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Left-orderability and Exceptional Dehn Surgery on Twist Knots

Masakazu Teragaito

Abstract. We show that any exceptional non-trivial Dehn surgery on a twist knot, except the trefoil, yields a 3-manifold whose fundamental group is left-orderable. This is a generalization of a result of Clay, Lidman, and Watson, and also gives a new supporting evidence for a conjecture of Boyer, Gordon, and Watson.

1 Introduction

A group is *left-orderable* if it admits a strict total ordering that is invariant under left-multiplication. It is well known that any knot group or link group is left-orderable (see [3]). More generally, many classes of 3-manifolds are known to have left-orderable fundamental groups.

Boyer, Gordon, and Watson [2] conjecture that an irreducible rational homology 3-sphere is an L-space if and only if its fundamental group is not left-orderable. Here, an L-space, introduced by Ozsváth and Szabó [12], is a rational homology sphere M whose Heegaard–Floer homology $\widehat{HF}(M)$ is a free abelian group of rank equal to $|H_1(M)|$. This conjecture is verified for Seifert fibered manifolds, Sol manifolds, and double branched covers of non-split alternating links in [2, 8, 9].

On the other hand, if a knot admits Dehn surgery yielding an L-space, referred to as an *L-space surgery*, then there are some constraints for the knot. For example, its Alexander polynomial has a specified form [12], and such a knot must be fibered [11]. Therefore, it is not going too far to say that most knots do not admit an L-space surgery. Thus we can expect that any non-trivial Dehn surgery on a hyperbolic knot, which does not admit an L-space surgery, yields a 3-manifold whose fundamental group is left-orderable.

In this direction, Boyer, Gordon, and Watson [2] show that if K is the figure-eight knot and $-4 < r < 4$, then r-surgery on K yields a 3-manifold with left-orderable fundamental group. Furthermore, Clay, Lidman, and Watson [6] show that this also holds for $r = \pm 4$. (Note that the figure-eight knot does not admit an L-space surgery.)

In this paper, we will examine the m-twist knot in the 3-sphere, illustrated in Figure 1. We adopt the convention that the horizontal twists are right-handed if m is positive, left-handed if m is negative. Thus, the 1-twist knot is the figure-eight knot,

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and the (-1)-twist knot is the right-handed trefoil. Also, if $|m| \geq 2$, then the m-twist knot is hyperbolic and non-fibered (see [5]).

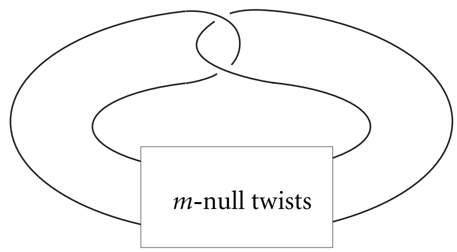

Figure 1

The purpose of this paper is to verify that 4-surgery on the m-twist knot with $|m| \geq 2$ yields a graph manifold whose fundamental group is left-orderable. Since such a twist knot is non-fibered, it does not admit an L-space surgery. (This fact also follows from the form of its Alexander polynomial.) Thus the following theorem provides new supporting evidence for the conjecture of Boyer, Gordon, and Watson mentioned above.

Theorem 1.1 *Let K be the m-twist knot with $|m| \geq 2$. Then 4-surgery on K yields a graph manifold whose fundamental group is left-orderable.*

Our argument follows that of Clay, Lidman, and Watson [6, Section 4] for the case of 4-surgery of the figure-eight knot. They make use of the Dubrovina–Dubrovin ordering for the braid group B_3 of order 3, which is isomorphic to the knot group of the trefoil, but we need some left-orderings for torus knot groups defined by Navas [10].

By combining with known results, we can immediately prove the following corollary.

Corollary 1.2 *Let K be a hyperbolic twist knot. Then any exceptional non-trivial Dehn surgery on K yields a 3-manifold whose fundamental group is left-orderable.*

2 Fundamental Group

Let K be the m-twist knot. We can assume that $m \neq 0, -1$. It is well known that 4-surgery on K yields a toroidal manifold. In fact, the manifold is a graph manifold. In this section, we will examine the structure of the manifold by using the Montesinos trick and get a presentation of its fundamental group.

As shown in Figure 2, put K in a symmetric position. By taking the quotient under the involution map, we have a tangle description of the knot exterior. This means that the double branched cover of the (outside) ball branched over the two strings recovers the knot exterior.

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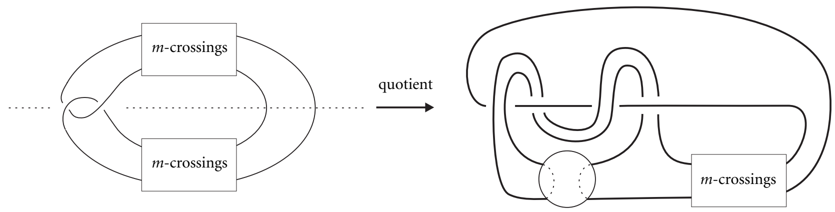

Figure 2

If we fill the ∞-tangle, as indicated by dotted lines in Figure 2, into the inner ball, then it gives an unknot. Here, we choose the framing so that the 0-tangle filling corresponds to 4-surgery. Figure 3 shows the 0-tangle filling yields a link with a trivial component. Let S be the 2-sphere illustrated there that gives an essential tangle decomposition.

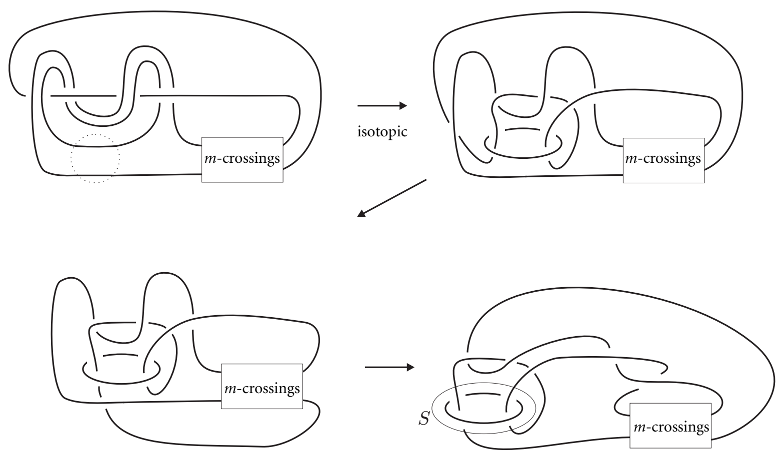

Figure 3

One side of S is the Montesinos tangle $M(-1/2, -m/(2m+1))$, and the other side is the Montesinos tangle $M(-1/2, 1/2)$. Then the double branched cover M_1 of $M(-1/2, -m/(2m+1))$ is the exterior of the torus knot of type $(2, 2m+1)$, and the double cover M_2 of $M(-1/2, 1/2)$ is the twisted I-bundle over the Klein bottle. Thus the resulting manifold M of 4-surgery on K is $M_1 \cup M_2$.

We have $\pi_1(M_1) = \langle a, b : a^2 = b^{2m+1} \rangle$. See Figure 4. The meridian μ is $b^{-m}a$, and the regular fiber h with respect to a (unique) Seifert fibration is a^2 $(= b^{2m+1})$.

It is well known that M_2 admits two Seifert fibrations. One is a fibration over the disk with two exceptional fibers of index 2, and the other is that over the Möbius band with no exceptional fiber. Then we can choose the generators $\{x, y\}$ of $\pi_1(M_2)$

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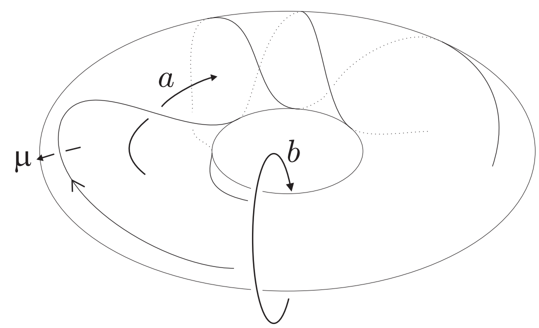

Figure 4

so that x corresponds to an exceptional fiber in the first fibration, and y corresponds to a regular fiber in the second fibration. Thus we obtain that

$$\pi_1(M_2) = \langle x, y : x^{-1}yx = y^{-1} \rangle,$$

and that $\pi_1(\partial M_2)$ is generated by x^2 and y.

To get a presentation of $\pi_1(M)$, we have to examine the identification between ∂M_1 and ∂M_2.

Lemma 2.1 *Under the identification between ∂M_1 and ∂M_2, μ and h on ∂M_1 correspond to y^{-1} and $y^{-1}x^2$ on ∂M_2, respectively.*

Proof Consider two loops z and w on the boundary of the Montesinos tangle $M(-1/2, 1/2)$ as illustrated in Figure 5. Then z and w lift to two copies of x^2 and y, respectively (see [5, Chapter 12]).

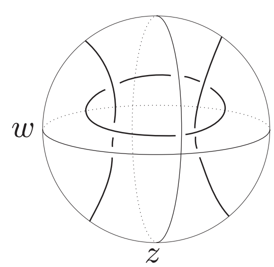

Figure 5

Next, we replace the Montesinos tangle $M(-1/2, 1/2)$ with the 0-tangle as shown in Figure 6, where we insert a narrow band f to chase z. The result is a trivial knot. By taking the double branched cover along this trivial knot, the band f lifts to a knotted annulus whose core forms the torus knot of type $(2, 2m + 1)$. (This proves that the

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double branched cover of $M(-1/2, -m/(2m+1))$ is the exterior of the torus knot of type $(2, 2m+1)$.) Also, the framing determined by f has slope $(4m+1)/1$. Recall that h has slope $(4m+2)/1$. Hence we can choose the orientations of x and y so that y^{-1} and x^2 correspond to the meridian μ and $\mu^{-1}h$, respectively. ∎

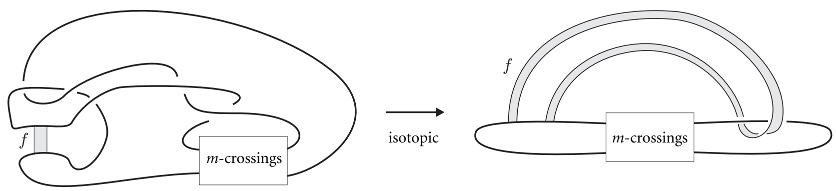

Figure 6

Thus we have shown the following proposition.

Proposition 2.2 *Let K be the m-twist knot with $m \neq 0, -1$. Then 4-surgery on K yields a graph manifold M that is the union of the twisted I-bundle over the Klein bottle and the knot exterior of torus knot of type $(2, 2m+1)$. Furthermore, its fundamental group has a presentation*

$$\pi_1(M) = \left\langle a, b, x, y : a^2 = b^{2m+1}, x^{-1}yx = y^{-1}, \mu = y^{-1}, h = y^{-1}x^2 \right\rangle,$$

where $\mu = b^{-m}a$ and h correspond to a meridian and a regular fiber of the torus knot exterior (with the Seifert fibration), respectively.

Remark 2.3 Our presentation in Proposition 2.2 is equivalent to that of [6] for the case $m = 1$.

3 Normal Families of Left-orderings

Let G be a left-orderable non-trivial group. This means that G admits a strict total ordering $<$ such that $a < b$ implies $ga < gb$ for any $g \in G$. This is equivalent to the existence of a *positive cone* P ($\neq \varnothing$), which is a semigroup and gives a disjoint decomposition $P \sqcup \{1\} \sqcup P^{-1}$. For a given left-ordering $<$, the set $P = \{g \in G \mid g > 1\}$ gives a positive cone. Any element of P (resp. P^{-1}) is said to be *positive* (resp. *negative*). Conversely, given a positive cone P, declare $a < b$ if and only if $a^{-1}b \in P$. This defines a left-ordering.

We denote by $\mathrm{LO}(G)$ the set of all positive cones in G. This is regarded as the set of all left-orderings of G as mentioned above. For $g \in G$ and $P \in \mathrm{LO}(G)$, let $g(P) = gPg^{-1}$. This gives a G-action on $\mathrm{LO}(G)$. In other words, for a left-ordering $<$ of G, an element g sends $<$ to a new left-ordering $<^g$ defined as follows: $a <^g b$ if and only if $ag < bg$. We say that $<$ and $<^g$ are *conjugate* orderings. Also, a family $L \subset \mathrm{LO}(G)$ is said to be *normal* if it is G-invariant.

Example 3.1 Let $G = \langle x, y : x^{-1}yx = y^{-1}\rangle$. This is the fundamental group of the Klein bottle. It is known that G admits exactly four left-orderings. We will define two normal families L_+ and L_- of left-orderings as follows. Consider a short exact sequence

$$1 \longrightarrow \langle y\rangle \longrightarrow G \xrightarrow{q} \langle x\rangle \longrightarrow 1.$$

For $g \in G$, define $1 <_{++} g$ if $q(g) = x^s$ with $s > 0$, or $q(g) = 1$ and $g = y^r$ with $r > 0$. Similarly, define $1 <_{+-} g$ if $q(g) = x^s$ with $s > 0$, or $q(g) = 1$ and $g = y^r$ with $r < 0$. Then we can easily prove that $L_+ = \{<_{++}, <_{+-}\}$ gives a normal family of $\mathrm{LO}(G)$.

Similarly, define $1 <_{-+} g$ if $q(g) = x^s$ with $s < 0$, or $q(g) = 1$ and $g = y^r$ with $r > 0$. And, define $1 <_{--} g$ if $q(g) = x^s$ with $s < 0$, or $q(g) = 1$ and $g = y^r$ with $r < 0$. Then $L_- = \{<_{-+}, <_{--}\}$ gives another normal family.

We need one more notion. For $i = 1, 2$, let G_i be a left-orderable group, let H_i be a subgroup of G_i, and let $L_i \subset \mathrm{LO}(G_i)$ be a family of left-orderings. Let $\phi\colon H_1 \to H_2$ be an isomorphism. We recall that ϕ is *compatible* for the pair (L_1, L_2) if for any $P_1 \in L_1$, there exists $P_2 \in L_2$ such that $h_1 \in P_1$ implies $\phi(h_1) \in P_2$ for any $h_1 \in H_1$.

Theorem 3.2 (Bludov–Glass [1]) *For $i = 1, 2$, let G_i be a left-orderable group and let H_i be a subgroup of G_i. Let $\phi\colon H_1 \to H_2$ be an isomorphism. Then the free product with amalgamation $G_1 * G_2$ $(H_1 \overset{\phi}{\cong} H_2)$ is left-orderable if and only if there exist normal families $L_i \subset \mathrm{LO}(G_i)$ for $i = 1, 2$ such that ϕ is compatible for (L_1, L_2).*

4 An Ordering of Torus Knot Group

For $n \geq 1$, let $\Gamma_n = \langle b, c : b = cb^n c\rangle$. Navas [10] proved that the semigroup generated by $\{b, c\}$ gives a positive cone, hence a left-ordering of Γ_n. In other words, an element $w \in \Gamma_n$ is positive (resp. negative) if w can be written in only positive (resp. negative) powers of b, c.

Let $\Delta = b^{n+1}$. Then $\Delta > 1$. It is easy to see that Δ is central. (In fact, Δ generates the center of Γ_n.) Also, $b^{-1} = b^n\Delta^{-1}$ and $c^{-1} = b^n cb^n\Delta^{-1}$. Thus, as Navas observes, every element $w \in \Gamma_n$ can be written in a form $u\Delta^\ell$ for some trivial or positive u and $\ell \in \mathbb{Z}$.

Furthermore, he shows that every element $w \in \Gamma_n$ has a *normal form*

$$w = c^{n_0}b^{m_1}c^{n_1}\cdots c^{n_{k-1}}b^{m_k}c^{n_k}\Delta^\ell = u\Delta^\ell,$$

with the properties

(i) $n_i > 0$ for $0 < i < k$, $n_0 \geq 0$, $n_k \geq 0$;
(ii) $m_i \in \{1, 2, \ldots, n-1\}$ for $1 < i < k$;
(iii) $m_1 \in \{1, 2, \ldots, n-1\}$ (resp. $\{1, 2, \ldots, n\}$) if $n_0 > 0$ (resp. $n_0 = 0$); similarly, $m_k \in \{1, 2, \ldots, n-1\}$ (resp. $\{1, 2, \ldots, n\}$) if $n_k > 0$ (resp. $n_k = 0$);

and $\ell \in \mathbb{Z}$.

The next lemma is proved in [10, Section 2].

Lemma 4.1 ([10]) *Let $w = u\Delta^\ell$ be a normal form of a non-trivial element $w \in \Gamma_n$.*

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(i) *If $u = 1$, then w is positive or negative, according to the sign of ℓ.*
(ii) *If $u \neq 1$ and $\ell \geq 0$, then w is positive. If $u \neq 1$ and $\ell < 0$, then w is negative.*

Lemma 4.2 *For any $w \in \Gamma_n$, there exists an integer ℓ such that $\Delta^\ell < w < \Delta^{\ell+1}$.*

Proof Let $w = u\Delta^\ell$ be a normal form, where u is trivial or positive. If $u = 1$, then the conclusion is clear. So, let $u > 1$. Recall that Δ is central. Then $\Delta^\ell < \Delta^\ell u$, and $\Delta^\ell u < \Delta^{\ell+1}$ by Lemma 4.1(ii). Thus we have $\Delta^\ell < w < \Delta^{\ell+1}$. ■

Let $G_{2m+1} = \langle a, b : a^2 = b^{2m+1} \rangle$. We are interested in the case where $|m| \geq 2$. This is isomorphic to the knot group of the torus knot of type $(2, 2m+1)$. It is well known (see [5]) that $h = a^2 = b^{2m+1}$ is a central element, which corresponds to a regular fiber of the torus knot exterior with a (unique) Seifert fibration, and the meridian μ is $b^{-m}a$.

Suppose $m > 0$. Then

$$G_{2m+1} = \langle a, b, c : a^2 = b^{2m+1}, c = ba^{-1} \rangle = \langle b, c : b = cb^{2m}c \rangle.$$

Thus this is Γ_{2m} in Navas's notation. We remark that $\Delta = h$.

Assume $m < 0$. Set $n = -m - 1$ (≥ 1). Then

$$G_{2m+1} = \langle a, b, c : a^2 = b^{-2n-1}, c = ab \rangle = \langle b, c : b = cb^{2n}c \rangle.$$

Hence $G_{2m+1} = \Gamma_{2n} = \Gamma_{-2m-2}$. We should remark that $\Delta = b^{2n+1} = b^{-2m-1} = h^{-1}$.

In either case, we can introduce Navas's left-ordering to G_{2m+1}, denoted by $<$, hereafter.

Lemma 4.3 *Both μ and h are either positive or negative, according to the sign of m.*

Proof Assume $m > 0$. Since $b^{-m}a = b^{m+1}a^{-1}$ and $c = ba^{-1}$,

$$\mu = b^{m+1}a^{-1} = b^m(ba^{-1}) = b^m c.$$

Thus μ is positive by Lemma 4.1. Also, $h = \Delta > 1$.

Assume $m < 0$, and set $n = -m - 1$ as before. Then

$$\mu = b^{n+1}a = b^{n+1}cb^{-1} = b^{n+1}cb^{2n}\Delta^{-1},$$

since $a = cb^{-1}$ and $b^{-1} = b^{2n}\Delta^{-1}$. Hence $\mu < 1$. Finally, $h = \Delta^{-1} < 1$. ■

Lemma 4.4 *For any integer r, $\mu^r < \Delta$.*

Proof Suppose $m > 0$. As in the proof of Lemma 4.3, $\mu = b^m c$. If $r > 0$, then $\Delta^{-1}\mu^r = \mu^r\Delta^{-1} = (b^m c)^r\Delta^{-1} < 1$ by the criterion of Lemma 4.1. Thus $\mu^r < \Delta$.

If $r < 0$, then set $k = -r$. Then $\mu^r = \mu^{-k} = (c^{-1}b^{-m})^k$. By $c^{-1} = b^{2m}cb^{2m}\Delta^{-1}$, we have $\mu^r = (b^{2m}cb^m)^k\Delta^{-k}$. When $k = 1$, this is a normal form, so $\mu^r < 1 < \Delta$. When $k > 1$, $\mu^r = (b^{2m}cb^m)(b^{2m}cb^m)\cdots(b^{2m}cb^m)\Delta^{-k} = b^{2m}cb^{m-1}\cdots cb^m\Delta^{-1}$. Again, $\mu^r < 1 < \Delta$.

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Now, assume $m < 0$. Set $n = -m - 1$ as before. As in the proof of Lemma 4.3, $\mu = b^{n+1}cb^{-1} = b^{n+1}cb^{2n}\Delta^{-1}$. If $r = 1$, then $\mu = b^{n+1}cb^{2n}\Delta^{-1} < 1 < \Delta$. If $r > 1$, then

$$\mu^r = (b^{n+1}cb^{-1})^r = b^{n+1}cb^n \cdots b^n cb^{-1} = b^{n+1}cb^n \cdots b^n cb^{2n}\Delta^{-1} < 1 < \Delta.$$

If $r < 0$, set $k = -r$. Then $\mu^r = \mu^{-k} = (bc^{-1}b^{-n-1})^k = (cb^{n-1})^k < \Delta$. ∎

The next lemma is proved by a similar argument to that of [6].

Lemma 4.5 *For any element $g \in G_{2m+1}$ and an integer r, $\Delta^{-1} < g^{-1}\mu^r g < \Delta$.*

Proof For a given g, there exists an integer ℓ such that $\Delta^\ell < g < \Delta^{\ell+1}$ by Lemma 4.2. Since Δ is central, we also have $\Delta^{-\ell-1} < g^{-1} < \Delta^{-\ell}$.

Here, assume that $\Delta < g^{-1}\mu^r g$ for contradiction. Then $\Delta = \Delta^\ell\Delta^2\Delta^{-\ell-1} < g(g^{-1}\mu^r g)^2 g^{-1} = \mu^{2r}$. This contradicts Lemma 4.4.

Assume $g^{-1}\mu^r g < \Delta^{-1}$. Then $\mu^{2r} = g(g^{-1}\mu^r g)^2 g^{-1} < \Delta^{\ell+1}\Delta^{-2}\Delta^{-\ell} = \Delta^{-1}$. So, $\Delta < \mu^{-2r}$, which contradicts Lemma 4.4 again. ∎

Unfortunately, Navas's ordering does not satisfy the so-called Property S, but we have a weaker result, which is sufficient to our purpose.

Lemma 4.6 *For any conjugate ordering $<^g$ of Navas's ordering $<$ of G_{2m+1}, assume $1 <^g \mu^r h^s$. Then we have the following.*

(i) *If $m > 0$, then*
  - (a) *$s > 0$; or*
  - (b) *$s = 0$ and $r > 0$ (resp. $r < 0$) if $g^{-1}\mu g > 1$ (resp. $g^{-1}\mu g < 1$).*

(ii) *If $m < 0$, then*
  - (a) *$s < 0$; or*
  - (b) *$s = 0$ and $r > 0$ (resp. $r < 0$) if $g^{-1}\mu g > 1$ (resp. $g^{-1}\mu g < 1$).*

Proof By definition, $1 < g^{-1}\mu^r h^s g$. Then $g^{-1}\mu^{-r} g < h^s$.

(i) Assume $m > 0$. Then $\Delta = h$. By Lemma 4.5, we have $s \geq 0$. So, suppose $s = 0$. If $g^{-1}\mu g > 1$, then $g^{-1}\mu^{-r} g < 1$ if and only if $r > 0$. Similarly, if $g^{-1}\mu g < 1$, then $g^{-1}\mu^{-r} g < 1$ if and only if $r < 0$.

(ii) Assume $m < 0$. Then $\Delta = h^{-1}$. By Lemma 4.5, we have $s \leq 0$. When $s = 0$, the argument is the same as above. ∎

5 Proofs

Proof of Theorem 1.1 Let M be the resulting manifold by 4-surgery on the m-twist knot. Let M_1 be the exterior of the torus knot of type $(2, 2m+1)$ and let M_2 be the twisted I-bundle over the Klein bottle. Also, let $G_i = \pi_1(M_i)$ and $H_i = \pi_1(\partial M_i)$. Then by Proposition 2.2, $\pi_1(M)$ is the free product with amalgamation $G_1 * G_2$ $(H_1 \overset{\phi}{\cong} H_2)$ where

$$G_1 = \langle a, b : a^2 = b^{2m+1}\rangle, G_2 = \langle x, y : x^{-1}yx = y^{-1}\rangle, \phi(\mu) = y^{-1}, \phi(h) = y^{-1}x^2.$$

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For LO(G_1), let L_1 be the (normal) family of all conjugate orderings of Navas's ordering. For LO(G_2), set L_2 to be the normal family $L_+ = \{<_{++}, <_{+-}\}$ or $L_- = \{<_{-+}, <_{--}\}$, defined in Example 3.1, according to the sign of m. To show that $\pi_1(M)$ is left-orderable, it is sufficient to verify that ϕ is compatible for the pair (L_1, L_2) by Theorem 3.2.

Let $<^g \in L_1$. Suppose $1 <^g \mu^r h^s$. Assume $m > 0$. According as $g^{-1}\mu g$ is positive or negative with respect to Navas's ordering, we choose $<_{+-}$ or $<_{++}$ from L_2, respectively. Since $\phi(\mu^r h^s) = y^{-r}(y^{-1}x^2)^s$, $q(\phi(\mu^r h^s)) = x^{2s}$. Then $\phi(\mu^r h^s)$ is positive by Lemma 4.6. When $m < 0$, we choose $<_{--}$ or $<_{-+}$ from L_2. ∎

Proof of Corollary 1.2 Let K be the m-twist knot. Then it is sufficient to consider the case where $|m| \geq 2$, because the conclusion for the figure-eight knot is settled by [2, 6]. According to the classification of exceptional Dehn surgery on 2-bridge knots [4], K admits exactly five exceptional (non-trivial) Dehn surgeries. More precisely, those slopes are $0, 1, 2, 3$, and 4. For $r = 1, 2$, or 3, r-surgery yields a small Seifert fibered manifold ([4]). Since K is not fibered, it does not admit an L-space surgery by [11]. Hence such a Seifert fibered manifold has left-orderable fundamental group by [2, Theorem 4]. For $r = 0$, the resulting manifold is prime ([7]) and has positive Betti number, so its fundamental group is left-orderable by [3]. Finally, our Theorem 1.1 solves the remaining case $r = 4$. ∎

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References

[1] V. V. Bludov and A. M. W. Glass, *Word problems, embeddings, and free products of right-ordered groups with amalgamated subgroup.* Proc. Lond. Math. Soc. (3) **99**(2009), no. 3, 585–608. http://dx.doi.org/10.1112/plms/pdp008 http://dx.doi.org/10.1112/plms/pdp008

[2] S. Boyer, C. McA. Gordon, and L. Watson, *On L-spaces and left-orderable fundamental groups*. arxiv:1107.5016.

[3] S. Boyer, D. Rolfsen, and B. Wiest, *Orderable 3-manifold groups.* Ann. Inst. Fourier (Grenoble) **55**(2005), no. 1, 243–288. http://dx.doi.org/10.5802/aif.2098

[4] M. Brittenham and Y.-Q. Wu, *The classification of exceptional Dehn surgeries on 2-bridge knots.* Comm. Anal. Geom. **9**(2001), no. 1, 97–113.

[5] G. Burde and H. Zieschang, *Knots*, de Gruyter Studies in Mathematics, 5, Walter de Gruyter & Co., Berlin, 2003.

[6] A. Clay, T. Lidman, and L. Watson, *Graph manifolds, left-orderability and amalgamation*. arxiv:1106.0486.

[7] D. Gabai, *Foliations and the topology of 3-manifolds. III.* J. Differential Geom. **26**(1987), no. 3, 479–536.

[8] J. E. Greene, *Alternating links and left-orderability.* arxiv:1107.5232.

[9] T. Ito, *Non-left-orderable double branched coverings*. arxiv:1106.1499.

[10] A. Navas, *A remarkable family of left-ordered groups: central extensions of Hecke groups.* J. Algebra **328**(2011), 31–42. http://dx.doi.org/10.1016/j.jalgebra.2010.10.020 http://dx.doi.org/10.1016/j.jalgebra.2010.10.020

[11] Y. Ni, *Knot Floer homology detects fibred knots.* Invent. Math. **170**(2007), no. 3, 577–608. http://dx.doi.org/10.1007/s00222-007-0075-9 http://dx.doi.org/10.1007/s00222-007-0075-9
[12] P. Ozsváth and Z. Szabó, *On knot Floer homology and lens space surgeries.* Topology **44**(2005), no. 6, 1281–1300. http://dx.doi.org/10.1016/j.top.2005.05.001 http://dx.doi.org/10.1016/j.top.2005.05.001

Department of Mathematics and Mathematics Education, Hiroshima University, Higashi-hiroshima, Japan 739-8524
e-mail: teragai@hiroshima-u.ac.jp