B. EXPERIMENTS

PAPER 6

MAGNETO-HYDRODYNAMIC EXPERIMENTS

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ABSTRACT

Comparisons are made between magneto-hydrodynamics on cosmical and on laboratory scale. Magneto-hydrodynamic waves, turbulence, the generation of magnetic fields and thermal convection are discussed and a review is given of earlier experimental investigations. The possibilities are examined of realizing cosmical phenomena of this type in the laboratory.

1. INTRODUCTION

It often happens that astrophysical problems have a degree of complication high enough to hamper a rigorous theoretical approach. An imaginable way to obtain a solution is provided by the method of 'scaling down' the astrophysical configuration to laboratory dimensions in proportions such as to conserve its main properties. Thus, besides forming valuable tests of theory, magneto-hydrodynamic experiments may serve the purpose of solving astrophysical problems directly in the laboratory.

The significance of such model experiments depends upon the possibility of realizing cosmical conditions in the laboratory. This question can be answered by means of similarity laws stated for sets of configurations of different linear dimensions and equal physical character. It should be stressed that the form of the laws depends upon the physical properties which are stated to be conserved in a transformation from cosmical to laboratory dimensions. As a matter of fact, similarity laws for gaseous discharges, where the potential difference per mean free path is kept constant, are inapplicable to magneto-hydrodynamic waves (Alfvén [1]). In an accompanying paper Dr Block will discuss the conditions for model experiments on gaseous discharges in a magnetic field and apply the results to the auroral phenomenon.

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2. CONDITIONS ON COSMICAL AND ON LABORATORY SCALE

The similarity laws of magneto-hydrodynamics are now formulated without inclusion of the Coriolis force. Relativistic effects and displacement- and convection currents are assumed to be negligible and the permeability to be nearly equal to its value *in vacuo*. The basic equations are

$$\frac{\partial \mathbf{B}}{\partial t} + \lambda \operatorname{curl}^2 \mathbf{B} = \operatorname{curl}(\mathbf{v} \times \mathbf{B}); \quad \lambda = 1/\mu\sigma, \tag{1}^*$$

$$\operatorname{div}(\rho \mathbf{v}) + \frac{\partial \rho}{\partial t} = 0, \tag{2}$$

$$\rho \frac{d\mathbf{v}}{dt} = \frac{1}{\mu}(\operatorname{curl} \mathbf{B}) \times \mathbf{B} - \nabla p - \rho \nabla \phi + \eta \nabla^2 \mathbf{v} + \tfrac{1}{3}\eta \nabla (\operatorname{div} \mathbf{v}), \tag{3}$$

$$\begin{aligned}\rho \frac{d}{dt}(e + \tfrac{1}{2}\mathbf{v}^2) = {} & -p \operatorname{div} \mathbf{v} + \mathbf{v}\,.\,[-\nabla p + \eta \nabla^2 \mathbf{v} + \tfrac{1}{3}\eta \nabla(\operatorname{div} \mathbf{v})] \\ & + \eta(\operatorname{curl} \mathbf{v})^2 + \tfrac{4}{3}\eta(\operatorname{div} \mathbf{v})^2 \\ & + 4\eta\left[\frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}\frac{\partial w}{\partial z} - \frac{\partial w}{\partial z}\frac{\partial u}{\partial x}\right] + W \end{aligned} \tag{4}$$

and

$$f(p, \rho, T) = 0, \tag{5}$$

where **B** is the magnetic field strength, σ the electric conductivity, ρ the density, p the pressure, ϕ the gravitation potential, η the viscosity, $e = e(\rho, T)$ the internal (thermal) energy per unit mass, $\mathbf{v} = (u, v, w)$ the velocity, and T the temperature. Equation (4) expresses the conservation of energy. The left-hand member is the change of internal and kinetic energy per unit time in a co-ordinate system which follows the motion of a fluid element. This is supplied by the right-hand member which is the sum of the rate at which the hydrodynamic stresses are doing work on an element of the fluid (Lamb [2]) and the rate W at which energy is supplied from other sources than these stresses. Eq. (5) is the equation of state. We have

$$W = \mathbf{i}^2/\sigma + \mathbf{v}\,.\,(\mathbf{i} \times \mathbf{B}) - \rho \mathbf{v}\,.\,\nabla \phi + \theta \nabla^2 T, \tag{6}$$

where $\mathbf{i}$ represents the current density and θ the thermal conductivity. The two first terms in Eq. (6) are the power $\mathbf{E}\,.\,\mathbf{i} = (\mathbf{i}/\sigma - \mathbf{v} \times \mathbf{B})\,.\,\mathbf{i}$ supplied by the electromagnetic field, the third is the work done per unit time by the gravitation field and the fourth is the heat supplied through thermal conduction.

We assume that the internal energy has the form

$$e = c_v T, \tag{7}$$

* In this paper Ohm's law is assumed to have the simple form $\mathbf{i} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

with c_v as the specific heat at constant volume. Eq. (7) holds for polytropic gases and with good approximation for experiments with liquid metals. We also introduce dimensionless variables according to the following transformations:

$$x_\alpha = L_c x'_\alpha; \quad t = t_c t'; \quad L_c/t_c = V_c, \tag{8}$$

$$B_\alpha = B_c B'_\alpha; \quad B_c(\mu\rho_c)^{-\frac{1}{2}} = V; \quad v_\alpha = V_c v'_\alpha \tag{9}$$

and

$$p = p_c p'; \quad \phi = \phi_c \phi'; \quad T = T_c T', \tag{10}$$

the indices (α), (c) and ($'$) implying an arbitrary component, characteristic quantities and dimensionless quantities, respectively. Further, expressions (6) and (7) and the conservation theorems, (2) and (3), of mass and momentum are introduced into the energy theorem (4) (compare Lehnert [3]). After introduction of transformations (8), (9) and (10) the basic equations (1), (2), (3) and (4) are easily shown to become

$$\frac{\partial \mathbf{B}'}{\partial t'} + \frac{\lambda}{V_c L_c} \operatorname{curl}' \mathbf{B}' = \operatorname{curl}' (\mathbf{v}' \times \mathbf{B}'), \tag{11}$$

$$\operatorname{div}' (\rho' \mathbf{v}') + \frac{\partial \rho'}{\partial t'} = 0, \tag{12}$$

$$\frac{d\mathbf{v}'}{dt'} = -\frac{p_c}{\rho_c V_c^2} \frac{1}{\rho'} \nabla' p' + \frac{V^2}{V_c^2} \frac{1}{\rho'} (\operatorname{curl}' \mathbf{B}') \times \mathbf{B}'$$
$$-\frac{\phi_c}{V_c^2} \nabla' \phi' + \frac{\nu}{V_c L_c} \frac{1}{\rho'} [\nabla'^2 \mathbf{v}' + \tfrac{1}{3} \nabla' (\operatorname{div}' \mathbf{v}')] \tag{13}$$

and

$$\frac{dT'}{dt'} = \frac{p_c}{\rho_c V_c^2} \frac{V_c^2}{c_v T_c} \frac{p'}{\rho'^2} \frac{d\rho'}{dt'} + \frac{\lambda}{V_c L_c} \frac{V^2}{V_c^2} \frac{V_c^2}{c_v T_c} \frac{1}{\rho'} (\operatorname{curl}' \mathbf{B}')^2$$
$$+ \frac{\nu}{V_c L_c} \frac{V_c^2}{c_v T_c} \frac{1}{\rho'} \Big\{ (\operatorname{curl}' \mathbf{v}')^2 + \tfrac{4}{3} (\operatorname{div}' \mathbf{v}')^2$$
$$+ 4 \left[\frac{\partial w'}{\partial y'} \frac{\partial v'}{\partial z'} + \frac{\partial u'}{\partial z'} \frac{\partial w'}{\partial x'} + \frac{\partial v'}{\partial x'} \frac{\partial u'}{\partial y'} - \frac{\partial u'}{\partial x'} \frac{\partial v'}{\partial y'} - \frac{\partial v'}{\partial y'} \frac{\partial w'}{\partial z'} - \frac{\partial w'}{\partial z'} \frac{\partial u'}{\partial x'} \right] \Big\}$$
$$+ \frac{\kappa}{V_c L_c} \frac{1}{\rho'} \nabla'^2 T'. \tag{14}$$

$\nu = \eta/\rho_c$ is the kinematic viscosity and $\kappa = \theta/\rho_c c_v$ the thermometric conductivity.* The basic equations contain the characteristic parameters V_c/V, $V_c L_c/\lambda$, $V_c L_c/\nu$, $V_c L_c/\kappa$, $V_c^2/c_v T_c$, $p_c/\rho_c V_c^2$ and ϕ_c/V_c^2. If the dimensionless variables x'_α, t', …, T' are given and these parameters are kept constant,

* After the completion of this manuscript the author has been informed that the generalization of the energy theorem to a form including viscous dissipation and heat diffusion has already been given by Dr Baños in the paper presented at this symposium.

a set of similar configurations of different linear dimensions can be constructed. The ratio between forces of different types is invariant at corresponding points within such a set.

Consider a strongly pronounced magneto-hydrodynamic phenomenon where the magnetic energy is of the same order of magnitude as the kinetic energy, i.e. $V^2 \approx V_c^2$. Dissipation of energy due to Joule heat, viscosity and heat conduction will be negligible if

$$B_c L_c/\lambda(\mu\rho_c)^{\frac{1}{2}} \gg 1, \quad (15)$$

$$B_c L_c/\nu(\mu\rho_c)^{\frac{1}{2}} \gg 1 \quad (16)$$

and

$$B_c L_c/\kappa(\mu\rho_c)^{\frac{1}{2}} \gg 1, \quad (17)$$

which relations are easily deduced from the forms of the characteristic parameters. Condition (15) implies that the magnetic flux is almost 'frozen' in the fluid (Lundquist [4]). Condition (16) corresponds to the same statement for vortex lines and condition (17) corresponds to a negligible 'slip' between matter and isothermal surfaces being caused by thermal conduction. When the compression work is negligible the isothermal surfaces will also be 'frozen' in the fluid.

Some examples are given in Table 1, where parts of the data have been taken from works by Lyon [5], Jackson [6], Elsasser [7, 8], Alfvén [1], Kuiper [9, 10] Chandrasekhar [11] and Allen [12]. The considerable increase of the effective thermometric conductivity caused by radiation at high temperatures has been taken into account. From Eq. (15) and Table 1 is seen that Joule dissipation plays an important role in magneto-hydrodynamic experiments. An exact similarity transformation of cosmical configurations to the laboratory is rendered impossible, even with the best available conductors such as liquid sodium, sodium-potassium alloy and ionized gases. For the characteristic linear dimensions given in Table 1 viscous dissipation is seen to be of minor importance in experiments and in most cosmical applications. This does not mean that viscous dissipation is negligible under all circumstances, which is readily seen from experiments on flow in ducts in transverse magnetic fields. In such cases a flow exists in narrow boundary layers with considerable viscous forces and the magnetic and kinetic energies are not necessarily of the same order of magnitude.

3. MAGNETO-HYDRODYNAMIC WAVES

The question now arises how significant is the departure of laboratory conditions from the real astrophysical situation. This may be discussed by means of a concrete, simple example. Consider a conducting incompressible

Table 1. *Some characteristic data for magneto-hydrodynamic experiments and for cosmical applications of magneto-hydrodynamics*

	Temperature T_c (° K)	Magnetic field B_c (Vs/m²)	Linear dimension L_c (m)	Density ρ_c (kg/m³)	Electro-magnetic viscosity† λ (m²/s)	Kinematic viscosity† ν (m²/s)	Thermo-metric conductivity† κ (m²/s)	$\frac{B_c L_c}{\lambda \sqrt{(\mu\rho_c)}}$‡	$\frac{B_c L_c}{\nu \sqrt{(\mu\rho_c)}}$‡	$\frac{B_c L_c}{\kappa \sqrt{(\mu\rho_c)}}$‡
				Experiments						
Mercury	293	1	0·1	1350	0·780	$1·14\times 10^{-7}$	$4·43\times 10^{-6}$	1·0	$6·8\times 10^{6}$	$1·6\times 10^{5}$
Sodium	373	1	0·1	928	0·0765	$6·31\times 10^{-7}$	$67·3\times 10^{-6}$	38	$4·6\times 10^{6}$	$4·2\times 10^{4}$
Gallium	308	1	0·1	6093	0·216	$3·11\times 10^{-7}$	$14·0\times 10^{-6}$	5·3	$3·7\times 10^{6}$	$7·4\times 10^{4}$
Tin	513	1	0·1	6910	0·378	$2·76\times 10^{-7}$	$20·0\times 10^{-6}$	2·8	$3·8\times 10^{6}$	$5·3\times 10^{4}$
Sodium-potassium (22 % Na, 78 % K)*	293	1	0·1	905	0·270	$7·86\times 10^{-7}$	$24·2\times 10^{-6}$	11·0	$3·8\times 10^{6}$	$1·2\times 10^{5}$
Ionized gas hydrogen	10^{5}	0·1	0·1	10^{-7}	15	10	10	2000	4000	4000
				Cosmical applications						
Earth's interior	10^{4}	10^{-3}?	2×10^{6}	10^{4}	1	10^{-6}?	2×10^{-5}	2×10^{4}	2×10^{10}	10^{9}
Sunspots	4×10^{3}	0·2	10^{7}	0·1	20	10^{-2}‖	10^{9}¶	10^{9}	2×10^{11}	10
Solar granulation	6×10^{3}	10^{-2}	10^{6}	10^{-4}	100§	10‖	10^{9}¶	10^{7}	10^{8}	1
Magnetic variable stars	10^{6}	1	10^{10}	10^{3}?	1	3×10^{-7}‖	10^{10}?¶	3×10^{11}	10^{18}	30
Interstellar space; more condensed regions	10^{4}	10^{-9}?	10^{20}	10^{-21}?	10^{3}?§	10^{17}‖	?	3×10^{21}	3×10^{7}	?
Interplanetary space	10^{5}	10^{-9}?	10^{13}	10^{-20}	10?§	10^{16}‖	?	10^{16}	10	?
Corona	10^{6}	10^{-4}?	10^{9}	10^{-15}	1?§	10^{13}‖	?	3×10^{15}	300	?

* Experiments with sodium-potassium alloy are being planned by Lochte-Holtgreven [13].

† The electromagnetic viscosity is defined by $\lambda = 1/\mu\sigma$, where σ is the electrical conductivity, the kinematic viscosity by $\nu = \eta/\rho_c$, where η is the coefficient of viscosity, and the thermometric conductivity by $\kappa = \theta/\rho_c c_v$, where θ is the coefficient of thermal conductivity and c_v the specific heat at constant volume.

‡ If the parameters of the third and second columns from the right are much greater than unity conditions become favourable for a strongly developed magneto-hydrodynamic phenomenon. In such a case there is almost no 'slip between matter and magnetic field lines', as well as between matter and vorticity lines, respectively. If, in addition, the parameter of the first column from the right is much greater than unity there is almost no 'slip' between matter and isothermal surfaces, provided that the compression work can be neglected.

§ Values given by the resistivity (the inverted 'parallel conductivity') have been chosen.

‖ Radiative viscosity is not included but may increase ν by a factor of about 100 (Cowling [9], p. 555). Turbulent viscosity is not included in ν.

¶ Effective value of the thermometric conductivity with inclusion of radiative exchange of heat. Figures give the order of magnitude. The conduction part is affected by the magnetic field.

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liquid between two parallel, infinitely conducting planes in a perpendicular, homogeneous external magnetic field $\mathbf{B}_0$ (Fig. 1). We assume a perturbation field $\mathbf{b} = (b, 0, 0)$ to exist as given by Fig. 1. From the basic equations is easily deduced the special solution

$$b(z, t) = \{b_1 \exp[j(\lambda - \nu) k^2(\zeta^2 - 1)^{\frac{1}{2}} t] + b_2 \exp[-j(\lambda - \nu) k^2(\zeta^2 - 1)^{\frac{1}{2}} t]\} \exp[-(\lambda + \nu) k^2 t] \sin kz, \quad (18)$$

with $k = 2\pi/L$, $\lambda > \nu$, $\zeta = 2B_0/(\mu\rho)^{\frac{1}{2}} k^2(\lambda - \nu)$ and $j = (-1)^{\frac{1}{2}}$.

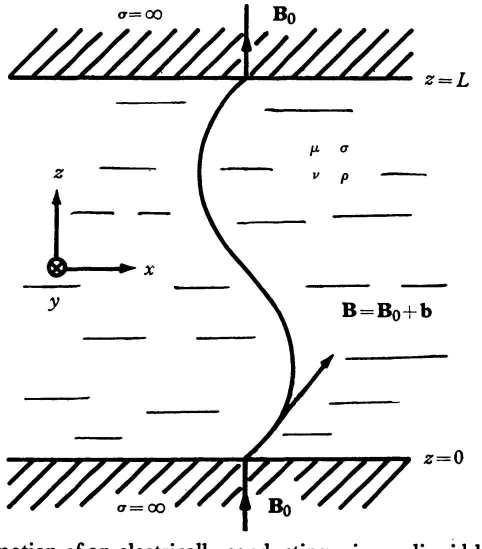

Fig. 1. Plane state of motion of an electrically conducting, viscous liquid between two infinitely conducting planes in a homogeneous, perpendicular magnetic field, $\mathbf{B}_0$.

The initial perturbation field is $b(z, 0) = (b_1 + b_2) \sin kz$. The magnetic field lines will perform damped oscillations if $\zeta > 1$, whereas the motion will be aperiodic if $\zeta \leqslant 1$ (Lehnert [14]). The critical aperiodic case, $\zeta = 1$, is given by

$$B_0 L/\lambda(\mu\rho)^{\frac{1}{2}} = \pi, \quad (19)$$

where we have put $\lambda \gg \nu$, which is a good approximation in most cases according to Table 1. The third column from the right in the same table shows that magneto-hydrodynamic waves exist in the laboratory only in the strongest available magnetic fields. A rough approach to the astrophysical situation of virtually undamped waves, i.e. waves where the damping length at least exceeds 10 wave-lengths, say, requires magnetic field strengths of 2 Vs/m² (= 20,000 gauss) within dimensions of 0·25 m for liquid sodium and of 1 m for sodium-potassium alloy.

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Investigations on forced magneto-hydrodynamic oscillations in mercury and in liquid sodium have already been carried out by Lundquist [15, 16] and Lehnert [17]. Torsional oscillations were fed into the lower end of a cylindrical column of liquid with a magnetic field directed along the axis of symmetry. The amplitude ratio and phase difference between the lower end and the upper free surface were studied as functions of frequency and magnetic field strength. In the case of high conductivity a strong resonance peak and a phase shift of 180° would be expected to occur at the free surface. The peak occurs when the length of the column corresponds to a quarter of a wave-length of transverse magneto-hydrodynamic waves. However, only a very flat maximum arises in the experiment with mercury; this is not a resonance phenomenon but merely a dispersion effect. The data of the mercury experiment correspond to the aperiodic range of free motion. Thus, the forced oscillations are 'degenerate waves', somewhat similar to the temperature 'wave' in a problem of heat conduction with periodically varying temperature at the boundary. In the liquid sodium experiment there occurs only a slightly developed resonance phenomenon, showing that Joule dissipation is of major importance even in this case. Consequently, both these experiments have to be regarded merely as verifications of theory and not as model experiments on cosmical physics.

An improvement can possibly be obtained with an ionized gas. Bostick and Levine [18, 19, 20] have observed oscillations in a toroidal plasma embedded in an annular stationary magnetic field. The oscillations are interpreted as standing magneto-hydrodynamic waves. An increase of the magnetic field strength gives an increasing number of oscillating modes. This could be explained by the passage of an increasing number of higher modes from the aperiodic range into the range of wave motion, provided that the damping is small enough. However, it should be pointed out that the configuration of this experiment is bounded by rigid walls; the oscillations do not necessarily consist purely of transverse Alfvén waves and the dissipation is mainly determined by the narrow cross section of the tube.

4. TURBULENCE

Turbulence is likely to be a common phenomenon in masses of cosmic dimensions. It is seen from Table 1 that a large part of the turbulent spectrum of wave-lengths will be situated in the periodic range of motion in most cosmical applications. In the simple case of Fig. 1 a turbulent field of cosmical dimensions will consist of a spectrum of waves, the decay time of which is not affected by an external magnetic field $\mathbf{B}_0$ (Eq. (18)). In the laboratory, on the other hand, the spectrum falls entirely within the aperiodic range of motion and a strong suppression of turbulence is caused by the external field, as found experimentally by Hartmann and Lazarus [21, 22].

In spite of this difference between cosmical and laboratory conditions experimental investigations may serve the purpose of deepening the understanding of such a complicated phenomenon as magneto-turbulence. The experiments by Hartmann and Lazarus show that the onset of turbulence in a channel is delayed by a transverse magnetic field. A plot of the critical velocity against the corresponding field strength gives a linear relationship (Lehnert [23]). This is interpreted theoretically in terms of the characteristic parameters mentioned earlier (Lehnert [23], Lundquist [4], Lock [24]). Murgatroyd [25, 26] has previously extended Hartmann's measurements to a wider range of data.

Experiments on magneto-hydrodynamic channel flow of liquid metals are of great technical importance. Further contributions in this field have been given by Kolin [27, 28], Arnold [29], Murgatroyd [30], Shercliff [31, 32, 33, 34, 35], Greenhill [36, 37], Robin [38], and Barnes [39].

5. GENERATION OF MAGNETIC FIELDS

The velocity field is capable of stretching and twisting the magnetic field lines in a fluid of high conductivity. The process, which may take place in the form of turbulence or as a regular macroscopic phenomenon, provides a mechanism for converting kinetic energy into magnetic. For the generation of the magnetic fields of the stars and of the earth a feed-back mechanism has been suggested in the form of a magneto-hydrodynamic dynamo. The possibility of making an experiment on such a dynamo may be discussed with the help of the simple configuration of Fig. 2 (Bullard and Gellman [40]). The condition for the dynamo to be self-excited becomes $v \approx 40\lambda/a$, where $v = \omega a$ is the velocity at the periphery of the rotating disc, a is the radius of the dynamo and the resistance has been put equal to about $10a/\sigma a^2$. For a volume of 100 litres of liquid sodium $a = 0{\cdot}3$ m and $v = 10$ m/sec. Besides being rigid and multiply connected this dynamo has the important feature of being asymmetric. It has been shown that a complete symmetry gives no feed-back of a singly connected fluid dynamo and that the feed-back is caused by asymmetric modes. Consequently, the fluid dynamo will be much less efficient than the simple dynamo in Fig. 2. Velocities considerably higher than 10 m/sec will probably be needed for liquid sodium in the example discussed and the experiment will be difficult to carry out.

A strong twisting of a magnetic field gives rise to an instability which may serve as a mechanism for an increase of magnetic energy (Alfvén [41], Lundquist [42]). The onset of the instability may possibly be studied in an experiment with liquid sodium. The field will be twisted to a high degree within a vessel of the size of 0·3 m if the differential velocity highly exceeds the diffusion velocity of the field lines, which is about 0·5 m/sec.

Another mechanism for the generation of magnetic fields is provided by the diffusion of charged particles in a turbulent field. This causes a separation of charge and gives rise to electric currents and magnetic fields (Biermann [43], Schlüter [44]). An experimental confirmation of the theory has been given with rotating flames and with mercury by Lochte-Holtgreven and Schilling [45, 46, 47] and by Burhorn, Griem and Lochte-Holtgreven [48, 49, 50].

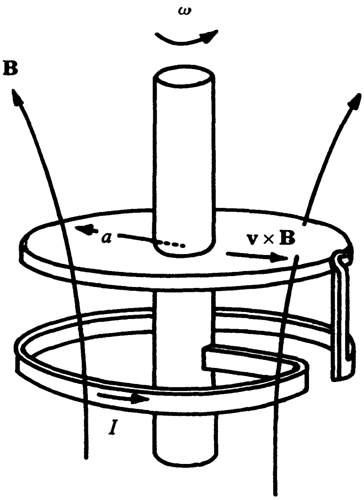

Fig. 2. A simple dynamo (Bullard and Gellman [40]).

6. INSTABILITY AND THERMAL CONVECTION

A complete description of the great variety of different types of magneto-hydrodynamic instability is not within the scope of the present paper. We may only mention an interesting type of torus-shaped disturbance in a plasma, which will be described in an accompanying paper by Dr Bostick (see also Bostick [51]). Some additional experiments on instability are given in the references of this paper (Colgate [52], Lehnert [53, 54]).

The following discussion will be limited to the onset of thermal convection in a magnetic field. In the laboratory cellular convection will be set up in a layer of fluid which is heated from below with a sufficiently large vertical temperature gradient. Table 1 shows that a cell size of 1 cm in mercury will correspond to the aperiodic range of magneto-hydrodynamic motion. Since $\kappa < \lambda$ the diffusion of isothermal surfaces occurs slowly as compared to the diffusion of magnetic field lines. The dissipation is increased considerably by the Joule heat and the onset of thermal convection is a sensitive function of the magnetic field strength as shown theoretically by Thompson [55] and by Chandrasekhar [11]. The theory agrees with experiments in mercury performed by Nakagawa [56], Jirlow [57] and Lehnert and Little [58].

However, thermal convection under astrophysical conditions differs fundamentally from that of the experiments. Radiative exchange of heat increases the effective thermometric conductivity and $\kappa \gg \lambda \gg \nu$. Dissipation due to Joule heat and gas viscosity is relatively small. A large range of wave-lengths of convective motion will correspond to the periodic case of virtually undamped magneto-hydrodynamic waves. For these wave-lengths the magnetic field lines are 'frozen' in matter, whereas the isothermal surfaces diffuse rapidly through it. Variations of the strength of an external magnetic field do not influence dissipation in such a case and the onset of convection is not critical with respect to the field strength, as shown by Chandrasekhar [11]. For wave-lengths in the periodic range the onset of convection occurs as over-stability, i.e. in the form of oscillations. This may be understood by the fact that the field lines are 'frozen' in matter and cannot be twisted indefinitely by a stationary cellular pattern.

7. CONCLUSIONS

The examples given in this paper clearly show the difficulties involved in making a reproduction of cosmical phenomena in the laboratory. Further progress in the problem of magneto-hydrodynamic model experiments can only be made with the largest achievable magnets or with ionized gases.

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