

both features within the text, possibly feeling that it is for the reader to decide for himself how he can best make use of the information.

The four main sections are essentially independent of each other, and each represents an excellent treatise on the topic in question. Perhaps the relationship between the largely alternative methods based on the operational calculus and the distribution theory approaches could have been discussed in more detail. Nearly all the proofs are given in their entirety, and are cast in an elementary, and sometimes rather lengthy form. However this does make less demands on the previous mathematical attainment of the reader, which is that to be expected of a fairly recent graduate in a scientific or engineering subject.

The book is a translation, with revisions, of the German edition of 1966. The standard of translation is very high, with only very infrequent lapses from an elegant style of English.

C. D. GREEN

COHEN, J. W., *The Single Server Queue* (North-Holland Publishing Company, Amsterdam, 1969), xiv + 653 pp.

This treatise deals thoroughly with an important area of applications of probability theory. The author, himself an active contributor to research in queueing over a number of years, has successfully drawn together a wide variety of complex problems in the field. In so doing, he has rendered us all a service: his book will provide a welcome guide in the proliferating literature of the subject.

The work consists of three parts. Part I is a 155-page review of topics in stochastic processes relevant to queueing theory. These include Markov chains in discrete and continuous time, birth and death processes, derived Markov chains (a topic originating with the author), renewal theory and regenerative processes, including elements of fluctuation theory.

In Part II, 205 pages are devoted to a detailed study of the single server queue. First the queueing model is developed, and Kendall's notation  $-/-/-$  for the (Inter-arrival time distribution)/(Service time distribution)/(Number of servers), with independent inter-arrival and service times, is explained. The cases  $M/M/1$ ,  $G/M/1$ ,  $M/G/1$ ,  $G/G/1$ , where  $M$  stands for a negative exponential and  $G$  for a general distribution, are considered in some detail. In every case, random variables such as the busy period, waiting time and queue length (among others) are discussed. Direct probabilistic and analytic methods are used for the first three queues; in the study of the  $G/G/1$  queue, Pollaczek's integral equation is introduced and this is then used to investigate the  $G/K_n/1$  and  $K_m/G/1$  systems. Here,  $K_n$  denotes a distribution whose Laplace-Stieltjes transform is a rational function with denominator of degree  $n$ . Finally some special methods for the treatment of queueing problems are outlined: these include Lindley's integral equation method, the phase method for Erlang distributions of inter-arrival and service times, and Takács's combinatorial method.

Part III, the longest section consisting of 255 pages, deals with variants of the single server queue. A chapter is devoted to queues where the customers may arrive in groups and be served in batches, or where priority disciplines are in force. The queue for which actual waiting times are limited to a fixed maximum is considered. This is followed by a study of the queue with bounded virtual waiting time, or in storage terminology, the finite dam. The  $M/G/1$  system with finite waiting room is then discussed; a final chapter on limit theorems for single server queues, including several for heavy traffic theory, concludes the book.

There follow a 7-page Appendix of useful mathematical theorems, some helpful explanatory notes on the literature, an impressive 9-page list of references on queueing, and finally author, notation and subject indices. Few misprints were noted.

The book is strongly recommended to all research workers interested in applications of probability theory; for specialists in queueing theory, it will become essential reading.

J. GANI

EIDEL'MAN, S. D., *Parabolic Systems* (North-Holland, Wolters-Nordhoff, 1969), v + 469 pp., £7.60.

Since the appearance in 1938 of a fundamental paper by Petrovskii parabolic equations and systems have been subjected to a great deal of scrutiny—not least by the Russian school. This book, an English translation by Scripta Technica of the Russian original first published in 1964, is devoted exclusively to the study of parabolic systems (mostly linear or near linear) and embodies the sizeable contribution of the author to that study over the two decades prior to its publication.

Using classical analytic methods throughout, the author first directs himself to the construction and analysis of the fundamental matrices of solutions of parabolic systems then applies them to the study of the Cauchy problem and the initial-boundary-value problem. Fundamental matrices of solutions thus constitute a unifying theme. A very adequate résumé of the contents can be found in Friedman's review (MR 26 #4998) of the Russian original. Specific omissions are the theories of quasi-linear second order parabolic equations, an active field of research at the time of writing, and of systems strongly parabolic in the sense of Vishik, where functional analytic techniques are appropriate.

This book in translation, although by now somewhat dated, must be a welcome addition to the very small number of specialized texts in English on this particular topic. Of major appeal is the fact that it gives, for English readers, ready access to the large corpus of Russian research on parabolic systems up to the beginning of the last decade. A good seventy-five per cent of the extensive bibliography refers to the Russian literature, for example.

Finally the translation and presentation appear adequate except for the rather annoying omission of an index.

D. DESBROW

ROGERS, C. A., *Hausdorff Measures* (Cambridge University Press, 1970), viii + 179 pp., £3.80.

The purpose of this book is to give an account of some of the research done on Hausdorff measures. Since the initial work by Hausdorff in 1919, this subject has been developing steadily, mainly as a result of the research work of Besicovitch and his students. The first chapter contains a systematic account of measures and their regularity properties in abstract spaces, topological spaces and metric spaces. (The author uses the terms "measure" instead of "outer measure" and "countably additive measure" for "measure".) Lebesgue measure in  $n$ -dimensional Euclidean space is discussed. There are also sections on metric measures on topological spaces, and the Souslin operation. In Chapter 2, various definitions of Hausdorff measure are given and their equivalence proved. Special Hausdorff measures arising in the theory of surface areas are discussed. There are sections on existence theorems, comparison theorems, Souslin sets, sets of non  $\sigma$ -finite measure, and the increasing sets lemma. §7 of this chapter contains some recent joint work of the author and Dr. R. O. Davies on the existence of comparable net measures and their properties, not previously published elsewhere. The final chapter contains a survey of the literature on applications of Hausdorff measures together with specific applications to (a) the theory of sets of real numbers defined in terms of their expansions in continued fractions and (b) the study of non-decreasing continuous functions on  $[0, 1]$ . There is also an