invertibility of A, the scheme applies to A precisely when the operators A_n are invertible for large enough n and the norms of the inverses are uniformly bounded.

The aim of the present book is to develop an abstract framework in which to study such approximation methods for various classes of convolution operators. In addition to operators of the form (1) but with the weaker assumption of piecewise continuity on the coefficient functions a and b, other operators related to singular integral operators are also studied such as Toeplitz and Wiener–Hopf operators, Hankel operators and Mellin convolutions. Singular integral operators associated with Lyapunov curves are also considered and as well as the classical Lebesgue L^p spaces of the real line certain weighted L^p spaces can also be treated by the authors' methods. Various different approximation methods are considered in the context of the more abstract theory and individual convergence criteria established for each.

The theory developed in the present book has its origins in work of I. B. Simonenko in the 1960s and A. Kozak in the 1970s, but their results applied to somewhat restricted classes of operators. However, a new approach was proposed by B. Silbermann (initially for a wider class of Toeplitz operators than it had previously been possible to handle) in 1981. Since then the subject has gone from strength to strength and the present work represents an up-to-date account of the current state of affairs, with many results appearing in print for the first time.

The book is clearly written and the authors have gone to great pains to present the material in as friendly a way as possible, so that long technical proofs are sometimes deferred in order not to interrupt the flow. There is also a useful introductory section on the necessary background from Banach algebra and operator theory. It has to be said, though, that the work is not for the faint-hearted! Someone new to the subject (the present reviewer classifies himself in this category) will find it tough going, but those who persevere will find much of interest.

T.A. GILLESPIE

Pisier, G., *Similarity problems and completely bounded maps* (Lecture Notes in Mathematics Vol. 1618, Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong, 1996), vii + 156 pp., 3 540 60322 0, (softcover) £20.50.

This book is concerned with three similarity problems involving operators on Hilbert space H.

Q1. Given a uniformly bounded continuous representation of a locally compact group $\pi : G \to B(H)$, when can one find an invertible operator $S \in B(H)$ so that $g \mapsto S^{-1}\pi(g)S$ is a unitary representation?

Q2. Is every bounded unital algebra homomorphism u from a C^*-algebra A to $B(H)$ similar to a $*$-representation?

Q3. Let T be a polynomially bounded operator so that $\|p(T)\| \leq C \sup\{|p(z)| : |z| = 1\}$ for all polynomials p. Can one find an invertible S for which $\|S^{-1}TS\| \leq 1$?

These problems turn out to be related by the important general concepts of lacunarity, complete boundedness and Schur multipliers. The author introduces these ideas in turn, showing that several partial answers to these questions can be viewed in a unified framework. Each chapter reaches a main idea rapidly and discusses its background and significance in remarks at the end.

A classical result of Dixmier shows that Q1 always has a positive answer when G is a discrete amenable group. In Chapter 2 the author considers a combinatorial criterion for amenability due to Hulanicki and Kesten. He uses this to show that there are uniformly bounded representations of the free group F_∞ not similar to unitary representations. Switching to Q3, he then uses Fournier's algorithm to obtain $T \in B(H)$ with $\|T^n\| \leq C$ for $n \geq 1$ which is not polynomially bounded.

Completely bounded maps become involved in the theory as the answers to Q2 and Q3 are affirmative when the homomorphism involved is completely bounded. Paulsen showed that $T \in B(H)$ is similar to a contraction if and only if the maps $p \mapsto p(T) \in M_n \bar{\otimes} B(H)$ are bounded,

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uniformly in n, for polynomials whose coefficients are $n \times n$ matrices. Such a T is said to be completely polynomially bounded. Since this book went to press, Pisier has obtained a negative answer to Q3 by constructing a polynomially bounded operator which is not completely polynomially bounded. His proof depends upon a deep multiplier theorem which extends the ideas of Chapter 6.

It is often difficult to check whether a given map is completely bounded. A frequently used tool is Grothendieck's Inequality, which is introduced in Chapter 5, together with some of its variants. One formulation is to say that the only bounded Schur multipliers $M_\varphi : [a_{ij}] \mapsto [\varphi(i,j)a_{ij}]$ of $B(H)$ are given by elements of the projective tensor product $\varphi(i,j) \in \ell_i^\infty \hat{\otimes} \ell_j^\infty$. Pisier obtained a non-commutative version of this result in which ℓ^∞ is replaced by any C^*-algebra A. In Chapter 7 this is used to verify significant special cases of Q2 such as Haagerup's Theorem. This asserts that if $u : A \mapsto B(H)$ has a cyclic vector ξ with $\{u(a)\xi : a \in A\}$ dense in H, then there is an invertible $S \in B(H)$ for which $a \mapsto S^{-1}u(a)S$ is a $*$-representation.

Chapter 8 features a discussion of completely bounded maps in the Banach space setting. The results are in the spirit of the author's earlier work on factorization of linear operators.

Despite the wealth of ideas and results presented in this short book, the text is not hard going and makes a good read. The author does not embark upon proofs requiring heavy harmonic analysis, although several are cited amongst two hundred references. This volume realises the author's objective of being a suitable basis for an advanced graduate course in functional analysis and may be strongly recommended. Several problems in this area remain to be solved.

G. BLOWER

ADAMS, D. R. and HEDBERG, L. I. *Function spaces and potential theory* (Grundlehren der mathematischen Wissenschaften, Band 314, Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong, 1996), xi + 366 pp., 3 540 57060 8, (hardcover) DM148.

A great deal of mediocre mathematics goes under the name of *function spaces*. This book, I am very pleased to say, is not about such mathematics. Indeed, it is an exposition of contemporary potential theory at the highest level, given by two of the foremost presences in the subject. It is in the honourable tradition of Gauss and later Frostman and is much influenced by Carleson [2] and Maz'ya.

The natural language for modern potential theory, as for much of modern analysis, is that of function spaces – principally the Lebesgue and Sobolev spaces and their variants. But the spaces are not the central objects of study in this book – merely the vehicle for the correct formulation of potential theory. (I once examined what was a very good Ph.D. thesis on function spaces; the student had developed a number of highly original and beautiful ideas. As a mere formality I asked him to explain why the Triebel–Lizorkin space $F_0^{p,2}$ coincided with the Lebesgue space L^p. He had no idea. He was awarded the degree.)

The point of view of the book under review is that (as is well-known) in the study of nonlinear problems in PDE it is often the case that the most important information is contained in L^p-based spaces rather than just L^2-based ones. Thus potential theory based upon say $\int |\nabla u|^p dx$ rather than $\int |\nabla u|^2 dx$ is required. This leads to corresponding potential operators which now have the form $\mu \mapsto G_\alpha * (G_\alpha * \mu)^{p'-1}$ (where G_α is, say, a standard Bessel potential kernel) and which are manifestly nonlinear when $p \neq 2$. (Of course, $1/p + 1/p' = 1$.) It is the study of such operators, their corresponding capacities and the function spaces upon which they act that forms the core of this book and as such there is relatively little overlap with other leading books in the field such as [1] or [3]. It turns out that the action on the so-called Besov and Triebel–Lizorkin spaces $B_\alpha^{p,q}$ and $F_\alpha^{p,q}$ is both nontrivial and unexpected, hence the prominence of the function spaces in the title. Indeed, consideration of these spaces is essential even to understand the action on the more classical Sobolev spaces. Work of Frazier and Jawerth and of Netrusov (the latter's influence pervades the entire book) on the atomic approach to $B_\alpha^{p,q}$ and $F_\alpha^{p,q}$ turns out