Macroeconomic Dynamics (2024), **29**, e3, pp. 1–23
doi:10.1017/S1365100524000063

ARTICLE

Unionization, industry concentration, and economic growth

Colin Davis[1], Ken-ichi Hashimoto[2], and Ken Tabata[3]

[1]The Institute for the Liberal Arts, Doshisha University, Kyoto, Japan
[2]Graduate School of Economics, Kobe University, Kobe, Japan
[3]School of Economics, Kwansei Gakuin University, Hyogo, Japan
Corresponding author: Colin Davis; Email: cdavis@mail.doshisha.ac.jp

Abstract

This paper examines how unionization affects economic growth through its impact on industry concentration in a two-country model of international trade and endogenous productivity growth. Knowledge spillovers link firm-level productivity in innovation with geographic patterns of industry ensuring a faster rate of output growth when industry is relatively concentrated in the country with the greater labor supply. We show that stronger bargaining power in the relatively large country increases the rate of output growth when labor unions are employment-oriented but decreases the rate of growth when unions are wage-oriented. We then calibrate the model using labor market data for the United Kingdom and France and study the effects of union bargaining power on industry location patterns, output growth, and national welfare.

Keywords: Labor union bargaining power; industry concentration; knowledge diffusion; endogenous productivity growth; endogenous market structure

1. Introduction

There is a great diversity in levels of unionization and collective bargaining over wages across countries. These differences in labor market institutions influence industry location patterns through their impact on production costs. Although the direct effect of unionization on economic growth is generally ambiguous, industry location patterns are known to have important implications for economic growth, with greater industry concentration generating knowledge spillovers that promote faster rates of innovation. In this paper, we consider how unionization affects economic growth through adjustments in industry location patterns.

We introduce an endogenous growth and endogenous market structure framework (Smulders and van de Klundert, 1995; Peretto, 1996, 2018; Young, 1998) that allows for a study of how unionization affects the geographic location of industry and economic growth. Building on Davis and Hashimoto (2014), our two-country framework consists of two industries. A final good sector employs labor and intermediate goods in the production of a homogeneous good. Firms in the intermediate sector employ final goods in the production of differentiated varieties and in process innovation aimed at reducing production costs. International trade costs on intermediate goods ensure that the country with the larger market (i.e., greater labor supply) hosts greater shares of the intermediate and final good industries, and has more productive intermediate firms. Imperfect knowledge diffusion between countries links the strength of knowledge spillovers with the geographic location of industry, generating a positive relationship between industry concentration and the rate of economic growth.

Following Chu et al. (2016), a key feature of our framework is the endogenous determination of national labor supplies through negotiation between a labor union and a federation of employers. Adopting the "managerial" labor union introduced by Pemberton (1988), the balance of internal power between leadership and membership determines union orientation, with important implications for the contract curve that arises from negotiation between the federation and the labor union. As a result, strengthening the wage orientation of the union lowers the national labor supply. An increase in union bargaining power, however, reduces the national labor supply if the union is wage-oriented and expands the labor supply if the union is employment-oriented (Pemberton, 1988; Chang et al. 2007; Chu et al. 2016).

We use the framework to explore how changes in labor union policy affect industry location patterns and the rate of economic growth. Beginning with union orientation, we find that strengthening the wage orientation of the labor union in the country with the larger labor supply decreases the large country's shares of intermediate and final good production and lowers the relative productivity of its intermediate firms, as the national labor supply contracts. The reduced concentration of industry weakens knowledge spillovers, slowing the rate of growth. In contrast, strengthening the wage orientation of the small country labor union improves knowledge spillovers by raising industry concentration in the large country, and thus accelerates economic growth.

As seen above, the effects of changes in union bargaining power depend on union orientation. Focusing on the labor union of the large country, if the union is wage-oriented, an increase in union bargaining power lowers the concentration of industry in the large country, weakening knowledge spillovers and slowing economic growth. Alternatively, if the union is employment-oriented, the increase in bargaining power raises industry concentration, thereby strengthening knowledge spillovers and hastening economic growth. These mechanisms generate similar, but opposite, results for the effects of an increase in labor union bargaining power in the small country.

We explore the welfare implications of unionization through a numerical analysis of the long-run equilibrium of our model that uses labor market data to calibrate union bargaining power for the UK and France between 2008 and 2019 through an adaptation of the method introduced by Chu et al. (2016). The numerical results suggest that unions in the UK and France were wage-oriented over the period of analysis. Further, the general decline in union bargaining power observed in the UK and the strengthening of bargaining power in France lead to a shift in manufacturing from France to the UK. The resulting increase in the geographic concentration of industry strengthens knowledge spillovers into innovation, raising the rate of output growth. In addition, the level of output expands in the UK and contracts in France. Overall, welfare improves in the UK, but may improve or deteriorate in France depending on whether there is a sufficient increase in the rate of output growth.

A broad empirical literature investigates the relationship between unionization and economic growth (Doucouliagos et al. 2017). Consider, for instance, a selection of studies examining data for OECD countries. On the one hand, Carmeci and Mauro (2003) and Nguyen-Van and Terraz (2021) report that increased union density and greater union centralization have negative effects on per capita income growth. On the other hand, Vernon and Rogers (2013) conclude that predominant union structure is important, with craft or general unions having negative, enterprise unions having negligible, and industrial unions having positive effects on labor productivity growth in manufacturing. Furthermore, the results of Storm and Naastepad (2009) suggest a positive role for labor unions through the effect of labor market regulation on labor productivity growth. More generally, however, the meta-analysis of Doucouliagos et al. (2017) concludes that there is no significant relationship between unionization and economic growth.

There is a diverse theoretical literature studying how unionization affects economic growth (Shister, 1954; Palokangas, 1996, 2005; Ramos Parreno and Sanchez-Losada, 2002; Irmen and

Wigger, 2002; Lingens, 2003, 2007; Montagna and Nocco, 2013). In particular, Chang et al. (2007) introduce a "managerial union" into a model of endogenous growth, and conclude that stronger union bargaining power reduces the labor supply and slows economic growth when the union is wage-oriented, but expands the labor supply and hastens growth when the union is employment-orientated. In a similar vein, Chang and Hung (2016) consider a wage-oriented managerial union in combination with an elastic household labor supply and show that greater union bargaining power promotes economic growth as a reduced number of workers increase their work hours in response to rising wages, expanding the effective labor supply. Chu et al. (2016) study the effects of managerial unions on economic growth in a two-country framework and show that an increase in the bargaining power of a wage-oriented union lowers the domestic labor supply, slowing economic growth and reducing domestic wages relative to the foreign country. These results are closely linked, however, with a scale effect, whereby the growth rate increases proportionately with the labor force. Indeed, in a scale-neutral endogenous growth and endogenous market structure framework, Ji et al. (2016) show that the economy absorbs changes in union bargaining power through adjustments in market entry, leaving no impact on the long-run growth rate. Our paper extends the growth literature through a consideration of the indirect effects of unionization on scale-neutral economic growth that result from the relationship between relative market size (i.e., labor supplies) and industry location patterns.

Accordingly, our paper is closely related to the new economic geography literature that explores the effects of unionization on industry location (Persyn, 2013; Egger and Etzel, 2014). In a two-country model of international trade, Picard and Toulemonde (2006) show that a symmetric increase in bargaining power across countries leads to a dispersed industry location pattern. Munch (2003) finds that industry tends to agglomerate in the country with weaker union bargaining power, although industry potentially agglomerates in the country with stronger union bargaining power over an intermediate range for international trade costs. Because these studies assume a negatively sloped or vertical contract curve, however, employment-oriented unions are not considered. Thus, our paper contributes to the literature with a study of how union orientation influences industry location patterns, with an emphasis on the implications of unionization for scale-neutral long-run growth.

The remainder of the paper is organized as follows. The next section introduces our two-country model of trade and endogenous productivity growth. Section 3 provides a characterization of the model's long-run equilibrium, and Section 4 studies how changes in the wage orientation and bargaining power of labor unions affect long-run economic growth. In Section 5, we calibrate the model using data for the UK and France and study the long-run welfare implications of changes in labor union bargaining power. Section 6 concludes. Proofs are provided in the appendices.

2. The model

This section develops a two-country model of trade and productivity growth. We refer to the two countries as home and foreign. A final good sector employs labor and intermediate goods in the production of a homogeneous good for sale to a competitive market. The intermediate goods sector consists of monopolistically competitive firms that employ final goods in the production of differentiated varieties and in process innovation that reduces production costs. The countries differ with respect to population size and labor market policy. In each country, negotiations between a federation of employers and a labor union determine national employment and the wage rate. In what follows, we focus on the home country as we introduce the model, but analogous conditions are derived for the foreign country.

2.1. Households

The lifetime utility of the representative dynastic household in the home country is

$$U = \sum_{t=0}^{\infty} \frac{\ln C_t}{(1+\rho)^t}, \tag{1}$$

where C_t denotes the consumption of final goods in period t, and $\rho > 0$ is the subjective discount rate. The household maximizes lifetime utility (1) subject to the following asset accumulation equation:

$$A_{t+1} = (1 + r_t)A_t + w_t L_t + b_t \left(\bar{L} - L_t\right) + \Pi_t - T_t - C_t, \tag{2}$$

where A_t is household assets, r_t is the real interest rate, w_t is the wage rate, $\bar{L}$ is the inelastic labor supply, L_t is the number of employed workers, b_t is the unemployment benefit provided to unemployed workers, and T_t is a lump-sum tax levied by the government. The household owns all domestic final good producers and receives Π_t in dividend income each period. In addition, all household members enjoy equal consumption levels, regardless of their employment status, eliminating consumption and employment uncertainty (Merz, 1995; Andolfatto, 1996). We adopt the final good as the model numeraire and set its price equal to one.

Solving the household's dynamic optimization problem, we derive the following Euler equation:

$$\frac{C_{t+1}}{C_t} = \frac{1 + r_{t+1}}{1 + \rho}. \tag{3}$$

Perfect international capital mobility ensures a common evolution for home and foreign household consumption, as the interest rate is equalized across countries: $C_{t+1}/C_t = C^*_{t+1}/C^*_t = (1 + r_{t+1})/(1 + \rho)$, where variables associated with foreign are indicated with an asterisk. The foreign household faces a similar utility maximization problem and therefore has similar demand conditions.

2.2. Government

A relatively passive role is set for the government: distributing unemployment benefits and levying a lump-sum tax T_t on the household to balance the fiscal budget. The balanced-budget condition is

$$T_t = b_t \left(\bar{L} - L_t\right). \tag{4}$$

To avoid degeneration of unemployment benefits along the balanced growth path, we assume that the unemployment benefit b_t is linked proportionately with final output Y_t; that is, $b_t = mY_t$, where $m > 0$ is a policy parameter that sets the ratio of the unemployment benefit to final good output in the home country. Similarly, the balanced-budget condition for the foreign government is $T^*_t = b^*_t \left(\bar{L}^* - L^*_t\right)$, with $b^*_t = m^* Y^*_t$, where $m^* > 0$. Note that we allow unemployment benefit policies, m and m^*, to differ across countries.

2.3. Final good production

Final goods are produced for a competitive international market characterized by free trade. Following Chang and Hung (2016), the mass of final good firms is fixed to one in each country. We adopt a Cobb-Douglas formation for the aggregate production function. For example, in the home country we have

$$Y_t = Z_t^{\alpha} L_t^{\beta}, \tag{5}$$

where Z_t is an intermediate composite and L_t is labor employment. The factor intensities associated with the intermediate composite and labor are $\alpha \in (0, 1)$ and $\beta \in (0, 1)$. Following Palokangas (1996, 2005) and Chang et al. (2007), we assume that the production function exhibits decreasing returns to scale, with $\alpha + \beta < 1$, thereby allowing final good producers to generate positive profits for the labor union and the employer federation to negotiate over.

The intermediate composite is formulated as a CES aggregator over the varieties available in period t:

$$Z_t = \left(\int_0^{n_t} x_{i,t}^{\epsilon} di + \int_0^{n_t^*} x_{j,t}^{\epsilon} dj \right)^{\frac{1}{\epsilon}}, \tag{6}$$

where $x_{i,t}$ is the demand for intermediate variety i of the n_t mass of varieties produced in home, $x_{j,t}$ is the demand for variety j of the n_t^* mass of varieties produced in foreign, and the constant elasticity of substitution between any given pair of varieties is measured by $1/(1-\epsilon)$, with $\epsilon \in (0, 1)$. Final good production exhibits "international" returns to scale, as introduced by Ethier (1982). For example, an increase in the mass of varieties produced in home expands Y_t by more than $n_t x_t$. We denote the total mass of intermediate varieties by $N_t \equiv n_t + n_t^*$.

The profit function of the representative final good producer in the home country is

$$\Pi_t = Y_t - w_t L_t - \int_0^{n_t} p_{i,t} x_{i,t} di - \int_0^{n_t^*} p_{j,t}^* \tau x_{j,t} dj, \tag{7}$$

where $p_{i,t}$ is the price of intermediate variety i produced in home, $p_{j,t}^*$ is the price of variety j imported from foreign, and $\tau > 1$ is an iceberg trade cost, under which τ additional units must be shipped for every unit sold in an export market.

Final good firms set their inputs of $x_{i,t}$ and $x_{j,t}$ to maximize profit (7), generating the following home country demand conditions for varieties produced in home and foreign:

$$x_{i,t} = p_{i,t}^{\frac{-1}{1-\epsilon}} P_{Z,t}^{\frac{\epsilon}{1-\epsilon}} \alpha Y_t, \qquad x_{j,t} = (\tau p_{j,t}^*)^{\frac{-1}{1-\epsilon}} P_{Z,t}^{\frac{\epsilon}{1-\epsilon}} \alpha Y_t, \tag{8}$$

where the home-country price index for intermediate varieties is

$$P_{Z,t} = \left(\int_0^{n_t} p_{i,t}^{-\frac{\epsilon}{1-\epsilon}} di + \varphi \int_0^{n_t^*} {p_{j,t}^*}^{-\frac{\epsilon}{1-\epsilon}} dj \right)^{-\frac{1-\epsilon}{\epsilon}}, \tag{9}$$

and $P_{Z,t} Z_t = \alpha Y_t$. We index the level of trade costs using the freeness of trade $\varphi \equiv \tau^{-\frac{\epsilon}{1-\epsilon}} \in (0, 1)$; that is, $d\varphi/d\tau < 0$, with $\varphi = 0$ describing prohibitively high trade costs and $\varphi = 1$ indicating free trade between countries. Final good producers in foreign solve a similar optimization problem generating analogous demand conditions for intermediate varieties.

2.4. *Labor unions and collective bargaining*

Central to the labor market in our framework are the negotiations between the labor union and the employer federation that determine national employment and the wage rate. As in Pemberton (1988), Chang et al. (2007), and Chu et al. (2016), we consider a managerial labor union, in which union members desire a high wage rate while union leaders aim for a large membership. Formally, the union's objective function in the home country is set as

$$O_t = (w_t - b_t)^{\chi} L_t, \tag{10}$$

where $\chi > 0$ measures the weight the union places on incremental wage income from employment (i.e., the wage rate minus the unemployment benefit).

The bargaining problem involves choosing w_t and L_t to maximize the following Nash product: $O_t^{\psi}\Pi_t{}^{1-\psi}$, where $\psi \in (0, 1)$ denotes the relative bargaining power of the labor union. Solving this bargaining problem, we obtain the following optimal conditions for the wage rate and employment:

$$w_t - b_t = \chi \left(w_t - \beta \frac{Y_t}{L_t} \right), \tag{11}$$

$$\frac{w_t L_t}{Y_t} = \beta + \psi(1 - \alpha - \beta), \tag{12}$$

where we have used (7) and (10). The contract curve (11) describes the locus of points for which the union's indifference curve and the firm's isoprofit curve are tangent in (w_t, L_t) space. The rent division curve (12) indicates the labor union's negotiated share of income.

The equilibrium level of employment is obtained through substitution of the rent curve (12) and $b_t = mY_t$ into the contract curve (11):

$$L = \frac{1}{m}\left[\beta + (1 - \chi)\psi(1 - \alpha - \beta)\right]. \tag{13}$$

Hence, we find that employment is decreasing in union wage orientation (χ). The relationship between union bargaining power and national employment is summarized as follows.

Lemma 1. *An increase in union bargaining power (ψ) contracts employment (L) for $\chi > 1$, but expands employment for $\chi < 1$.*

Following Chu et al. (2016), we use χ to describe the relative importance that the union places on wages over membership. When $\chi > 1$, the contract curve has a negative slope, and we refer to the union as wage-oriented. In this first case, a rise in union bargaining power shifts the rent curve, causing a contraction in national employment. Alternatively, when $\chi < 1$, the contract curve has a positive slope, and we refer to the union as employment-oriented. In this second case, an increase in bargaining power leads to an expansion in national employment.

Analogous conditions are obtained for the foreign labor market: $w_t^* - b_t^* = \chi^*(w_t^* - \beta Y_t^*/L_t^*)$, $w_t^* L_t^*/Y_t^* = \beta + \psi^*(1 - \alpha - \beta)$, and $L^* = (1/m^*)\left[\beta + (1 - \chi^*)\psi^*(1 - \alpha - \beta)\right]$, where we allow union wage orientation (χ, χ^*) and bargaining power (ψ, ψ^*) to differ between countries.[1]

2.5. Intermediate good production

Intermediate firms employ final goods in the production of horizontally differentiated varieties for supply to the home and foreign markets under monopolistic competition (Dixit and Stiglitz, 1977). Each firm survives for two periods. In the first, firms invest in market entry and process innovation, and in the second they produce an intermediate product variety.

The production technologies of intermediate firms located in home and foreign are

$$X_{i,t} = \theta_{i,t}^{\gamma} I_{x,i,t}, \qquad X_{j,t}^* = \theta_{j,t}^{*\,\gamma} I_{x,j,t}^*, \tag{14}$$

where $X_{i,t}$ and $X_{j,t}^*$ are firm-level outputs, $I_{x,i,t}$ and $I_{x,j,t}^*$ are firm-level inputs of final goods, $\theta_{i,t}$ and $\theta_{j,t}^*$ are firm-specific productivity coefficients, and $\gamma \in (0, 1)$ is the output elasticity of productivity. Each intermediate firm produces to meet the combined demands from final good firms in home and foreign. For example, a home-based intermediate firm sets supply equal to $X_{i,t} = x_{i,t} + \tau x_{i,t}^*$.

Under monopolistic competition, the large mass of intermediate varieties sold in each market eliminates strategic interaction between firms as they choose their optimal production levels. Thus, each firm maximizes operating profit on sales, $\pi_{i,t} \equiv p_{i,t}X_{i,t} - I_{x,i,t}$, by setting price equal to a constant markup over unit cost; that is,

$$p_{i,t} = \frac{1}{\epsilon \theta_{i,t}^{\gamma}}, \qquad p_{j,t}^{*} = \frac{1}{\epsilon \theta_{j,t}^{*\,\gamma}}, \tag{15}$$

for home and foreign firms. Accordingly, optimal operating profit on sales can be obtained as $\pi_{i,t} = (1-\epsilon)p_{i,t}X_{i,t} = [(1-\epsilon)/\epsilon]I_{x,i,t}$. Substituting the demand conditions (8), the pricing rules (15), and $X_{i,t} = x_{i,t} + \tau x_{i,t}^{*}$ into $\pi_{i,t} = (1-\epsilon)p_{i,t}X_{i,t}$ yields the following expressions for the optimal operating profits

$$\pi_{i,t} = \alpha(1-\epsilon)p_{i,t}^{-\frac{\epsilon}{1-\epsilon}} \left(P_{Z,t}^{\frac{\epsilon}{1-\epsilon}} Y_t + \varphi P_{Z,t}^{*\,\frac{\epsilon}{1-\epsilon}} Y_t^{*} \right),$$

$$\pi_{j,t}^{*} = \alpha(1-\epsilon)p_{j,t}^{*\,-\frac{\epsilon}{1-\epsilon}} \left(\varphi P_{Z,t}^{\frac{\epsilon}{1-\epsilon}} Y_t + P_{Z,t}^{*\,\frac{\epsilon}{1-\epsilon}} Y_t^{*} \right), \tag{16}$$

of home and foreign intermediate firms.

2.6. Process innovation

Intermediate firms employ final goods both in preparation for market entry and in process innovation. Specifically, to produce intermediate variety i with productivity $\theta_{i,t}$ in home in period t, a firm must invest a fixed f units of final goods in market entry and a variable $I_{R,i,t-1}$ units of final goods in process innovation in period $t-1$.

The productivities associated with the production technologies of home and foreign intermediate firms are generated by investment in process innovation as follows:

$$\theta_{i,t} = \xi K_{t-1} I_{R,i,t-1}, \qquad \theta_{j,t}^{*} = \xi K_{t-1}^{*} I_{R,j,t-1}^{*}, \tag{17}$$

where K_{t-1} and K_{t-1}^{*} are the productivities of home and foreign firms in R&D, and $\xi > 0$. Following the in-house process innovation literature (Smulders and van de Klundert, 1995; Peretto, 1996, 2018), we assume that technical knowledge accumulates within the production technology of each firm. Specifically, national stocks of knowledge are captured by the average productivity of the intermediate production technologies employed in each country: $\theta_t \equiv (1/n_t)\int_0^{n_t} \theta_{i,t} di$ and $\theta_t^{*} \equiv (1/n_t^{*})\int_0^{n_t^{*}} \theta_{j,t}^{*} dj$.

We then model the productivities of home and foreign firms in process innovation as the weighted average of the productivities of observable technical knowledge in period $t-1$, generating an intertemporal knowledge spillover from production to innovation. Knowledge spillovers in home and foreign are therefore

$$K_{t-1} = s_{t-1}\theta_{t-1} + \delta s_{t-1}^{*}\theta_{t-1}^{*}, \qquad K_{t-1}^{*} = s_{t-1}^{*}\theta_{t-1}^{*} + \delta s_{t-1}\theta_{t-1}, \tag{18}$$

where $s_{t-1} \equiv n_{t-1}/N_{t-1}$ and $s_{t-1}^{*} = 1 - s_{t-1} \equiv n_{t-1}^{*}/N_{t-1}$. The degree of international knowledge diffusion is regulated by $\delta \in (0, 1)$, with knowledge spillovers that are completely national in scope for $\delta = 0$, and perfect international knowledge diffusion for $\delta = 1$.

In period $t-1$, firms borrow from households to finance the costs of market entry and process innovation. As in Young (1998), the net present values of home and foreign firms in period $t-1$ are

$$V_{i,t-1} = \frac{\pi_{i,t}}{1+r_t} - \left(I_{R,i,t-1} + f\right), \qquad V_{j,t-1}^{*} = \frac{\pi_{j,t}^{*}}{1+r_t} - \left(I_{R,j,t-1}^{*} + f\right), \tag{19}$$

where we assume that fixed entry costs are symmetric across countries ($f = f^{*}$).

Firms choose their optimal levels of investment in innovation, $I_{R,i,t-1}$ and $I_{R,j,t-1}^{*}$ to maximize firm value (19) subject to the technology constraint (17). From the first-order conditions (i.e.,

$\partial V_{t-1}/\partial I_{R,j,t-1} = \partial V^*_{t-1}/\partial I^*_{R,j,t-1} = 0$), we obtain

$$I_{R,i,t-1} = \frac{\eta \pi_{i,t}}{1+r_t}, \qquad I^*_{R,j,t-1} = \frac{\eta \pi^*_{j,t}}{1+r_t}, \tag{20}$$

with $\eta \equiv \gamma\epsilon/(1-\epsilon)$. These expressions imply that firms located in each country have the same productivity level, and therefore set the same prices and employment levels for production and innovation. Henceforth, we omit the indices i and j, with $\theta_{i,t} = \theta_t$ and $\theta^*_{j,t} = \theta^*_t$. Moreover, substituting (20) into (19), we rewrite the net present values of home and foreign firms in period $t-1$ as

$$V_{t-1} = \frac{(1-\eta)\pi_t}{1+r_t} - f, \qquad V^*_{t-1} = \frac{(1-\eta)\pi^*_t}{1+r_t} - f. \tag{21}$$

We assume that $\eta \equiv \gamma\epsilon/(1-\epsilon) < 1$ in order to satisfy the second-order condition for the maximization of firm value.

2.7. Market entry

Firm value drives market entry and exit in the intermediate sector in each country. Specifically, with no costs incurred in the introduction of new product designs, firms enter the intermediate sector when firm value is positive ($V_{t-1} > 0$) and exit when firm value is negative ($V_{t-1} < 0$). Referencing (9), (16), and (21), firm value responds correctly to market entry and exit ($\partial V_{t-1}/\partial n_t < 0$ and $\partial V^*_{t-1}/\partial n^*_t < 0$), and thus, the level of market entry in each country adjusts immediately, forcing firm value to zero ($V_{t-1} = V^*_{t-1} = 0$) at all moments in time, in the spirit of Novshek and Sonnenschein (1987).

A key implication of free market entry and exit, in combination with the integrated financial market, is common scales of production and process innovation across firms in both countries.[2] Substituting (20) and (21) into $V_{t-1} = V^*_{t-1} = 0$ yields equilibrium firm-level investment in process innovation as $I_R \equiv I_{R,t-1} = I^*_{R,t-1} = \eta f/(1-\eta)$. As such, using (18) with (17), the productivity growth rates of intermediate firms in home and foreign are

$$g_\theta(\tilde{\theta}_{t-1}) \equiv \frac{\theta_t}{\theta_{t-1}} = \frac{K_{t-1}}{\theta_{t-1}}\xi I_R = \left[s_{t-1} + \delta(1-s_{t-1})\tilde{\theta}^{-1}_{t-1}\right]\xi I_R,$$

$$g^*_\theta(\tilde{\theta}_{t-1}) \equiv \frac{\theta^*_t}{\theta^*_{t-1}} = \frac{K^*_{t-1}}{\theta^*_{t-1}}\xi I_R = \left[1 - s_{t-1} + \delta s_{t-1}\tilde{\theta}_{t-1}\right]\xi I_R, \tag{22}$$

where $\tilde{\theta}_t \equiv \theta_t/\theta^*_t$ denotes the international productivity differential. These expressions show that firm-level productivity growth depends solely on knowledge spillovers (K_t/θ_t and K^*_t/θ^*_t) and is therefore closely linked with national shares of intermediate production (s_t) and the international productivity differential ($\tilde{\theta}_t$).

2.8. Market equilibrium

We now characterize the market clearing conditions for the intermediate and final good sectors. First, with perfect capital mobility between countries, the total asset holdings of households equals total lending to home and foreign firms: $A_{t+1} + A^*_{t+1} = (I_R + f)n_{t+1} + (I^*_R + f)n^*_{t+1}$. Combining this expression with (19) and the free entry conditions $V_{t-1} = V^*_{t-1} = 0$ implies that $(1 + r_{t+1})(A_{t+1} + A^*_{t+1}) = \pi_{t+1}n_{t+1} + \pi^*_{t+1}n^*_{t+1}$.

Next, the market clearing condition for intermediate goods is given by

$$\alpha Y^w_t = n_t p_t X_t + n^*_t p^*_t X^*_t, \tag{23}$$

where $Y_t^w \equiv Y_t + Y_t^*$. The left-hand side is the sum of home and foreign final good firms' expenditure on intermediate goods. The right-hand side is the total revenue of home and foreign intermediate firms.

Lastly, the market clearing condition for final goods is given by

$$Y_t^w = C_t^w + N_t I_{x,t} + N_{t+1}(I_R + f), \tag{24}$$

where $C_t^w \equiv C_t + C_t^*$. Thus, the world supply of final goods is set to meet household consumption, total employment in intermediate production, and total employment in process innovation and market entry.

3. Long-run equilibrium

In this section, we derive two implicit conditions for the determination of national shares of intermediate firms (s_t) and the international productivity differential ($\tilde{\theta}_t = \theta_t/\theta_t^*$) in order to characterize the steady-state equilibrium of the model. We then derive the long-run rate of productivity growth along with national welfare levels. Because a balanced growth path requires a constant allocation of final goods across sectors, we focus on steady states that exhibit a constant international productivity differential and a common rate of productivity growth across countries ($g_\theta = g_\theta^*$) and thereby generate a constant rate of growth in household consumption (C_{t+1}/C_t).

3.1. Steady-state industry location patterns

Adapting the equilibrium concept of Davis and Hashimoto (2014), the first condition for the determination of s_t and $\tilde{\theta}_t$ is obtained by recalling that free market entry and exit reduces the value of intermediate firms to zero ($V_{t-1} = V_{t-1}^* = 0$). Then, the integrated financial market implies that all firms have the same level of operating profit; that is, $\pi_t = \pi_t^*$ at all moments in time. In Appendix A, we combine $\pi_t = \pi_t^*$ with $Y_t/Y_t^* = (Z_t/Z_t^*)^\alpha (L_t/L_t^*)^\beta$, $Y_t/Y_t^* = (P_{Z,t}Z_t)/(P_{Z,t}^* Z_t^*)$, (9), (15), and (16) to solve for the equilibrium share of intermediate firms based on home at time t as

$$s_{R,t} = \frac{(\tilde{\theta}_t^\eta - \varphi)y_t - \varphi(1-\varphi\tilde{\theta}_t^\eta)}{(\tilde{\theta}_t^\eta - \varphi)(1-\varphi\tilde{\theta}_t^\eta)(1+y_t)}, \qquad y_t \equiv \frac{Y_t}{Y_t^*} = \left(\frac{L}{L^*}\right)^{\frac{\beta\epsilon}{\epsilon-\alpha}} \left(\frac{\tilde{\theta}_t^\eta - \varphi}{1-\varphi\tilde{\theta}_t^\eta}\right)^{\frac{\alpha(1-\epsilon)}{\epsilon-\alpha}}, \tag{25}$$

where $\epsilon > \alpha$ is required to ensure that productivity growth generates a positive rate of growth in final good output (see Section 3.2). This expression is illustrated by the s_R curve in Figure 1 and has a strictly positive slope: $\partial s_{R,t}/\partial\tilde{\theta}_t > 0$.[3] An increase in the international productivity differential ($\tilde{\theta}_t$) generates an expansion in the home share of intermediate firms through two channels. The first is a direct effect, whereby an improvement in the relative productivity of home-based firms expands their market share ($\partial s_{R,t}/\partial\tilde{\theta}_t > 0$). The second channel is an indirect effect, wherein international trade costs ensure that an increase in $\tilde{\theta}_t$ raises the home country's share of final good production ($\partial y_t/\partial\tilde{\theta}_t > 0$) as the relative cost of sourcing home produced intermediate goods falls. The expansion of the home market relative to foreign leads to a greater share of intermediate good producers for home ($\partial s_{R,t}/\partial y_t > 0$). Because the home share of intermediate firms is bounded ($s_t \in (0, 1)$), however, there are limits on the range of values for the international productivity differential, outside of which an equilibrium with active intermediate sectors in both countries is not feasible; that is, $\tilde{\theta}_t \in (\tilde{\theta}_l, \tilde{\theta}_u)$.

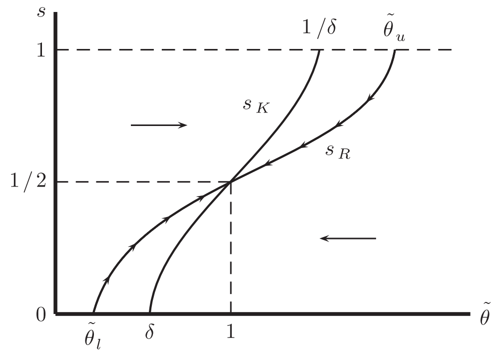

Figure 1. Determination of the international productivity differential ($\tilde{\theta}$).

Next, we derive a differential equation to describe the dynamics of the international productivity differential. Referencing (22), we have

$$\frac{\tilde{\theta}_{t+1}}{\tilde{\theta}_t} = \frac{g_\theta(\tilde{\theta}_t)}{g_\theta^*(\tilde{\theta}_t)} = \frac{K_t/\theta_t}{K_t^*/\theta_t^*} = \frac{s_t + \delta(1-s_t)\tilde{\theta}_t^{-1}}{1 - s_t + \delta s_t \tilde{\theta}_t}. \tag{26}$$

Thus, we naturally find that a constant productivity differential ($\tilde{\theta}_{t+1}/\tilde{\theta}_t = 1$) is synonymous with equal productivity growth rates for home and foreign intermediate firms ($g_\theta = g_\theta^*$).

With all firms employing the same level of final goods in process innovation ($I_R = I_R^*$), the constant productivity differential and equalized productivity growth rates require that firms have access to the same level of knowledge spillovers ($K_t/\theta_t = K_t^*/\theta_t^*$), allowing us to derive the following steady-state condition for the home country's share of intermediate firms using (18):

$$s_K(\tilde{\theta}) = \frac{1-\delta\tilde{\theta}^{-1}}{2-\delta\tilde{\theta}-\delta\tilde{\theta}^{-1}}, \tag{27}$$

where we now drop the time script to indicate that steady-state values are constant. As shown by the s_K curve in Figure 1, when the international productivity differential ($\tilde{\theta}$) increases, a greater share of firms is required for home to offset the improved access of foreign-based firms to technical knowledge ($\partial s_K/\partial\tilde{\theta} > 0$). Note that $\tilde{\theta} \in (\delta, 1/\delta)$ is required for $s_K \in (0, 1)$. In addition, the slope of the s_K curve is closely linked to the degree of knowledge diffusion. When there is no international knowledge diffusion ($\delta = 0$), the s_K curve becomes a horizontal line. Alternatively, with perfect knowledge diffusion ($\delta = 1$), the s_K curve becomes a vertical line at $\tilde{\theta} = 1$, and the international productivity differential vanishes.

In Appendix B, we study the local dynamics around the steady state characterized by the intersection of the s_K and s_R curves in Figure 1 and obtain the following proposition.

Proposition 1. *The international productivity differential ($\tilde{\theta}$) converges to a long-run equilibrium with $s \in (0, 1)$ when $\partial s_K/\partial\tilde{\theta} > \partial s_R/\partial\tilde{\theta}$.*

Proof: See Appendix B.

As a state variable, the international productivity differential is rising (falling) for values of $\tilde{\theta}$ to the left (right) of the s_K curve. Therefore, a stable steady state requires $\partial s_K/\partial\tilde{\theta} > \partial s_R/\partial\tilde{\theta}$ to ensure that the economy moves along the s_R curve towards the long-run equilibrium, as depicted in the phase diagram of Figure 1.[4] Examining (25), we find that $\lim_{\varphi\to 0}\tilde{\theta}_l = 0$ and $\lim_{\varphi\to 0}\tilde{\theta}_u = \infty$. Accordingly, we assume that the freeness of trade (φ) is small enough to satisfy $\tilde{\theta}_l < \delta < 1/\delta < \tilde{\theta}_u$, ensuring the existence of at least one steady state with the slope ranking outlined in Proposition 1.[5]

Hereafter, we focus on equilibria that satisfy these conditions for the existence of a stable long-run equilibrium.

The long-run international productivity differential ($\tilde{\theta}$) is implicitly linked with the relative employment level of the home country (L/L^*). Referencing (25), an increase in L/L^* expands the relative market size of the home country for intermediate producers ($\partial y/\partial(L/L^*) > 0$), causing an upward shift in the s_R curve in Figure 1 ($\partial s_R/\partial y > 0$). As a result, the home country's shares of intermediate firms (s) and final good production (y) both increase. The rise in s coincides with an increase in the international productivity differential that returns knowledge spillovers back to equality across countries ($K/\theta = K^*/\theta^*$), as the economy converges to a new long-run equilibrium. These results are summarized in the following lemma.

Lemma 2. *An increase in the relative employment of home (L/L^*) expands the home country's shares of intermediate firms (s) and final good production (y), while raising the international productivity differential ($\tilde{\theta}$).*

From the results of Lemma 2 and the mechanics of Figure 1, referencing (25) and (27), it becomes clear that equality of national employment levels generates a symmetric equilibrium with $y = 1$, $s = 1/2$ and $\tilde{\theta} = 1$. Accordingly, when home employment rises above foreign employment ($L/L^* > 1$), the home country has greater shares of intermediate firms ($s > 1/2$) and final good production ($y > 1$), and relatively productive intermediate firms ($\tilde{\theta} > 1$). When the foreign country has a larger relative employment level ($L/L^* < 1$), however, it has greater shares of intermediate firms ($s < 1/2$) and final good production ($y < 1$), and foreign-based intermediate firms are relatively productive ($\tilde{\theta} < 1$).

As we have seen, knowledge spillovers equalize across countries in the long run. Substituting (27) back into (18), and reorganizing the result, yields the steady-state level of knowledge spillovers as follows:

$$\frac{K}{\theta} = \frac{K^*}{\theta^*} = \frac{1-\delta^2}{2-\delta\tilde{\theta}-\delta\tilde{\theta}^{-1}}. \tag{28}$$

Therefore, we determine that K/θ is convex in the international productivity differential ($\tilde{\theta}$) with a minimum at $\tilde{\theta} = 1$; that is, $\partial(K/\theta)/\partial\tilde{\theta} = -[\delta(1-\tilde{\theta}^2)/(1-\delta^{-2})\tilde{\theta}^2](K/\theta)^2$. For $\tilde{\theta} < 1$, an increase in $\tilde{\theta}$ lowers knowledge spillovers, and for $\tilde{\theta} > 1$, the increase in $\tilde{\theta}$ raises knowledge spillovers. Consequently, we find that firm-level productivity in R&D is directly linked with national shares of intermediate production, with an increase in the geographic concentration of industry in one country raising the level of knowledge spillovers.

Lastly, before leaving this section, we derive the equilibrium operating profit associated with the intermediate firms based in home and foreign as

$$\pi_t = \frac{\alpha(1-\epsilon)Y_t^w}{N_t}, \tag{29}$$

where we have used $\pi_t = (1-\epsilon)p_t X_t$ and $\pi_t = \pi_t^*$ in the market clearing condition for intermediate goods (23).

3.2. Economic growth

We now turn to the long-run rate of output growth. First, combining (3), (21), and (29) with $V_{t-1} = V_{t-1}^* = 0$, we rewrite the dynamics of household consumption as

$$\frac{C_{t+1}}{C_t} = \frac{\alpha(1-\epsilon)(1-\eta)}{(1+\rho)f}\frac{Y_{t+1}^w}{N_{t+1}}. \tag{30}$$

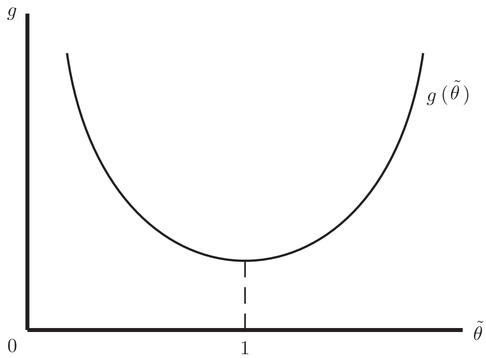

Figure 2. Long-run output growth.

Because we are interested in balanced growth paths that feature a constant rate of growth in consumption for home and foreign households, this expression implies that final good output and the mass of intermediate firms grow at the same rate. In addition, given that $y \equiv Y_t/Y_t^*$ is constant in a steady state with a constant international productivity differential ($\tilde{\theta}_{t+1}/\tilde{\theta}_t = 1$), we find that $Y_{t+1}^w/Y_t^w = Y_{t+1}/Y_t = Y_{t+1}^*/Y_t^* = N_{t+1}/N_t$.

Then, substituting (5), (9), (15), (17), and (28) with $\alpha Y_t = P_{Z,t}Z_t$ into Y_{t+1}/Y_t delivers the long-run rate of output growth as follows:

$$g \equiv \frac{Y_{t+1}^w}{Y_t^w} = g_\theta^{\frac{\alpha\epsilon\gamma}{\epsilon-\alpha}} = \left[\frac{(1-\delta^2)\xi I_R}{2-\delta\tilde{\theta}-\delta\tilde{\theta}^{-1}}\right]^{\frac{\alpha\epsilon\gamma}{\epsilon-\alpha}}, \tag{31}$$

where $I_R = \eta f/(1-\eta)$. A casual examination of this expression makes it clear that knowledge spillovers are the key channel through which intermediate and final good production patterns affect the steady-state rate of output growth. As such, referencing (28), we find that the output growth rate is convex in the international productivity differential ($\tilde{\theta}$) with a minimum at $\tilde{\theta} = 1$, as depicted in Figure 2. In addition, (31) indicates that the long-run growth rate is scale-invariant as proportionate changes in the labor forces of home and foreign leave the international productivity differential and the rate of output growth unchanged.

The mechanics of Figure 2, combined with Lemma 2, yield the following result.

Lemma 3. *The long-run rate of output growth (g) is convex in the relative employment of the home-country (L/L^*) with a minimum at $L/L^* = 1$.*

Lemma 3 describes the key mechanism in our framework linking national labor supplies with long-run output growth through the relationship that arises between industry location patterns (s) and the strength of knowledge spillovers (K/θ) from production into innovation. Returning to Lemma 2, an increase in the relative employment of home (L/L^*) expands the home-country shares of final production (y) and intermediate firms (s), while raising the relative productivity of home-based firms ($\tilde{\theta}$). Thus, starting from $L/L^* < 1$, where initially the foreign country has a greater share of industry ($s < 1/2$), an increase in L/L^* reduces the geographic concentration of industry, as the foreign share of intermediate firms falls, weakening knowledge spillovers (K/θ) and slowing output growth (g). The minimum growth rate is reached when home and foreign have equal shares of industry at $L/L^* = 1$, $s = 1/2$, and $\tilde{\theta} = 1$, as illustrated in Figure 2. Further increases in L/L^* then lead to the concentration of industry in the home country, strengthening knowledge spillovers and accelerating output growth. Thus, the rate of output growth is faster when industry is concentrated in a single country than when industry is dispersed equally across countries.

The level of market entry in the intermediate sector (N_t) depends on the international productivity differential ($\tilde{\theta}$) and the rate of output growth (g). To emphasize this point, we first use (5), (9), (15), and (25) to derive final good output in home and foreign as

$$Y_t = \frac{(\alpha\epsilon\theta_t^{\gamma})^{\frac{\alpha}{1-\alpha}}(1-\varphi^2)^{\nu} N_t^{\nu} L^{\frac{\beta}{1-\alpha}}}{[(1-\varphi\tilde{\theta}^{\eta})(1+1/y)]^{\nu}}, \qquad Y_t^* = \frac{(\alpha\epsilon\theta_t^{\gamma})^{\frac{\alpha}{1-\alpha}}(1-\varphi^2)^{\nu} N_t^{\nu} L^{*\frac{\beta}{1-\alpha}}}{[(\tilde{\theta}^{\eta}-\varphi)(1+y)]^{\nu}}, \tag{32}$$

where $\nu \equiv \alpha(1-\epsilon)/((1-\alpha)\epsilon) < 1$. An examination of these expressions shows that $N_{t+1}/N_t = Y_{t+1}/Y_t = Y_{t+1}^*/Y_t^* = g$ in the long-run equilibrium. A corollary of this result is that $C_{t+1}/C_t = g$, and referencing (3) the steady-state interest rate is therefore constant: $1 + r = (1+\rho)g$.

Substituting national output levels into $\pi = \alpha(1-\epsilon)Y_t^w/N_t = (1+\rho)fg/(1-\eta)$, we then obtain the total mass of intermediate firms along the balance growth path as follows:

$$N_t = \theta_t^{\frac{\alpha\epsilon\gamma}{\epsilon-\alpha}} \left[\frac{\Gamma}{(1+\rho)gf}\right]^{\frac{1}{1-\nu}} \left\{\frac{L^{\frac{\beta}{1-\alpha}}}{[(1-\varphi\tilde{\theta}^{\eta})(1+1/y)]^{\nu}} + \frac{L^{*\frac{\beta}{1-\alpha}}}{[(\tilde{\theta}^{\eta}-\varphi)(1+y)]^{\nu}}\right\}^{\frac{1}{1-\nu}}, \tag{33}$$

with $\Gamma \equiv \alpha^{1/(1-\alpha)}(1-\eta)(1-\epsilon)\epsilon^{\alpha/(1-\alpha)}(1-\varphi^2)^{\nu}$. Hence, we find that for a given level of home productivity (θ), our model features the standard tradeoff that arises between market entry and economic growth in endogenous growth and endogenous market structure frameworks: given the productivity differential ($\tilde{\theta}$) and relative final good output (y), a rise in g necessitates a fall in N. In addition, the mass of intermediate firms adjusts to absorb proportionate changes in the labor forces of home and foreign, ensuring a scale-invariant rate of output growth.

Although productivity growth drives market entry and the expansion of aggregate output along the balance growth path, adjustments in the international productivity differential ($\tilde{\theta}$) generally have an ambiguous effect on market entry, as an increase in $\tilde{\theta}$ expands the final good output of home, but contracts the final good output of foreign, while generating an ambiguous effect on the rate of output growth (g), as seen in Lemma 3.

3.3. *Social welfare*

Lastly, we solve for the steady-state welfare levels of households in each country. Using (1) and (3), we express the lifetime welfare of home and foreign households along the balanced growth path as

$$U_0 = \frac{1+\rho}{\rho}\left(\ln C_0 + \frac{1}{\rho}\ln g\right), \qquad U_0^* = \frac{1+\rho}{\rho}\left(\ln C_0^* + \frac{1}{\rho}\ln g\right). \tag{34}$$

Household welfare derives from the current level of consumption (C_0) and the growth of consumption (g). In Appendix C, we derive steady-state consumption as follows:

$$C_0 = (1-\alpha)Y_0 + \frac{\rho f N_0 g}{2(1-\eta)}, \qquad C_0^* = (1-\alpha)Y_0^* + \frac{\rho f N_0 g}{2(1-\eta)}, \tag{35}$$

where we have set the initial value of home-firm productivity to unity ($\theta_0 = 1$), and we have assumed that initial asset wealth is the same for home and foreign ($A_0 = A_0^*$). With Y_0, Y_0^*, and N_0 determined by (32) and (33), from the above expressions, we find that labor union policy influences welfare through its effects on domestic output levels (Y_0 and Y_0^*), market entry (N_0), and the rate of output growth (g).

4. Labor unions

This section investigates how changes in the wage orientation and bargaining power of the home-country labor union affect output growth.

4.1. Wage orientation of labor unions

We begin with an examination of the effects of an increase in the wage orientation (χ) of the home-country labor union.[6] The results are summarized in the following proposition.

Proposition 2. *The long-run rate of output growth (g) is convex in the wage orientation of the home-country labor union* (χ) *with a minimum at* $\chi = 1 + (\beta - mL^*)/(\psi(1 - \alpha - \beta))$.

The intuition behind Proposition 2 lies in the link between national labor supplies, the location of industry, and the strength of knowledge spillovers from production to innovation. Referring back to (13), a rise in the wage orientation of the home-country union (χ) leads to lower employment in home ($dL/d\chi < 0$), with the union's push for higher wages depressing labor demand. Suppose that the home country initially has a larger labor supply ($L/L^* > 1$) and therefore has greater shares of final good production ($y > 1$) and intermediate firms ($s > 1/2$), and higher productivity ($\tilde{\theta} > 1$). An increase in χ then reduces the home country's share of industry (s) and lowers the international productivity differential ($\tilde{\theta}$) as the relative labor supply (L/L^*) falls. Consequently, referencing Lemma 3, the rise in χ weakens knowledge spillovers (K/θ) and slows output growth (g) for $\tilde{\theta} > 1$, and strengthens knowledge spillovers and accelerates output growth for $\tilde{\theta} < 1$. The rate of output growth is minimized when $\chi = 1 + (\beta - mL^*)/(\psi(1 - \alpha - \beta))$, where $L/L^* = 1$, $s = 1/2$, and $\tilde{\theta} = 1$.

4.2. Bargaining power of labor unions

Next, we consider how adjustments in the bargaining power (ψ) of the home-country labor union influence output growth (g). Returning to (13), we find that the relationship between bargaining power and national employment depends critically on the slope of the contract curve, as described by the wage orientation (χ) of the union, with $dL/d\psi < 0$ for $\chi > 1$ and $dL/d\psi > 0$ for $\chi < 1$. In the former case, the wage-oriented union demands higher wages as its bargaining power increases, thereby depressing labor demand. In the latter case, the employment-oriented union negotiates greater employment by allowing lower wages and expanding labor demand.

The relationship between union bargaining power and national employment has important implications for the rate of output growth, as summarized in the following proposition.

Proposition 3. *The long-run rate of output growth (g) is convex in the bargaining power of the home-country labor union* (ψ) *when the union is wage-oriented* ($\chi > 1$) *and concave in* ψ *when the union is employment-oriented* ($\chi < 1$), *with either a minimum or a maximum at* $\psi = (\beta - mL^*)/((\chi - 1)(1 - \alpha - \beta))$.

As we have seen, the rate of output growth is linked with national labor supplies through the role of industry location patterns in determining the strength of knowledge spillovers from production to innovation. When the home-country is union is wage-orientated ($\chi > 1$), an increase in union bargaining power (ψ) reduces the home labor supply, causing a fall in relative employment (L/L^*) that decreases the home country share of intermediate firms. Then, knowledge spillovers (K/θ) and the rate of output growth (g) fall for $L/L^* > 1$, but rise for $L/L^* < 1$, with the growth rate minimized at $\psi = (\beta - mL^*)/((\chi - 1)(1 - \alpha - \beta))$ where $L/L^* = 1$, $s = 1/2$, and $\tilde{\theta} = 1$. In contrast, when the home-country union is employment-oriented ($\chi < 1$), an increase in bargaining power (ψ) expands the home-country labor supply, raising relative employment (L/L^*) and the home-country share of firms (s). In this case, knowledge spillovers (K/θ) and the output growth rate (g) fall for $L/L^* < 1$, but rise for $L/L^* > 1$. The growth rate is maximized at $\psi = (\beta - mL^*)/((\chi - 1)(1 - \alpha - \beta))$.

Table 1. Parameter values

Parameter	Description	Value
ρ	Discount rate	0.024
α	Intermediate intensity in final production	0.1
β	Labor intensity in final production	0.25
ϵ	Elasticity of substitution $1/(1-\epsilon)$	0.8
δ	Degree of knowledge diffusion	0.3
τ	Trade cost	1.650
f	Fixed operating cost	1
γ	R&D elasticity of labor	0.21
ξ	R&D efficiency	0.457
ψ	UK labor union bargaining power	0.471→0.458
ψ^*	France labor union bargaining power	0.564→0.576
χ	UK union wage orientation	1.304
χ^*	France union wage orientation	1.162
m	UK unemployment benefits	0.148
m^*	France unemployment benefits	0.209
$\bar{L}$	UK labor force	1.126
$\bar{L}^*$	France labor force	1

5. Numerical analysis

The complex nature of our framework renders a theoretical study of welfare intractable. As an alternative, we calibrate the long-run equilibrium of the model using data for the income share of labor and employment in the UK and France over the period from 2008 to 2019 with the aim of investigating how changes in labor union bargaining power affect national welfare by inducing shifts in industry location patterns. We set the UK as the home country and France as the foreign country.

Because the numerical results are sensitive to changes in parameter values, we select parameters to generate initial values that match with three data points for the UK and France. First, as the active labor forces of the UK and France were 31,414,000 and 27,897,000 in 2008, we set the labor forces to $\bar{L} = 1.126$ and $\bar{L}^* = 1$ (OECD Labor Market Statistics, 2023). Second, we adopt GDP per person employed, \$86,768 for the UK and \$96,152 for France in 2015 US dollars, to generate a target value of $(Y^*/L^*)/(Y/L) = 1.108$ for the relative final output per worker of France in 2008 (OECD Productivity Statistics, 2023). Third, we target a benchmark output growth rate of $g = 1.011$ to match the average rate of real GDP growth for the UK and France between 2008 and 2019 (OECD Productivity Statistics, 2023).

Table 1 provides a summary of the model parameters. The discount rate is fixed to $\rho = 0.024$ following Jones et al. (1993). In the final goods sector, we assume values of $\alpha = 0.1$ and $\beta = 0.25$ for the respective factor intensities of intermediate goods and labor.[7] We set the elasticity of substitution across intermediate varieties to $1/(1-\epsilon) = 5$, yielding a price-cost markup of $1/\epsilon = 1.25$, which is in the range of estimates presented by Britton et al. (2000) and Gali et al. (2007). In the intermediate sector, the degree of knowledge diffusion is fixed at $\delta = 0.3$, and the trade cost is set to $\tau = 1.650$, based on the estimates of Anderson and van Wincoop (2004) and Novy (2013), generating a value of $\varphi = 0.135$ for the freeness of trade. With respect to process innovation, we normalize the fixed cost of market entry to $f = 1$ and set the intermediate output elasticity of productivity to $\gamma = 0.21$ and the efficiency parameter for process innovation to $\xi = 0.457$, targeting

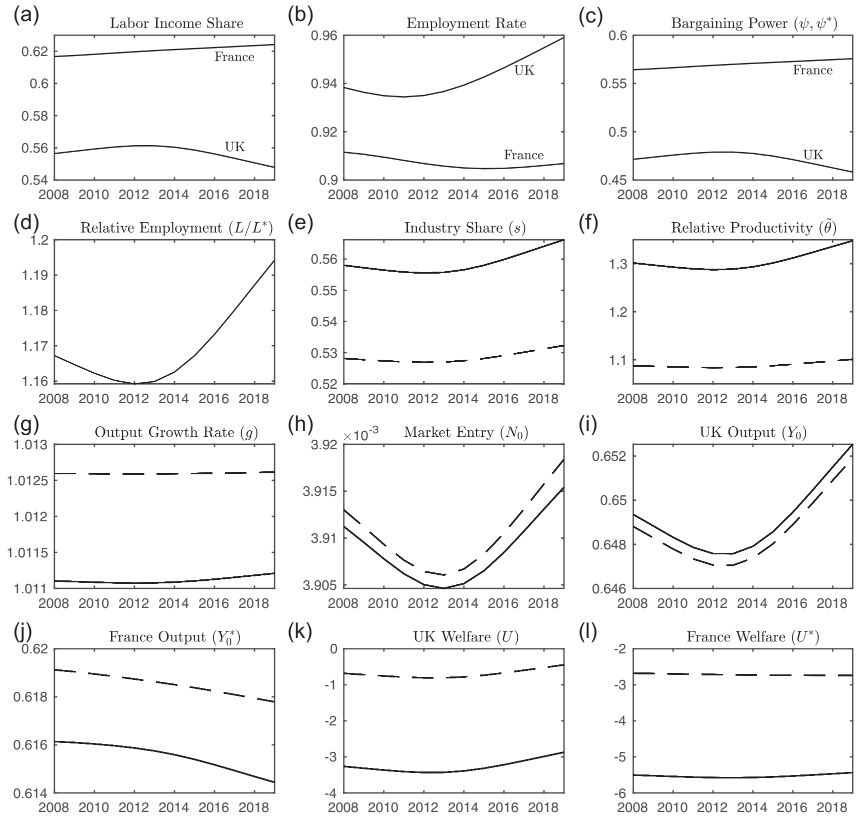

Figure 3. The effects of changes in bargaining power (ψ, ψ^*). Panels (a) and (b) plot HP-filtered trends for labor income shares of GDP and national employment rates. The calibrated values for union bargaining power are shown in Panel (c). In Panels (d) through (l), the solid lines plot values for the indicated variable for the benchmark data set with $\delta = 0.3$. The dashed lines in Panels (e) through (l) plot values for the indicated variable with $\delta = 0.4$. Data source: The labor income share data are the share of labor compensation in GDP at current national prices reported in the Penn World Table 10.01. The employment rate is calculated using the unemployment rates provided in the World Bank Development Indicators.

the benchmark growth rate ($g = 1.011$), and satisfying the second-order condition for optimal investment in R&D ($\eta = 0.84 < 1$).

Adapting the methodology of Chu et al. (2016), we use (12) with the labor income share data shown in Figure 3(a) to generate time series data for union bargaining power in the UK (ψ) and France (ψ^*). The calibrated data are plotted in Figure 3(c). Over the period of analysis, ψ falls from 0.471 to 0.458 in the UK and ψ^* rises from 0.564 to 0.576 in France, illustrating that adjustments in union bargaining power follow the trends in the labor income share data. The greater level of union bargaining power in France, relative to the UK, is consistent with the empirical results of Dumont et al. (2006). Although a smaller proportion of workers are part of a union in France, the proportion of workers covered by a collective bargaining agreement is significantly larger than in the UK (Tippet et al., 2022).

We calculate best fit lines for the relationship between the national employment rate data shown in Figure 3(b) and the union bargaining power data shown in Figure 3(c) to calibrate parameter values for union wage orientation and unemployment benefits in each country.[8] The resulting values are $\chi = 1.3039$ and $m = 0.1477$ for the UK, and $\chi^* = 1.1623$ and $m^* = 0.2093$ for France, suggesting that labor unions are wage-oriented in both countries. This result matches with

Table 2. Simulated effects on growth and welfare

Variables	UK ($\delta = 0.3$)	France ($\delta = 0.3$)	UK ($\delta = 0.4$)	France ($\delta = 0.4$)
Change in Output Growth: $(g_{2019} - g_{2008})$	0.011pp	0.011pp	0.002pp	0.002pp
Change in Welfare: $(U_{2019} - U_{2008})/U_{2008}$	12.054%	1.227%	34.495%	−2.244%
Domestic Output Channel (Y_0):	6.383%	−2.137%	29.833%	−3.430%
Investment Income Channel $(N_0 g)$:	0.0008%	0.0005%	0.0044%	0.0012%
Output Growth Channel (g):	5.667%	3.363%	4.657%	1.185%

empirical evidence presented by Dumont et al. (2006), where most of the industries examined for the UK and France feature wage-oriented unions.[9]

Referencing (13), the union bargaining power data and calibrated parameters yield the time series data for relative employment (L/L^*) plotted in Figure 3(d). While union bargaining power initially rises, but then falls in the UK, it rises continuously in France, generating a U-shaped trend in L/L^* across the period of analysis. As unions are wage-oriented in both countries, the weaker union bargaining power of the UK ($\psi < \psi^*$) results in a greater relative labor supply ($L/L^* > 1$). Thus, the UK has larger shares of final good production ($y > 1$) and intermediate good ($s > 1/2$) production, following Lemma 2. The higher productivity of intermediate firms in the UK ($\tilde{\theta} > 1$) aligns with empirical evidence that the geographic concentration of production increases productivity (Melo et al., 2009).

The solid lines in Figures 3(e) to 3(h) plot the effects of changes in union bargaining power on the long-run values of the key variables of our framework. Between 2008 and 2019, the UK's share of intermediate production (s) and the international productivity differential ($\tilde{\theta}$) increase, matching the adjustment in the relative labor supply. As a result of greater industry concentration in the UK, knowledge spillovers are strengthened, generating a 0.011 percentage point (pp) increase in output growth, as shown in Figure 3(g) and summarized in Table 2. Although the empirical literature has generally produced mixed results on how the geographic concentration of industry influences economic growth (Gardiner et al. 2011), Desmet and Rossi-Hansberg (2009, 2014) find a positive relationship between spatial agglomeration and economic growth. The UK decline in union bargaining power, and the corresponding increase in the national labor supply, generates an expansion in UK output (Y_0), as shown in Figure 3(i), while the strengthening of union bargaining power in France leads to a decrease in the national labor supply and a contraction in domestic output (Y_0^*), as shown in Figure 3(j). Overall, Figure 3(h) indicates a rise in the level of market entry (N_0).

Turning now to national welfare, Figures 3(k) and 3(l) indicate that for the benchmark parameter set changes in union bargaining power lead to welfare improvements in both the UK and France. In order to obtain a clearer understanding of the mechanisms driving these welfare improvements, we decompose welfare adjustments into three channels. In the home country, for example, taking the total derivative of steady-state utility (34) yields

$$dU_0 = \frac{(1+\rho)(1-\alpha)}{\rho C_0} dY_0 + \frac{(1+\rho)f}{2(1-\eta)C_0} d(N_0 g) + \frac{1+\rho}{\rho^2 g} dg.$$

The first term on the right-hand side captures the effect of a change in domestic final output (Y_0), the second term describes the effect of a change in investment income ($N_0 g$), and the third term describes the effect of a change in the rate of output growth (g).

We quantify the annual effect of each of the channels described above and then aggregated across the period of analysis to obtain the welfare decompositions shown in Table 2. For the benchmark parameter set with $\delta = 0.3$, the results show that welfare improved by 12.054% in the UK and 1.227% in France over the period of analysis. On the one hand, the fall in union bargaining power (ψ) causes an expansion in domestic output in the UK. On the other hand, the rise in union bargaining power (ψ^*) results in a contraction in domestic output in France. Although the investment channel is small, the output growth channel is large and ensures that France experiences a welfare improvement.

The welfare results obtained from our benchmark numerical example are sensitive to adjustments in the degree of knowledge diffusion (δ). As such, we present a second numerical illustration using a higher degree of knowledge diffusion ($\delta = 0.4$), as shown by the dashed plots in Figures 3(e) to 3(l). The increase in the degree of knowledge diffusion has level effects on all of the variables in our analysis. Of key interest, however, is the impact of greater knowledge diffusion on the relationship between union bargaining power and national welfare levels. The rise in the degree of knowledge diffusion weakens the positive effect of industry concentration on knowledge spillovers, dampening the overall increase in the rate of output growth to 0.002 percentage points over the period of analysis. Indeed, as shown in Table 2, although the UK still experiences a welfare improvement (34.495%), for France the positive growth channel is no longer sufficient to counter the negative domestic output channel, indicating that adjustments in union bargaining power over the period of analysis now lead to a welfare deterioration (−2.244%).

6. Conclusion

This paper has examined how unionization affects economic growth through its influence on industry location patterns. We introduce a two-country model of endogenous growth and endogenous market structure. Intermediate firms invest in process innovation to lower their production costs, and supply differentiated varieties to final good producers. International trade costs on intermediate goods ensure that the country with the larger market (i.e., greater labor supply) hosts a greater share of industry. In addition, imperfect knowledge diffusion between countries links the strength of knowledge spillovers with the geographic concentration of industry generating a positive relationship between industry concentration and the rate of economic growth.

We use the framework to study the effects of changes in labor union orientation and union bargaining power on the output growth rate. An increase in the wage orientation of the labor union in the large country reduces its labor supply, lowering geographic concentration of industry and slowing the rate of economic growth. The effect of an increase in union bargaining power, however, depends on the orientation of the union. When the union is wage-oriented the labor supply contracts, reducing industry concentration and slowing output growth. And, when the union is employment-oriented, the labor supply expands, increasing industry concentration and accelerating economic growth.

In order to explore the welfare implications of unionization, we calibrate our model using labor market data for the UK and France between 2008 and 2019. Our numerical analysis suggests that a general decline in union bargaining power in the UK and a strengthening of bargaining power in France results in a shift in industry from France to the UK. The increase in the geographic concentration of industry then strengthens knowledge spillovers into innovation, raising the rate of output growth. The increased concentration of industry generates a welfare improvement for the UK, but the welfare of France may improve or deteriorate depending on whether there is a sufficient rise in the rate of output growth.

Acknowledgements. We are grateful for helpful comments from Tetsugen Haruyama, Yoshiyasu Ono, Takumi Naito, two anonymous reviewers, and participants of the 81st Annual Meeting of the Japanese Society of International Economics at Kindai University, the 2022 Japanese Economic Association Autumn Meeting at Keio University, and seminars at Kobe University and Okayama University. This research was financially supported by the Joint Usage/Research Center at ISER (Osaka University) and KIER (Kyoto University), and the Japan Society for the Promotion of Science: Grants-in-Aid for Scientific Research 19K01638, 20H05631, 20K01569, 20K01631, and 22K01511.

Notes

1 In this paper, wages are determined through union bargaining. Alternatively, one could consider a search and matching mechanism and or an efficiency wage framework. Davis and Hashimoto (2022) consider how search frictions influence the relationship between industry location patterns and economic growth. Zenou (2009) examines the role of efficiency wages in urban models.
2 Substituting $\pi_t = [(1-\epsilon)/\epsilon]I_{x,t}$ into (21) and setting $V_{t-1} = V^*_{t-1} = 0$ delivers the equilibrium scale of production as $I_{x,t} = I^*_{x,t} = [\epsilon/(1-\epsilon)][(1+r)f/(1-\eta)]$.
3 The derivative of (25) with respect to the international productivity differential is

$$\frac{\partial s_{R,t}}{\partial \tilde{\theta}_t} = \left[\frac{y_t}{(1-\varphi\tilde{\theta}_t^{\eta})^2} + \frac{1}{(\varphi - \tilde{\theta}_t^{\eta})^2} \right] \frac{\eta\varphi\tilde{\theta}_t^{\eta-1}}{1+y_t} + \frac{(1-\varphi^2)\tilde{\theta}_t^{\eta}}{(\tilde{\theta}_t^{\eta}-\varphi)(1-\varphi\tilde{\theta}_t^{\eta})(1+y_t)^2} \frac{\partial y_t}{\partial \tilde{\theta}_t} > 0,$$

with $\partial y_t / \partial \tilde{\theta}_t > 0$.
4 See Baldwin et al. (2003) for an in-depth discussion of the use of phase diagrams in the examination of the dynamics associated with industry location patterns.
5 Note that as the s_K curve is horizontal and the slope ranking is not satisfied when $\delta = 0$, a positive level of knowledge diffusion is required for a stable long-run equilibrium with manufacturing located in both countries. Davis and Hashimoto (2014) provide a full characterization of the interior and corner solutions that arise for different values of the degree of knowledge diffusion and trade costs in this class of models.
6 In our framework, raising unemployment benefits (m) has the same effect as increasing union wage orientation (χ).
7 From (12), the lower and upper bounds on the labor income shares are $\beta = 0.25$ and $1-\alpha = 0.9$.
8 Specifically, we calculate the following best fit line for each country: $L_t/\bar{L} = B_0 + B_1\psi_t$. Referencing (13), the calibrated values of B_0 and B_1 yield unemployment benefits as $m = \beta/(B_0\bar{L})$ and union wage orientation as $\chi = 1 - (mB_1\bar{L})/(1-\alpha-\beta)$.
9 The one industry for which Dumont et al. (2006) report evidence of an employment union orientation is paper, printing, and publishing in France.

References

Anderson, J. and E. van Wincoop (2004) Trade costs. *Journal of Economic Literature* 42(3), 691–751.
Andolfatto, D. (1996) Business cycles and labor-market search. *The American Economic Review* 86(1), 112–132.
Baldwin, R., R. Forslid, P. Martin, G. Ottaviano and R. Robert-Nicoud (2003) *Economic Geography and Public Policy*. New Jersey: Princeton University Press.
Britton, E., J. Larsen and I. Small (2000) Imperfect competition and the dynamics of mark-ups, *Bank of England Working Papers*, vol. 110, Bank of England.
Carmeci, G. and L. Mauro (2003) Imperfect labor market and convergence: theory and evidence in some OECD countries. *Journal of Policy Modeling* 25(8), 837–856.
Chang, J. and H. Hung (2016) Trade unions, unemployment, economic growth, and income inequality. *Macroeconomic Dynamics* 20(1), 404–428.
Chang, J., M. Shaw and C. Lai (2007) Managerial trade union and economic growth. *European Economic Review* 51(2), 365–384.
Chu, A., G. Cozzi and Y. Furukawa (2016) Unions, innovation and cross-country wage inequality. *Journal of Economic Dynamics and Control* 64(C), 104–118.
Davis, C. and K. Hashimoto (2014) Patterns of technology, industry concentration, and productivity growth without scale effects. *Journal of Economic Dynamics and Control* 40, 266–278.
Davis, C. and K. Hashimoto (2022) Productivity growth, industry location patterns, and labor market frictions. *Regional Science and Urban Economics* 97, 1–13.
Desmet, K. and E. Rossi-Hansberg (2009) Spatial growth and industry age. *Journal of Economic Theory* 144(6), 2477–2502.
Desmet, K. and E. Rossi-Hansberg (2014) Spatial development. *American Economic Review* 104(4), 1211–1243.
Dixit, A. and J. Stiglitz (1977) Monopolistic competition and optimum product diversity. *The American Economic Review* 67(3), 297–308.
Doucouliagos, H., R. Freeman and P. Laroche (2017) *The Economics of Trade Unions: A Study of a Research Field and Its Findings*. New York: Routledge.
Dumont, M., G. Rayp and P. Willemé (2006) Does internationalization affect union bargaining power? An empirical study for five EU countries. *Oxford Economic Papers* 58(1), 77–102.
Egger, H. and D. Etzel (2014) Union wage-setting and international trade with footloose capital. *Regional Science and Urban Economics* 48, 56–67.
Ethier, W. (1982) National and international returns to scale in the modern theory of international trade. *The American Economic Review* 72(3), 389–405.

Gali, J., M. Gertler and D. López-Salido (2007) Markups, gaps, and the welfare costs of business fluctuations. *Review of Economics and Statistics* 89(1), 44–59.

Gardiner, B., R. Martin, P. Tyler (2011) Does spatial agglomeration increase national growth? Some evidence from Europe. *Journal of Economic Geography* 11(6), 979–1006.

Irmen, A. and B. U. Wigger (2002) Trade union objectives and economic growth. *FinanzArchiv, Public Finance Analysis* 59(1), 49–67.

Ji, L., J.-J. Chang and C.-Y. Huang (2016) Unionization, market structure, and economic growth. *Southern Economic Journal* 82(3), 935–951.

Jones, L., R. Manuelli and P. Rossi (1993) Optimal taxation in models of endogenous growth. *Journal of Political Economy* 101(3), 485–517.

Lingens, J. (2003) The impact of a unionised labour market in a Schumpeterian growth model. *Labor Economics* 10(1), 91–104.

Lingens, J. (2007) Unions, wage setting, and economic growth. *Economic Modelling* 24(1), 167–188.

Melo, P., D. Graham and R. Noland (2009) A meta-analysis of estimates of urban agglomeration economies. *Regional Science and Urban Economics* 39(3), 332–342.

Merz, M. (1995) Search in the labor market and the real business cycle. *Journal of Monetary Economics* 36(2), 269–300.

Montagna, C. and A. Nocco (2013) Unionization, international integration, and selection. *Canadian Journal of Economics* 46(1), 23–45.

Munch, J. (2003) The location of firms in unionized countries. *Scandinavian Journal of Economics* 105(1), 49–71.

Van & Terraz (2021)Nguyen-Van, P. and I. Terraz (2021) When union's activity matters: the impact of union centralization on economic growth in OECD countries. *Applied Economics* 53(21), 2433–2446.

Novshek, W. and H. Sonnenschein (1987) General equilibrium with free entry: a synthetic approach to the theory of perfect competition. *Journal of Economic Literature* 115(3), 1281–1306.

Novy, D. (2013) Gravity redux: measuring international trade costs with panel data. *Economic Inquiry* 51(1), 101–121.

OECD Labor Market Statistics (2023) This information is retrieved from the OECD labour market statistics: http://stats.oecd.org/.

OECD Productivity Statistics (2023) This information is retrieved from the OECD productivity statistics: https://doi.org/10.1787/pdtvy-data-en.

Palokangas, T. (1996) Endogenous growth and collective bargaining. *Journal of Economic Dynamics and Control* 20(5), 925–944.

Palokangas, T. (2005) International labor union policy and growth with creative destruction. *Review of International Economics* 13(1), 90–105.

Pemberton, J. (1988) A 'managerial' model of the trade union. *The Economic Journal* 98(392), 755–771.

Peretto, P. (1996) Sunk costs, market structure, and growth. *International Economic Review* 37(4), 895–923.

Peretto, P. (2018) Robust endogenous growth. *European Economic Review* 108, 49–77.

Persyn, D. (2013) Union wage demands with footloose firms and agglomeration forces. *Regional Science and Urban Economics* 43(1), 142–150.

Picard, P. M. and E. Toulemonde (2006) Firms agglomeration and unions. *European Economic Review* 50(3), 669–694.

Ramos Parreno, J. M. and F. Sanchez-Losada (2002) The role of unions in an endogenous growth model with human capital. *Journal of Macroeconomics* 24(2), 171–192.

Shister, J. (1954) Trade-unionism, collective bargaining, and economic growth. *The American Economic Review* 44(2), 214–227.

Smulders, S. and T. van De Klundert (1995) Imperfect competition, concentration and growth with firm-specific R&D. *European Economic Review* 39(1), 139–160.

Storm, S. and C. W. M. Naastepad (2009) Labor market regulation and productivity growth: evidence for twenty OECD countries (1984-2004. *Industrial Relations* 48(4), 629–654.

Tippet, B., 'O. Onaran and R. Wildauer (2022) The effect of labor's bargaining power on wealth inequality in the UK, USA, and France. *The Review of Income and Wealth*, forthcoming.

Vernon, G. and M. Rogers (2013) Where do unions add value? Predominant organizing principle, union strength and manufacturing productivity growth in the OECD. *British Journal of Industrial Relations* 51(1), 1–27.

Young, A. (1998) Growth without scale effects. *Journal of Political Economy* 106(1), 41–63.

Zenou, Y. (2009) *Urban Labor Economics*. Cambridge: Cambridge University Press.

APPENDIX A

In this appendix, we derive the equilibrium industry location pattern ($s_{R,t}$) described by (25). First, from (21), with free market entry reducing firm value to zero ($V_t = V_t^* = 0$), from (21) we find that profit is the same for all firms: $\pi_t = \pi_t^*$. We rewrite this condition using (16) with (15), $y_t \equiv Y_t/Y_t^*$, $\tilde{\theta}_t \equiv \theta_t/\theta_t^*$ and $\eta \equiv \gamma\epsilon/(1-\epsilon)$ to obtain

$$(1-\varphi\tilde{\theta}_t^{\eta}) = y_t(\tilde{\theta}_t^{\eta}-\varphi)\left(\frac{P_{Z,t}}{P_{Z,t}^*}\right)^{\frac{\epsilon}{1-\epsilon}}, \tag{A.1}$$

where, referencing (9), the price indices for intermediate varieties in home and foreign are

$$P_{Z,t} = p_{i,t}N_t^{-\frac{1-\epsilon}{\epsilon}}(s_t+\varphi s_t^*\tilde{\theta}_t^{-\eta})^{-\frac{1-\epsilon}{\epsilon}}, \qquad P_{Z,t}^* = p_{i,t}N_t^{-\frac{1-\epsilon}{\epsilon}}(\varphi s_t + s_t^*\tilde{\theta}_t^{-\eta})^{-\frac{1-\epsilon}{\epsilon}}. \tag{A.2}$$

Substituting (A.2) with $s_t^* = 1-s_t$ into (A.1) delivers (25). Next, we derive $y_t \equiv Y_t/Y_t^*$ in (25) as follows. Combining (5), $\alpha Y_t = P_{Z,t}Z_t$ and $\alpha Y_t^* = P_{Z,t}^*Z_t^*$, we obtain

$$\frac{Y_t}{Y_t^*} = \left(\frac{Z_t}{Z_t^*}\right)^{\alpha}\left(\frac{L}{L^*}\right)^{\beta}, \qquad \frac{Y_t}{Y_t^*} = \frac{Z_t}{Z_t^*}\frac{P_{Z,t}}{P_{Z,t}^*}.$$

Then, removing Z_t/Z_t^* from the above two equations yields the following expression:

$$y_t \equiv \frac{Y_t}{Y_t^*} = \left(\frac{L}{L^*}\right)^{\frac{\beta}{1-\alpha}}\left(\frac{P_{Z,t}}{P_{Z,t}^*}\right)^{-\frac{\alpha}{1-\alpha}},$$

which can be rewritten using (A.1) to obtain y_t as shown in (25).

APPENDIX B

We reduce the model to a dynamic system in three variables: the international productivity differential ($\tilde{\theta}_t$), the ratio of the productivity of the home intermediate production technology in final good output to the mass of intermediate firms ($\lambda_t \equiv \theta_t^{\alpha\epsilon\gamma/(\epsilon-\alpha)}/N_t$), and the ratio of household consumption to the mass of intermediate firms ($\mu_t \equiv C_t^w/N_t$). We then use the reduced system to derive a sufficient condition for the stability of long-run equilibrium.

A key feature of the system is that the dynamic path of $\tilde{\theta}$ is determined independently of λ_t and μ_t. Evaluating a linear expansion of (26) around the steady-state at $\tilde{\theta}_{t+1}/\tilde{\theta}_t = 1$ yields

$$\frac{\partial\tilde{\theta}_{t+1}}{\partial\tilde{\theta}_t} = \frac{(1-\delta\tilde{\theta}_t)(1-\delta\tilde{\theta}_t^{-1})}{1-\delta^2} + \frac{(2-\delta\tilde{\theta}-\delta\tilde{\theta}_t^{-1})^2\tilde{\theta}_t}{1-\delta^2}\frac{\partial s_{R,t}}{\partial\tilde{\theta}_t}, \tag{B.1}$$

where we have used $1-s+\delta s\tilde{\theta} = (1-\delta^2)/(2-\delta\tilde{\theta}-\tilde{\theta}^{-1})$, and the slope of the s_R curve in Figure 1 determines the effect of changes in $\tilde{\theta}_t$ on the home share of intermediate firms (s_t), as (25) is satisfied at all moments in time. Setting $\tilde{\theta}_t$ as a state variable, stability requires $\partial\tilde{\theta}_{t+1}/\partial\tilde{\theta}_t < 1$. Thus, using (B.1), we rewrite the stability condition as

$$\frac{\partial\tilde{\theta}_{t+1}}{\partial\tilde{\theta}_t} - 1 = -\frac{(2-\delta\tilde{\theta}-\delta\tilde{\theta}_t^{-1})^2\tilde{\theta}_t}{1-\delta^2}\left(\frac{\partial s_K}{\partial\tilde{\theta}} - \frac{\partial s_R}{\partial\tilde{\theta}}\right), \tag{B.2}$$

which provides the slope ranking $\partial s_K/\partial\tilde{\theta} > \partial s_R/\partial\tilde{\theta}$ that is required for stability, as illustrated in Figure 1 and outlined in Proposition 1.

Next, we derive two differential equations to describe the evolutions of λ_t and μ_t, and show that these variables jump immediately and permanently to their steady-state values. First, using (32), we obtain the ratio of final good output to the mass of intermediate firms as

$$\frac{Y_t^w}{N_t} = \Omega\lambda_t^{1-\nu}, \tag{B.3}$$

where $\Omega \equiv (\alpha\epsilon)^{\alpha/(1-\alpha)}(1-\varphi^2)^{\nu}\{L^{\beta/(1-\alpha)}/[(1-\varphi\tilde{\theta}^{\eta})(1+1/y)]^{\nu} + L^{*\beta/(1-\alpha)}/[(\tilde{\theta}^{\eta}-\varphi)(1+y)]^{\nu}\}$ is constant for a given steady-state value of $\tilde{\theta}$. Linking $\pi_t = (1-\epsilon)p_tX_t = [(1-\epsilon)/\epsilon]I_{X,t}$

with (23) gives $N_t I_{X,t} = \alpha\epsilon Y_t^w$, which in turn is substituted with (B.3) into (24) to arrive at

$$\frac{N_{t+1}}{N_t} = \frac{1-\eta}{f}\left[(1-\alpha\epsilon)\Omega\lambda_t^{1-\nu} - \mu_t\right]. \tag{B.4}$$

Then, referencing (31), we express the dynamics of λ_t as follows:

$$\lambda_{t+1} = \frac{gf}{(1-\eta)[(1-\alpha\epsilon)\Omega\lambda_t^{1-\nu} - \mu_t]}\lambda_t, \tag{B.5}$$

where the rate of output growth (g) assumes a steady-state value determined by $\tilde{\theta}$.

Turning to the dynamics of μ_t, we use (30), (B.1), (B.2), and (B.3) to obtain the following differential equation:

$$\mu_{t+1} = \frac{\alpha(1-\epsilon)\Omega(gf)^{1-\nu}\lambda_t^{1-\nu}}{(1+\rho)(1-\eta)^{1-\nu}[(1-\alpha\epsilon)\lambda_t^{1-\nu} - \mu_t]^{2-\nu}}\mu_t. \tag{B.6}$$

Together (B.5) and (B.6) describe the dynamics of λ_t and μ_t. We evaluate a linear expansion of the system around a steady state for which $\lambda_{t+1}/\lambda_t = \mu_{t+1}/\mu_t = 1$, and thus, $C_{t+1}^w/C_t^w = N_{t+1}/N_t = g$. From (B.4) and (B.5), the steady-state values of λ and μ are

$$\lambda = \left[\frac{(1+\rho)fg}{\alpha(1-\epsilon)(1-\eta)\Omega}\right]^{\frac{1}{1-\nu}}, \qquad \mu = \frac{[1-\alpha+(1-\alpha\epsilon)\rho]fg}{\alpha(1-\epsilon)(1-\eta)}.$$

Taking a linear expansion of (B.3) and (B.4) around λ and μ, we obtain the following Jacobian matrix

$$J = \begin{pmatrix} 1 + \frac{(2-\nu)[1-\alpha+(1-\alpha\epsilon)\rho]}{\alpha(1-\epsilon)} & -\left[1+(1-\nu)(1-\alpha\epsilon) - \frac{\alpha(1+\epsilon\rho)}{1+\rho}\right]\frac{(1-\nu)(1+\rho)\mu}{\alpha(1-\epsilon)\lambda} \\ \frac{(1-\eta)\lambda}{fg} & 1 + \frac{\epsilon(1-\nu)(1+\rho)}{(1-\epsilon)} \end{pmatrix},$$

where the trace and the determinant associated with J are

$$tr(J) = 2 + \frac{\epsilon(1-\nu)(1+\rho)}{(1-\epsilon)} + \frac{(2-\nu)[1-\alpha+(1-\alpha\epsilon)\rho]}{\alpha(1-\epsilon)},$$

$$|J| = \left\{1 + \frac{(2-\nu)[1-\alpha+(1-\alpha\epsilon)\rho]}{\alpha(1-\epsilon)}\right\}\left[1 + \frac{\epsilon(1-\nu)(1+\rho)}{(1-\epsilon)}\right] + \left[1+(1-\nu)(1-\alpha\epsilon) - \frac{\alpha(1+\epsilon\rho)}{1+\rho}\right]\frac{(1-\nu)(1-\eta)(1+\rho)\mu}{\alpha(1-\epsilon)fg}.$$

As λ and μ are jump variables, we require two eigenvalues, ω_1 and ω_2, with absolute values that are greater than one for saddle point stability. The values of ω_1 and ω_2 are determined as a solution to the characteristic equation $\Upsilon(\omega) \equiv \omega^2 - tr(J)\omega + |J| = 0$. Substituting $tr(J)$, $|J|$, and $\omega = 1$ into the characteristic equation, we obtain

$$\Upsilon(1) = \frac{\epsilon(1-\nu)(2-\nu)[1-\alpha+(1-\alpha\epsilon)\rho](1+\rho)}{\alpha(1-\epsilon)^2} + \left[1+(1-\nu)(1-\alpha\epsilon) - \frac{\alpha(1+\epsilon\rho)}{1+\rho}\right]\frac{(1-\eta)(1-\nu)(1+\rho)\mu}{\alpha(1-\epsilon)fg} > 0.$$

As $\Upsilon(1) = (1-\omega_1)(1-\omega_2) > 0$ and $tr(J) = \omega_1 + \omega_2 > 2$, we find that $\omega_1 > 1$ and $\omega_2 > 1$, and conclude that λ and μ jump immediately and permanently to their long-run values. Thus, the slope ranking $\partial s_K/\partial\tilde{\theta} > \partial s_R/\partial\tilde{\theta}$ is a sufficient condition for saddle path stability.

APPENDIX C

Substituting (4), (12) and $\Pi_t/Y_t = (1-\psi)(1-\alpha-\beta)$ into (2), and rearranging, we obtain the following intertemporal budget constraint:

$$\sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} = (1-\alpha) \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t} + (1+r)A_0. \tag{C.1}$$

Along the balanced growth path $C_{t+1}/C_t = g$. Thus, substituting $C_t = C_0 g^t$, $Y_t = Y_0 g^t$, and $(1+r) = (1+\rho)g$ into (C.1), we obtain

$$C_0 = (1-\alpha)Y_0 + \frac{\rho}{1+\rho}(1+r)A_0. \tag{C.2}$$

Because initial assets are the same for home and foreign ($A_0 = A_0^*$) and the total asset holdings of households equals total lending to home and foreign firms, $A_0 + A_0^* = (I_R + f)n_0 + (I_R^* + f)n_0^*$, noting that $I_R = I_R^* = \eta f/(1-\eta)$, we obtain $A_0 = A_0^* = fN_0/(2(1-\eta))$. Substituting this expression into (C.2) yields (35).

Cite this article: Davis C, Hashimoto K and Tabata K (2024). "Unionization, industry concentration, and economic growth." *Macroeconomic Dynamics* **29**(e3), 1–23. https://doi.org/10.1017/S1365100524000063