Appendix L
Josephson effect

Josephson (1962) proposed that there should be a contribution to the current through an insulating barrier between two superconductors which would behave like direct tunnelling of condensed pairs from one condensed gas of bound pairs at the Fermi surface to the other. The measurement of such an effect has provided a beautiful scenario where the collective rotational degree of freedom in gauge space manifests itself, let alone some of the most accurate measurements of the electron charge (Anderson (1964)).

Before proceeding further let us briefly discuss a technical detail which, aside from being essential to microscopically understanding the mechanism which is at the basis of the effect, also clarifies the long-range order induced by pairing correlations. Because one is interested in calculating the tunnelling of Cooper pairs across the barrier separating the two superconductors, it is natural to start by assuming that it is the pairing interaction that is the source of this transfer, by annihilating a pair in superconductor 2 and creating a pair in superconductor 1 (see Fig. L.1). Although this is what effectively happens, it can be shown that the pairing interaction leads to a negligible contribution to pair transfer, and that essentially all the transfer proceeds through the single-particle mean field acting twice. Note that this reaction mechanism leading to a (successive) two-particle tunnelling does not destroy the correlation existing between the pair of fermions of a Cooper pair participating in the condensate. In fact, aside from the fact that ξ is much larger than typical particle distances (see equation (1.32) and (1.39)), successive transfer mediated by the single-particle field is essentially equivalent to simultaneous transfer, being only one of the different choices of representations used to describe the process to properly take into account the non-orthogonality of the wavefunctions describing the motion of the fermions in each of the superconductors: prior-prior, post-prior, post-post representations (see Cohen *et al*. (1962), Prange (1963), Anderson and Rowell (1963), Götz *et al*. (1975), Broglia and Winther (1991)). Let us now come back to the main subject of this appendix, i.e. the Josephson effect.

Owing to the macroscopic number of paired electrons which are present in a superconductor, it is not possible to observe so directly as in the case of a finite system like the nucleus the individual states of the (pairing) rotational spectrum (in gauge space) shown, for example, in Fig. 4.2. The so-called Josephson junction consists of two

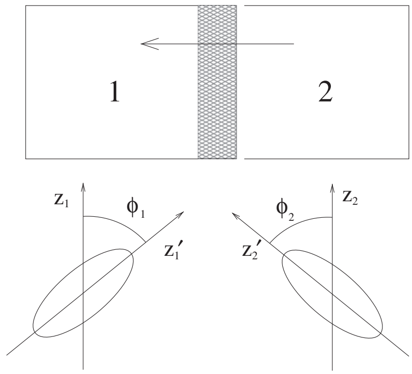

Figure L.1. Schematic representation of a Josephson junction.

superconductors which are separated by a thin dioxide (insulating) layer, through which the electrons can penetrate. Each of the superconductors can, because of the analogy discussed in connection with Fig. 4.1 (see also Sections 1.2, 3.8, 4.2, equation (4.14), as well as Appendix I), be thought of as a rotor (see Fig. L.1). These two rotors are coupled together through the exchange of pairs

$$P_1^\dagger P_2 = \mathrm{e}^{2\mathrm{i}\phi_1} P_1'^\dagger \mathrm{e}^{-2\mathrm{i}\phi_2} P_2', \tag{L.1}$$

where

$$a_\nu'^\dagger = \mathcal{G}(\phi) a_\nu^\dagger \mathcal{G}^{-1}(\phi) = \mathrm{e}^{-\mathrm{i}\phi} a_\nu^\dagger, \tag{L.2}$$

and thus

$$a_\nu^\dagger = \mathrm{e}^{\mathrm{i}\phi} a_\nu'^\dagger. \tag{L.3}$$

This implies

$$P_1^\dagger = \sum_{\nu_1>0} a_{\nu_1}^\dagger a_{\bar{\nu}_1}^\dagger = \mathrm{e}^{2\mathrm{i}\phi} P_1'^\dagger, \tag{L.4}$$

and similarly for P_2.

Consequently, the coupling between the superconductors is

$$\begin{aligned} H_{\mathrm{coupl}} &\sim \mathrm{e}^{2\mathrm{i}\phi_1}\mathrm{e}^{-2\mathrm{i}\phi_2}\mathrm{e}^{\mathrm{i}\delta} + \mathrm{h.c.} \\ &\sim \cos(2(\phi_1 - \phi_2) + 2\delta), \end{aligned} \tag{L.5}$$

where ϕ_1 and ϕ_2 are the gauge phases of the superconductors and δ a phase shift, associated with barrier penetration. The rate at which the quanta are exchanged between the two superconductors is thus given by

$$\begin{aligned} \dot{N}_1 = (-\dot{N}_2) = \frac{\mathrm{i}}{\hbar}[H, N_1] = \frac{\mathrm{i}}{\hbar}\left(\mathrm{i}\frac{\partial H}{\partial \phi}\right) = -\frac{1}{\hbar}\frac{\partial H}{\partial \phi} \\ \sim \sin\left(2(\phi_1 - \phi_2) + 2\delta\right). \end{aligned} \tag{L.6}$$

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The rotational frequency of the rotors corresponds to the chemical potential of the superconductors (see Appendix I, Section I.3)

$$\dot{\phi}_1 = \frac{1}{\hbar}\frac{\partial H}{\partial N_1} = \frac{1}{\hbar}\lambda_1 \tag{L.7}$$

and

$$\dot{\phi}_2 = \frac{1}{\hbar}\frac{\partial H}{\partial N_2} = \frac{1}{\hbar}\lambda_2. \tag{L.8}$$

Introducing $\phi = \dot{\phi}t = \frac{\lambda}{\hbar}t$ in equation (L.6) one obtains

$$\dot{N}_1 \sim \sin\left(\frac{2}{\hbar}(\lambda_1 - \lambda_2)\,t + 2\delta\right). \tag{L.9}$$

This means that if there is a difference in chemical potential between the two superconductors, which can be obtained by applying an external voltage, there will be an oscillating current running between the superconductors. In terms of the voltage differential $V_1 - V_2$, equation (L.9) can be written as

$$\dot{N}_1 \sim \sin\left(\frac{2e}{\hbar}(V_1 - V_2)\,t + 2\delta\right). \tag{L.10}$$

This shows that the frequency of the oscillating current is determined by the applied voltage, the carriers having charge $2e$. Note that to make this point evident we have used the function $\mathcal{G}(\phi) = e^{-i\hat{N}\phi}$ to induce a gauge transformation (see equation (L.2)), and not $e^{i\frac{\hat{N}}{2}\phi}$ as introduced in equation (4.12).

The remarkable confirmation of the picture of deformation and of rotation in gauge space provided by the Josephson effect is an example of the general fact that, arguably, the most successful approach to physics is a combination of phenomenology with microscopic theory, and of experiment with both. From this kind of approach one can arrive at a degree of understanding of phenomena which essentially amounts to certainty. Superconductivity and superfluidity are likely to belong to this category of phenomena, of whose basic nature one is virtually certain, primarily because of the large variety of phenomena which can be correlated by one form or another of BCS theory.

In general, a condensation phenomenon is characterized by a new parameter in the condensed phase leading to emergent properties which were not present in the original system nor in the particles which compose it. For example, below its Curie point a ferromagnet has magnetization in the absence of a field. The long-range order of a solid is not present in the liquid. The order parameter of a superconductor is the energy gap itself.

All these systems and their order parameters have an important feature in common: the condensed system does not have the full symmetry of the Hamiltonian describing it. Superfluidity and superconductivity can be considered particular examples of this general theory, letting the order parameter be $\langle \mathrm{BCS}|GP^\dagger|\mathrm{BCS}\rangle = e^{2i\phi}\Delta$ and fixing the magnitude and the phase ϕ. Then, it is gauge invariance which is violated in the superconductor.

Clearly, general gauge invariance is not violated, but from the point of view of individual fermions it is, in the sense that the phase of the field operator with which we insert additional particles is relevant.

It is of course physically obvious that the full symmetry of the original Hamiltonian still governs the system, in the sense that it is only the state of the system which is taken to be non-invariant, and one considers all other states to which the assumed state can be carried by symmetry operations as degenerate with a given one (see Section 4.2.1, in particular equation (4.14)).

These ideas seem rather evident and general. Now, however, one comes to the real distinction among the different situations. In a few cases – ferromagnetism being an obvious example – the order parameter is a constant of motion. Then, of course, the non-invariant states are, rigorously, degenerate eigenstates of H, and no serious questions of principle arise: all the consequences of the true symmetry of H can be retained in the most direct fashion.

More common is the opposite case: the order parameter is not a quantum-mechanical constant of the motion. The orientation of the solid in space, for instance, and its position, are not constants of the motion; the correct constants are total momentum and angular momentum. In the superconductor we find the phase variable is not only not a constant of motion, but is normally assumed to be meaningless.

In the cases of the solid or the ferroelectric one can understand the physics of the situation. What happens is that the condensation has given the system one form or another of long-range order, so that $\approx 10^{23}$ different atoms must move as a unit rather than individually. Under such circumstances the system is so large that its behaviour is essentially classical, and one may fix the value of the order parameter even though it is not a constant of motion – the coordinate or orientation of the solid, for instance. There is indeed zero-point motion of a macroscopic solid, but it is so small that one does not need to deal with it.

Another aspect of the situation is that in general the usual type of condensed system finds itself in the presence of external fields which fix the order parameter at some preferred value. Because of the long-range order, only a very small external force is necessary to do this. A small external field can align a ferromagnet, a small external force pin down the orientation and position of a crystal (see final paragraph of Section 4.2.4, Weinberg's chair).

In actual fact one seldom deals with condensed systems in the absence of external fields, so that one is accustomed to think of such systems as having definite values of such order parameters as the orientation. But this is because we are accustomed to working with measuring instruments which are themselve rigid, i.e. have a long-range positional order. Thus it does not seem extraordinary that a solid has a fixed position and orientation. In the case of magnets, again one is used to instruments which violate time-reversal symmetry themselves, and thus we do not find it unusual for a system to have a definite value of ferromagnetic order.

In the case of superconducting systems things are quite different. The internal long-range order parameter – the phase – is not a parameter for which suitable measurement instruments exist. A superconductor, or a superfluid, has rather perfect internal phase order, but as has been shown in Section 4.2.1 (equation (4.39)) (see also Appendix I), the zero-point motion of the total order parameter of an isolated superconductor is large and rather rapid.

The importance of the Josephson effect is that it provides for the first time an instrument which can act like a clamp for a solid: it can pin down the order parameter, making superfluidity and superconductivity one more example of condensation phenomena.

Summing up, condensation is a self-consistent choice by the system of a state – and a corresponding mean self-consistent field – which does not have the full symmetry of the Hamiltonian. Fluctuations of the order parameter will, in the absence of asymmetric external forces, restore the original symmetry. The external forces needed to 'pin down' the quantum fluctuations can only come from systems which themselves violate the given symmetry: in the case of a superconductor, another superconductor.

The possibility to study the transfer of Cooper pairs between superfluid nuclei in a heavy ion collision (transient Josephson junction), has been extensively discussed (see e.g. von Oertzen (1994), Broglia and Winther (1991) and references therein)