Series solutions for the profiles of
axisymmetric fluid-liquid interfaces

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When a solid meets the interface between two fluids, the resulting
surface may assume a variety of shapes, depending on certain
characteristics of the solid, the contiguous fluids, and the net body
force, gravity. If the solid is rotationally symmetric and has its axis
vertical, then the resulting meniscal surface is said to be axisymmetric.

Truncated series approximations, describing the profiles and other
properties of axisymmetric menisci, have been in use since the beginning of
the nineteenth century, when the governing differential equations were
first formulated by Young and Laplace. In recent years it has become
popular to produce tabulated solutions of meniscus profiles using numerical
algorithms with computer assistance.

This thesis commences its analytic presentation with the derivation of
recurrence relations for the coefficients of forward and reverse Taylor
series in Cartesian coordinates, valid in the neighbourhood of the vertex
of a sessile or pendant drop. Elementary series analysis techniques are
used to obtain information describing the dominant singularities of the
forward series, facilitating their removal. The modified series have
improved convergence properties. The techniques of stretching and
straining of variables are used in deriving forward and reverse
perturbation series, also in Cartesian coordinates, for small sessile and
pendant drops. Asymptotic approximations for the characteristic dimensions
of the drops are easily obtained from the perturbation series for these

profiles.

A pair of matching perturbation series is obtained in reverse form to describe large external menisci. One of these series is obtained by an iteration method.

The singular perturbation series are parameter expansions, except the series derived by iteration, which must be considered a coordinate expansion. Under certain conditions of distribution, the families of profiles exhibit envelope curves. The equations for three of these envelopes are obtained by exploiting the straining equations.

It is argued that both the systematic development of techniques, and the interrelationships of the ensuing series, are of interest in their own right. Numerically, the series generate accurate profile points, offering an alternative to the use of tabulated profiles within the domains of validity of the series. Furthermore, certain analytical characteristics of the solutions may be ascertained from the serics.

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