noting that the value of 12′ for $\delta\theta$ applies to the experimental conditions which are considered to have been rather better than the general case at sea. The observers had a steady platform and unlimited time for their observations. The stations observed were the well defined vertical side of a rectangular tower and a flag pole. Some errors were due to the sextant being held in other than a horizontal plane.

In Fig. 4, taking $\delta\theta$ to be 12′, representative contours are drawn for an area within which the angle (θ) subtended by the two stations (S and T) is greater than 10°. The values of position line displacement (e) for which the contours are drawn are given as decimals of the baseline length so that the diagram may be used for any required length of baseline. Thus, for a baseline of 1000 metres, the 0·005, 0·010 and 0·015, 0·020 and 0·050 contours would represent the 5 metre, 10 metre, 15 metre, 20 metre and 50 metre contours respectively.

The contours discussed in this paper are useful for defining the accuracy coverage of horizontal angle position lines as affected by random observational errors. The effect of systematic errors in observed angles should be dealt with by other methods. In particular, the Admiralty Manual of Hydrographic Surveying gives a very full treatment of errors due to the stations observed being on different levels.

REFERENCE

1 Cotter, C. H. (1972). A brief history of the method of fixing by horizontal angles. This *Journal*, **25**, 528

Martelli's Tables

Charles H. Cotter

CAPTAIN S. T. L. Lecky, in his famous *Wrinkles*,[1] wrote with derision in his reference to so-called 'short-methods'. He warned his readers to '*beware*' of these:

> They generally only *look* short [wrote Lecky] because good care is taken to apply the various corrections *beforehand*, and the unsuspecting reader is deceived of this device.

As a case in point Lecky considered the 'small but expensive pamphlet by Mr. Martelli', to support his derogatory remarks:

> When his so-called 'short-method' is overhauled and compared with Raper, we get the following startling result:—Martelli, 56 figures and five logarithms,

https://doi.org/10.1017/S0373463300021627 Published online by Cambridge University Press

against Raper's 59 figures and five logarithms, required to produce the same result. So that by the first method we have the enormous (!!!) gain of *three* figures. Furthermore Mr. Martelli's pamphlet contains several glaring errors which makes one rather dubious about the general correctness of the tables, although (for all the writer (Lecky) knows to the contrary) the mathematical principle of his method may be correct enough.

It appears that the name and nationality of the author of *Martelli's Tables*[2] are not, with any certainty, known. According to Hopkins,[3] the publishers of the tables (in 1937) had no record of him. Hopkins put forward the view that a French editor named G. Pouvreau was the original author who used 'Martelli' as a *nom-de-plume*. G. Pouvreau is listed in H. Bencker's Regimen of the Sea, *Hydrographic Review*, 1943, as the author of *Nouvelles Tables de mer pour le calcùl de la hauteur, de l'heure et de l'azimut*, Paris, 1885. Lecky, writing in 1894, stated that the pamphlet to which his caustic remarks particularly applied, first appeared 'at least thirty years ago'. This would date its origin at *c.* 1864. According to Lecky a revised edition was brought out in 1887, and Lecky's pronouncement on this was: '... it is not much better than the original production....' This remark appears to have stemmed from an unfavourable review which appeared in the *Nautical Magazine* of 1891.

Professor Pes, of the Royal Naval College at Genoa, and a leading authority on nautical matters, was less severe than Lecky and others in their treatment of Martelli's Tables. Writing in *Rivista Marittima* in June 1906 he disapproved of the mystery surrounding the construction of the tables but he did not approve of the harshness of censure and the emphatic condemnation which the tables met 'in some quarters'. Pes informs us that the mystery of the construction of the tables had been solved by his compatriots Professor Cevasco and others, and the solution given in *Rivista Marittima* in January 1904.

A small pamphlet of 49 pages of log tables, under the authorship of G. F. Martelli, was published in New Orleans in 1873—*vide American Practical Navigator* (H.O. Pub. No. 9) Washington, 1966, p. 524. Several editions of the same pamphlet were published in Great Britain: that dated 1919[4] under the joint imprint of D. McGregor and Co., of Glasgow; Imray, Laurie, Norie and Wilson, of London; J. D. Potter of London; and Simpkin, Marshall and Co., of London. It appears that McGregor and Co. acquired the copyright of the tables from Messrs. Pike of Liverpool some time before 1912. The earlier editions were devised to provide a:

> Short, Easy and Improved Method of finding the Apparent Time at Ship (Rapid Calculation of apparent time for finding the longitude).

Later editions (1934, 1936, 1940, 1944 and 1948) gave examples for finding altitude for use with the Marcq Saint Hilaire method. The 1944 and 1948 editions made provision (the earlier editions did not) for finding azimuth. Martelli's method became exceedingly popular on board American, British, German and Danish ships.

Captain F. H. Trap of the Royal Danish Navy, writing in *Tideskrift for Sovaesen*, stated that:

> The tables suffer from one shortcoming in that they do not indicate which formula is used.

And, indeed, the Tables were known generally as 'Martelli's Mysteries'. The well-known nautical teacher Captain Charles H. Brown,[5] of Glasgow, wrote:

> It seems like the man has tried to hide his formula as if he had committed some unpardonable sin.

The remarks of Edward J. Willis[6] are also interesting:

> It is a curiosity, for not since mediaeval ages, when men of science had to conceal their ideas to avoid religious persecutions, has one taken so much trouble to mask a method.

The tables are five in number, and the relatively short and quick method for finding hour angle requires six book-openings, six table-entries, and four simple arithmetical steps. The rules for using the method are simple and interpolation is negligible.

According to Cevasco, whose description of Martelli's principle seems to have been adopted by Willis,[6] the method employs the formula:

$$\operatorname{cosec}^2 \frac{P}{2} = \frac{2 \cos l \cos d}{\cos (l \pm d) - \sin a}$$

and that:

Table I gives ($\log \cos \theta + 0{\cdot}5$) with the decimal point removed.

Table II gives ($\cos \theta + 0{\cdot}200$) with the decimal point moved three places and expressed in minutes and seconds of time.

Table III gives the complement of natural sine θ (that is $\cos \theta$) with the decimal point moved three places, and expressed in minutes and seconds of time.

Table IV gives the co-log of $\frac{1}{2}(\cos (l \pm d) - \sin a)$ and should start with the co-log of $\frac{1}{2}$, that is log 2 which is 0·3010. The data really start at $20^m\ 02^s$ with 3·0334 and end at $36^m\ 40^s$ with 0·0334. The respondents in Table IV are, therefore, 0·0334 too high. This and the two 0·5s from the double use of Table I are corrected in Table V.

Table V gives values of ($\log \operatorname{cosec}^2 P/2 + 1{\cdot}0334$).

According to Trap (*vide supra*) the five tables provide collectively the solution to the formula:

$$2 \sin^2 \frac{P}{2} = \frac{\cos (l \pm d) - \sin a}{\cos l \cos d}$$

in which P, l, d and a are hour angle, latitude, declination and altitude, respectively. Trap's explanation is as follows:

Inverting the original formula we have:

$$\frac{1}{2 \sin^2 P/2} = \frac{\cos l \cos d}{\cos (l \pm d) - \sin a}$$

Now

$$2 \sin^2 P/2 = \operatorname{vers} P$$

TABLE I
LOG OF LAT AND DEC

′		4°		40°	
32	....	....	....	3808	
46	....	4985			

TABLE II
SUM OR DIFFERENCE

′		45° m. s.	
18	....	15 3·7	

TABLE III
ANGLE OF ALTITUDE

′		36° m. s.	
17	....	6 48·1	

TABLE IV
AUXILIARY LOGARITHM

21 minutes

secs		·8s	
51	....	1·2866	

TABLE V

Hour Angle—West—21 Hours

m	00s	5s	10s		55s
53	2·1594	....	....	....	2·1655
54	2·1661				

https://doi.org/10.1017/S0373463300021627 Published online by Cambridge University Press

so that:

$$\frac{1}{\text{vers } P} = \frac{\cos l \cos d}{\cos (l \underset{\sim}{+} d) - \sin a}$$

	log cos l and log cos d	answer to Table I
	log cos $(l \underset{\sim}{+} d)$	answers to Table II
	log sin a	,, ,, ,, III
	log $1/(\cos (l \underset{\sim}{+} d) - \sin a)$	,, ,, ,, IV
and	log $1/\text{vers } P$	,, ,, ,, V

When one compares the five tables with the respective functions it at once becomes apparent that the author, in order to avoid negative logs and to facilitate the use of his method, introduces certain constants, as follows: Table I gives values of log cos $\theta - 9{\cdot}5$. Thus under argument latitude (or declination), say 48° 15′, we find 3234, this being log cos 48° 15′ (9·8234) − 9·5.

Table II gives values of $20^{\text{m}}\ 00^{\text{s}} - (1 - \cos (l \underset{\sim}{+} d))1000/60$. Thus under argument 'Sum or Difference' (that is the sum of latitude and declination when of different names, and their difference when of the same name), say 60° 00′, we find 11m 40s, which is $20^{\text{m}}\ 00^{\text{s}} - (1 - \cos 60°)1000/60$.

Table III gives values of $(1 - \sin a)1000/60$. Thus, under argument 'Angle of Altitude', say 28° 10′ we find $8^{\text{m}}\ 48{\cdot}0^{\text{s}}$, which is $(1 - \sin 28°\ 10')1000/60$.

Table IV gives values of (log $(\cos (l \underset{\sim}{+} d) - \sin a) - 10 - 0{\cdot}0334$). These are called 'Auxiliary Logarithms'.

Table V gives values of $(10 - \log \text{vers } P - 1{\cdot}0334)$, under the heading of the Table'Log of Hour Angle'.

According to the anonymous author of the 1954 edition of *The Admiralty Manual of Navigation*,[7] the Martelli method is based on a re-arranged cosine-haversine formula thus:

$10{\cdot}8/\text{hav } P = (1{\cdot}08\sqrt{}(\cos l \cos d))/(\text{have } z - \text{hav } (l \underset{\sim}{+} d)$ and that:

Table I gives values of $10^4\ (0{\cdot}5 + \log \cos x)$ where x is latitude or declination.

Table II gives values of $10^4\ (1{\cdot}2 - 2 \text{ hav } (l \underset{\sim}{+}))$ in units of seconds of time. Thus, for $(l \underset{\sim}{+} d) = 0°$ the tabulated value is 1200 secs or $20^{\text{m}}\ 00{\cdot}0^{\text{s}}$.

Table III gives values of $2(10^4)$ hav z in minutes and seconds of time. The table is entered with altitude, which is $(90 - z)$.

Table IV, which is entered with the combination of quantities extracted from Tables II and III, gives values of log $(1{\cdot}08/(\text{hav } z - \text{hav } (l \underset{\sim}{+} d))$.

Table V, which is entered with the combination of the quantities obtained from Tables I and IV, gives P. The tabulated quantities here, then, are logs of $10{\cdot}8/\text{hav } P$.

It is not without interest to compare the above three explanations of the construction of Martelli's Tables.

To illustrate the relationship between the formula used by Trap in his investigation of the Martelli mystery, and the tabular arrangement, let us consider the following example:

Example: Find the hour angle (P), given lat = 40° 32′ N, dec. = 4° 46′ S, altitude = 36° 17′.

$$2\sin^2\frac{P}{2}=\frac{\cos(l\pm d)-\sin a}{\cos l\cos d}$$

lat	= 30° 32′ N	log cos	= 9·8808
dec	= 4° 46′ S.	log cos	= 9·9985
$(l\pm d)$	= 45° 18′ nat cos 0·7036	log denom	= 9·8793
alt	= 36° 17′ nat sin 0·5918		
	2)0·1118		
numer	= 0·0559	log numer	= 8·7468
P	= 2h 06m 01s	log $\sin^2 P/2$	= 8·8675

By Martelli's Method (Refer to facsimiles of sections of Martelli's five tables illustrated in Fig. 1):

lat	40° 32′ N.	(Tab I) =	3808		
dec	4° 46′ S.	(Tab I) =	4985		m s
$(l+d)$	45° 18′	sum =	8793	(Tab II) =	15 03·7
alt	36° 17′			(Tab III) =	6 48·1
			1·2866	(Tab IV) =	21 51·8
P = 2h 06m 02s = (Tab V)			2·1659		

When using Martelli's tables for the Intercept Method, the sum or difference of the logs extracted from Table I is subtracted from the respondent in Table V corresponding to the given hour angle. Table IV is then entered with this quantity as argument. From the respondent from Table IV is subtracted the respondent from Table II (using $(l\pm d)$ as argument). This result is used in Table III as argument to find the required altitude.

The later editions of Martelli's Tables provide for azimuths through the medium of Table I, the respondents in which are, essentially, log cosines. The basis of the method is the spherical sine formula which, for solving Z in the PZX-triangle given declination, hour angle and altitude, is:

$$\sin Z=\frac{\sin PX\sin P}{\sin ZX}$$

Expressing this relationship in terms of cosines, we have:

$$\cos(90-Z)=\frac{\cos(90-P)\cos\text{dec}}{\cos\text{alt}}$$

The defect of this formula is that the computed azimuth is either Z or $(180-Z)$. The ambiguity is of particular importance in the case of an observed body lying near to the prime vertical circle of an observer.

Martelli's Tables are small in compass. They are logically arranged for time-sights, although a little inconvenient for solving altitudes as in the Marcq Saint Hilaire method. The arithmetic is simple and interpolation is trivial, but the use of four-figure logarithms may lead to serious error when the tables are employed for reducing sights of bodies having small azimuths. This follows because error in longitude due to error in latitude varies as the cotangent of the azimuth and that error in longitude due to error in altitude varies as the cosecant of the azimuth.

Remarks made by Goodwin[8] in respect of Martelli's Tables are interesting:

> To this little work attaches the somewhat unique experience that, more or less violently assailed by the critics, it has contrived to survive the ordeal, has passed through edition after edition, and seems to have firmly established itself in the favour of practical seamen of various nationalities. . . .

Goodwin's remarks were written in 1914, but they could well have been written at least forty years later, at which time Martelli's Tables were still extensively used; and, no doubt, there are still navigators afloat who have used them to this very day.

REFERENCES

1 Lecky, S. T. L. (1894). *Wrinkles in Practical Navigation*, 9th edn. London.
2 Martelli, G. F. (1873). *Tables of Logarithms*. New Orleans.
3 Hopkins, H. V. (1937). Martelli's Tables, *Nautical Mag*, Vol. 138. Glasgow.
4 Anon. (1919). *Short, Easy and Improved Method of finding the Apparent Time at Ship*. Glasgow, London.
5 Brown, C. H., Written communication to Hopkins (*op. cit.*).
6 Willis, E. J. (1925). *The Methods of Modern Navigation*. New York.
7 Anon. (1955). *Admiralty Manual of Navigation*. Vol. III, London.
8 Goodwin, H. B. (1914). Methods of mystery in nautical astronomy. *Nautical Mag.*, Vol. 91.

'A Brief History of the Method of Fixing by Horizontal Angles'

from J. Dickson

READERS of Captain Cotter's very interesting article 'A Brief History of the Method of Fixing by Horizontal Angles', may be interested to know that the possible use of the resection or three-point problem was appreciated by English mathematical practitioners of the seventeenth century.

Explanations have been given by John Collins, F.R.S. (1625–83), and Edmond Halley, F.R.S., Master and Commander, later Captain, Royal Navy and Astronomer Royal. It seems probable that Halley actually made some use of the principle when he charted the English Channel.

Dr. Angus Armitage and Dr. A. H. W. Robinson have explained Halley's method.

https://doi.org/10.1017/S0373463300021627 Published online by Cambridge University Press