April 1883 he returned to Bonchurch, Isle of Wight, where he rapidly grew worse, and died on Saturday, 21st April, surrounded by nearly all his family. He was buried in the New Churchyard, Bonchurch, on 28th April.

Mr Rumble was twice married, and has left a large family to mourn his loss. In business Mr Rumble was straightforward and unerringly honest to his employers, often nervous about small matters, but without fear in cases of grave import, when he was always calm and self-possessed. The rapidity and clearness of his perceptive faculties amounted almost to the gift of second sight, and led him to form swift conclusions which rarely proved false. His firmness in dealing with faults in those under his charge was moderated by great kindness to his men when suffering under any affliction, illness, or distress. He was considered by them always more as a friend than master, and they showed their appreciation of his goodness by presenting him with a testimonial on the celebration of his 50th birthday, 26th December 1882. In private life Mr Rumble's genial spirits, shrewd observations, and witty remarks, endeared him to a large circle of friends. Indeed, his critical condition was almost to the last concealed by his courageous efforts to appear better than he was, and thus relieve the anxiety of his family. He possessed a most retentive memory, and had the faculty of readily assimilating those portions of the books he read which were likely to be useful to him in his professional work. His travels over the greater part of Europe and America naturally enlarged his ideas, and he drew full benefit from the varied experience thus acquired. He had deeply studied the legal as well as the technical points of his profession, and so was particularly well fitted to fill the various appointments he held during his lifetime.

Joseph Liouville. By Professor Chrystal.

Joseph Liouville was born at St Omer on the 24th March 1809. He came of a family of Lorrainers, more than one of whom were distinguished for talents beyond the common. Liouville's father held a public office under the Empire, and an elder brother, Felix Silvestre Jean Baptiste, was a distinguished Parisian advocate. Joseph gave early indications of mathematical ability, and entered upon that stereotyped course of training which has been famous for the nurture of so many Frenchmen of genius. At the age of sixteen he entered the École Polytechnique, and on leaving it in 1827 was classed as an engineer in the Department of Roads and Bridges. After two years he forsook engineering for the cultivation of the higher mathematics. He speedily distinguished himself in his chosen career; for as early as 1829 we find a paper of his ("Démonstration d'un Théorème d'Éléctricité Dynamique") in the *Annales de Chimie et de Physique;* and in 1831 he became a repétiteur, and seven years later a professor, in the Polytechnic School. In the interval he had performed perhaps his greatest service to his favourite science by starting in 1836 *The Journal de Mathématiques Pures et Appliquées.* This journal came most opportunely to fill the gap left by the discontinuance of the *Annales de Gergonne;* but it could scarcely have attained its brilliant success had it not been for the many excellent qualities of its editor, whose critical discernment, that enabled him to enter so readily into the spirit of the works of other mathematicians, and to assist at the *debut* of so many men of distinction,—whose amiability, candour, and freedom from national prejudice,—whose own inexhaustible powers as a contributor of original memoirs, all combined to fit him uniquely for the post which he filled so admirably for nearly forty years.

In 1839 Liouville was elected a member of the Academy of Sciences in succession to Lalande, and the year following he was put upon the Board of Longitude, in whose proceedings he took a lively interest to the end of his life. In 1852 he became a professor in the College de France, and continued to lecture in that capacity until about a year before his death.

If we except his continually recurring successes as a teacher and as an investigator in the most recondite of all the sciences, and the honours accorded to him by the scientific world in token of their appreciation, Liouville's public career was uneventful, as the career of a devoted man of science usually is. On one occasion, however, he departed from the "even tenor of his way." In 1848, a year of much tribulation for France, he received a flattering mark of widely spread popular esteem by being elected a member of the "Constituent Assembly." He promptly answered this call of public duty, and served his alloted time with efficiency if without special distinction. When, however, his mandate expired, instead of seeking re-election, he betook himself once more to the uninterrupted pursuit of the career for which his abilities best fitted him.

Liouville was as fortunate in his private life as he was successful in his public career. He lived to a good old age in the happiest domestic circumstances, until a cruel accident deprived him of his wife. His son, a councillor in the Court of Nancy, died soon afterwards, and the aged mathematician never completely recovered from the effects of this double bereavement. Although his health gave way, his intellect remained unclouded; and it was only in the beginning of 1882 that he gave up his favourite work of lecturing at the College de France. He still continued, however, to attend the meetings of the Academy, but expressed to his friends his consciousness that the end was near. He died on the 8th September 1882, as he himself said, "in his turn"; for, since the death of Chasles, he had been the patriarch among European mathematicians.

Some idea of the extent of Liouville's mathematical writings may be obtained by consulting *The Catalogue of Scientific Memoirs* published by the Royal Society of London. The entries under Liouville's name number 379, and cover some twelve pages. Many of these are merely remarks made on contributions to his journal, or notes appended to works by other mathematicians which he edited; yet, brief as they are, they frequently contain matter of much importance. As specimens of this part of his work, we may mention his "Notes on Two Letters of Mr Thomson relative to the Employment of a New System of Orthogonal Coordinates in certain Problems in the Theories of Heat and Electricity, and in the Problem of the Distribution of Electricity on the Segment of a Spherical Shell of Infinite Thinness" (*Jour. d. Math.*, xii. 1847), in which he draws attention to the analytical and geometrical importance of the method of Inversion, which had just been brought under the notice of mathematicians by the brilliant use that Thomson had made of it in his physical researches.

In another note (*Jour. d. Math.*, xv. 1850) he enunciates the important theorem that the equation

$$dx^2 + dy^2 + dz^2 = \lambda(d\alpha^2 + d\beta^2 + d\gamma^2),$$

where x, y, z, λ are functions of α, β, γ, has for its unique solution the stereographic or inversion transformation.

The following rough analysis will give some idea of the territory covered by his more elaborate memoirs :—

One of the earliest subjects that engaged his attention was Generalised Differentiation ("Différentielles à Indices Quelconques"). The subject is developed at considerable length in five memoirs printed in the 13th and 15th volumes of the *Journal de l'École Polytechnique* (1832–37).

Some of his most important work relates to the Integral Calculus, more particularly that part of it which deals with the theory of elliptic and other transcendental functions.

The earliest memoirs on the subject are two in the *Journal de l'École Polytechnique* (xiv. Cah. 1833, see also *Comptes Rendus*, 1837), "On the Determination of Integrals whose value is Algebraical." He here follows up the researches of Abel on the same subject; and arrives, *inter alia*, at the following important results :—

1. If χ be any rational function of x, then $\int dx \sqrt[m]{\chi}$, if algebraically expressible at all, can be expressed in the form $P\sqrt[m]{\chi}$, P being rational. And, farther, that the integral $\int dx \sqrt[m]{\chi}$ can always be reduced to the form $\theta / \sqrt[m]{T}$, where T is a known rational integral function, and θ a rational integral function whose coefficients have to be determined. This theorem enables us at once to find the value of the integral, if it is algebraically expressible; or else to show that it has no finite algebraical value.

2. If y be an algebraical function of x, *i.e.*, connected with x by means of an equation $F(x, y) = 0$, which is rational and integral in both x and y, then, if the integral $\int y dx$ is expressible explicitly in finite terms by means of algebraic, exponential, or logarithmic functions, it will be expressible in the form

$$\int y dx = t + A \log u + B \log v + \ldots\ldots + C \log w ;$$

where AB . . . . C are constants, and t, u, v, ... w algebraic functions of x.

Among the other memoirs on the present subject may be mentioned the following :—

"On the Elliptic Transcendents of the First and Second Species,

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considered as Functions of their Amplitude." *Jour. Éc. Polytech.*, xiv., 1834, and *Jour. de Math.*, v., 1840.

"On the Integration of a Class of Transcendent Functions." *Jour. de Math.*, xiii., 1835.

"On a New Use of Elliptic Functions in Celestial Mechanics." *Jour. de Math.*, i., 1836.

"On the Classification of Transcendents, and on the Impossibility of Expressing the Roots of certain Equations as a Finite Function of their Coefficients." *Jour. de Math.*, ii., 1837.

"On a very Extensive Class of Quantities whose value is neither Algebraic nor reducible to Algebraic Irrationals." *Jour. de Math.*, xvi., 1851.

Relating to the theory of differential equations, we have the following:—

"On the Equation of Riccati." *Jour. de l'Éc. Polytech.*, xiv., 1833.

"On a Question in the Calculus of Partial Differences." *Jour. de Math.*, i., 1836.

"On the Development of Functions or Parts of Functions in Series, whose various terms satisfy the same Differential Equation of the Second Order containing a Variable Parameter." Three Memoirs. *Jour. de Math.*, i. and ii., 1836–37.

"On the Integration of the Equation $\frac{du}{dt}=\frac{d^3u}{dx^3}$." *Jour. de l'Éc. Polytech.*, xv., 1837.

"On the Theory of Linear Differential Equations, and on the Development of Functions in Series." *Jour. de Math.*, iii., 1838.

"On the Integration of a Class of Differential Equations of the Second Order explicitly Infinite Terms." *Jour. de Math.*, iv., 1839.

Some of Liouville's most important work was in the department of applied mathematics. When, in 1834, Jacobi enunciated his theorem that an ellipsoid with three unequal axes is a possible figure of equilibrium for a mass of rotating fluid, and challenged the French mathematicians to give a proof, Liouville at once published one in the *Journal de l'École Polytechnique*, xiv., 1834. He afterwards returned to the problem, and, in continuation of the work of Meyer on the same subject, showed that Jacobi's form is not possible unless the ratio of the angular momentum to the mass exceeds a certain limit.—*Comptes Rendus*, xvi., 1843; *Jour. de Math.*, 1851.

In the *Journal de Mathématiques* for 1855 we have a farther contribution to this branch of hydrodynamics in the memoir entitled "General Formulæ relating to the question of the Stability of the Equilibrium of a mass of Homogeneous Liquid rotating uniformly about an Axis." The memoir, "On a passage of the *Mécanique Celeste* relating to the Theory of the Figure of the Planets" (*Jour. de Math.*, ii., 1837), in which he points out and corrects an error of Laplace, should also be mentioned.

On dynamics we have three memoirs in vols. xi., xii., and xiv. of the *Journal de Mathématiques*, dealing with certain cases in which the equations of motion of a material point, or of a system of such, can be integrated. The equations are transformed by the substitution of various systems of generalised coordinates (mostly elliptic coordinates), and then the form of the Force Function (Potential) is so specified that integration in finite terms shall be possible. The third of these memoirs, which deals with a system of material particles, is interesting mainly as regards the theory of Abelian integrals. In addition to these there are memoirs, "On a particular case of the Problem of Three Bodies," *Jour. de Math.*, i., 1856; and "On Developments of a chapter in Poisson's *Mécanique*," *Jour. de Math.*, iii., 1858.

Liouville made several contributions to Planetary Theory, among which which we may specially mention his memoir, "On the Secular Variations of the Angles between the straight lines that form the Intersections of the Orbits of Jupiter, Saturn, and Uranus." *Jour. de Math.*, iv., 1839.

In a variety of scattered notes are to be found some very important additions to our knowledge of Theoretical Dynamics. Perhaps the most striking of these is that "On a Remarkable Expression of the Quantity which in the Movement of a System of Material Particles connected in any way is a minimum in virtue of the principle of Least Action." *Jour. de Math.*, 1856. If we take the case of a single free particle, and use Cartesian coordinates, Liouville's result for the form of the integral which expresses the action is—

$$A = \int d\theta \sqrt{\left\{ 1 + \frac{\left(\dot{y}\frac{d\theta}{dz} - \dot{z}\frac{d\theta}{dy}\right)^2 + \left(\dot{z}\frac{d\theta}{dx} - \dot{x}\frac{d\theta}{dz}\right)^2 + \left(\dot{x}\frac{d\theta}{dy} - \dot{y}\frac{d\theta}{dx}\right)^2}{\dot{\theta}^2} \right\}}$$

where θ is a function of xyz, which satisfies the equation

$$\left(\frac{d\theta}{dx}\right)^2 + \left(\frac{d\theta}{dy}\right)^2 + \left(\frac{d\theta}{dz}\right)^2 = 2(E - V).$$

Liouville gives this theorem in terms of generalised coordinates for any system of particles, and points out that it opens up a new method of treatment leading readily to all the known results of Theoretical Dynamics.

During the latter part of his life, Liouville's researches were almost entirely directed to the Theory of Numbers. From 1857 to 1873 we have a list of over 200 notes and memoirs on this subject, all published in his own journal. A few occur with earlier dates, for examples the following :—

"On the equation $Z^{2n} - Y^{2n} = 2X^n$." *Jour. de Math.*, v., 1840.

"On a Theorem of the Indeterminate Analysis." *Comptes Rendus*, x., 1840.

"On the Two Forms $x^2 + y^2 + z^2 + t^2$, $x^2 + 2y^2 + 3z^2 + 6t^2$." *Jour. de Math.*, x., 1845.

The most important of all the memoirs on this subject are the series entitled "On some General Formulæ which may be useful in the Theory of Numbers." *Jour. de Math.*, vols. iii.–viii., New Ser., 1858–1863.

Very few of the longer memoirs are devoted to Pure Geometry; but many interesting and novel geometrical theorems occur incidentally in Liouville's mathematical writings. A full account of these will be found in the third chapter of Chasles' "Report on the Progress of Geometry in France."—*Recueil de Rapports sur l'État des Lettres et les Progrès des Sciences en France*, Paris, 1870.

We may mention here some of the results arrived at in two memoirs, "On certain general Geometrical Propositions, and on the Theory of Elimination in Algebraical Equations (*Jour. de Math.*, vi., 1841), and "Developments of a Geometrical Theorem" (*Jour. de Math.*, 1844). The following results among others are arrived at :—

1. The points of contact of a geometrical surface with all the

tangent planes parallel to a given plane have a fixed centre of mean position whose position is independent of the direction of the given plane.

2. The centre of mean position of the meeting points of two algebraical curves is also the centre of mean position of the meeting points of the asymptotes of one of them with the other or with its asymptotes.

3. If through the points of intersection of a curve and a circle normals to the curve be drawn, these normals intercept on a transversal through the centre of the circle segments measured from the centre, which are such that the sum of their reciprocals is zero.

If the circle be drawn to touch the curve at P, and we take for the transversal the normal at P, this proposition gives us a construction for the centre of curvature at P.

4. Considering all the tangents to a curve parallel to a given line. The centre of mean position of the points of contact is the centre of mean position of the interventions of the asymptotes.

The centres of curvature corresponding to the points of contact have the same centre of mean position as the points of contact themselves; the sum of all the corresponding radii of curvature is zero, and the same is true of the sum of their inverses.

5. Considering all tangent planes to a surface parallel to a given plane, the sum of the principal radii of curvature at the points of contact is zero, and the same is true of the sums of their reciprocals.

The work of the scientific teacher is scarcely less important than that of the scientific investigator, although the record of the former is more perishable, being at best an oral tradition handed over by the immediate disciples of the master. It would appear that in this walk Liouville was worthy to rank with his illustrious predecessor Monge, whose pupils shed such lustre on the French school of mathematicians. M. Faye, in his funeral oration, says, "M. Liouville was one of the most brilliant professors that ever lectured. So lively was my youthful impression of his lectures that to this day I have a vivid recollection of the captivating clearness that was so peculiarly his own. Accordingly, when in later years I had the good fortune to hear him speak at the Institute, I was the less surprised at the effect which his words produced on my colleagues, who marvelled at being able, for a moment, under his guidance, to penetrate the most difficult questions of the higher analysis. No one, with the exception perhaps of Arago, ever produced this effect in the same degree."

His lectures at the College de France were attended by the *élite* of French mathematicians, and doubtless did much to keep alive the ardent spirit of pure mathematical research which still lives among his countrymen. Among those who either were his pupils or were indebted to his encouragement and patronage may be reckoned Le Verrier, Hermite, Bertrand, Serret, Bour, Bonnet, Mannheim, all of whom are or have been pillars of French science.

If we compare Liouville as an investigator with other great contemporaries whose rolls of achievement like his own are already closed, we can scarcely put him in the highest rank of all, along with Abel and Jacobi, whose fortune it was in the course of their discoveries to open up new fields of research and create new branches of the analytic art. Nevertheless, so profound are some of his isolated contributions, and so elegant is all his mathematical writing, that it will be long before the traces of his handiwork vanish from the fabric of mathematical science; and it seems certain that future generations will accord him all but the highest rank in the temple of mathematical fame.

Robert Wilson. By Professor Fleeming Jenkin, F.R.SS. L. and E.

Mr Robert Wilson was born in 1803 at Dunbar. In 1810 he lost his father, who was connected with the royal and mercantile navies. This brave man, after having twice reached the wreck of the "Pallas" frigate in the Dunbar life-boat, was drowned in the third attempt to reach the ship and rescue the remainder.

Mr Robert Wilson was apprenticed to a joiner, and, like many other distinguished Scotchmen of the same generation, he owed his high standing as a mechanical engineer almost entirely to his natural genius, since he does not appear to have received any special advantages in respect of education.

During his apprenticeship, and at a date considerably prior to the successful introduction of the screw propeller into our navy, he

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