

## 47. COSMOLOGY (COSMOLOGIE)

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### 1. HUBBLE CONSTANT, DECELERATION PARAMETER AND THE LARGE SCALE DISTRIBUTION OF MATTER IN THE UNIVERSE

(M. S. Longair)

#### A. $H_0$ and $\Omega$

A major procedural advance in the determination of the  $H_0$  and  $\Omega$  has been that the problem is now being attacked from many different points of view and to some extent the observations are converging on preferred values of  $H_0$  and  $\Omega$  ( $\Omega = \text{density parameter} = 8\pi G\rho_0/3H_0^2$  where  $\rho_0$  is the mean density of matter in the Universe and  $H_0$  is the Hubble constant;  $\Omega = 2q_0$  where  $q_0$  is the deceleration parameter). The classical approaches through the redshift-magnitude relation for the most massive galaxies in clusters suggest a value of  $H_0 = 60 \text{ km s}^{-1} \text{ Mpc}$  (see the review by Tammann in *IAU Symp.* 63).

The classical approach to the determination of  $\Omega$  is through the extension of the redshift-magnitude relation to large redshifts and the major contributions have been made by Sandage and by Gunn and Oke. Of particular importance has been the use of the multichannel scanner on the 200-in. telescope to determine the absolute spectra of galaxies. Two important advances are:

(i) the redshift range 0.3 to 0.5 has been filled out by the addition of about 17 new galaxies which are the brightest members of clusters to the  $M-Z$  plot;

(ii) the problem of K-corrections is eliminated since the absolute spectrum of each galaxy is measured. One result of immediate importance is that 3C295, which is at a redshift of 0.461 and which up till recently was the only object at such a large redshift, appears to be intrinsically brighter than other first ranking galaxies in clusters. Gunn and Oke (*Astrophys. J.* 195, 255, 1975) find that the best fitting value of  $q_0$  is small,  $q_0 = 0.31 \pm 0.68$  (3C295 included) or  $q_0 = -0.15 \pm 0.54$  (3C295 excluded) without making any allowance for cosmological evolution. Formally, if allowance is made for the evolution of the luminosity of the most massive galaxies in clusters, the best fit value of  $q_0$  is *negative*,  $q_0 = -0.43 \pm 0.54$  (3C295 included) or  $q_0 = -1.27 \pm 0.62$  (3C295 excluded), i.e. the Universe is accelerating. A literal interpretation of this result suggests that the cosmological constant  $\Lambda$  is non-zero. However, what has become increasingly apparent even with the much improved data-base, is the great sensitivity of  $q_0$  to very small corrections. Sandage's approach has been to investigate all possible correlations between the properties of massive galaxies in clusters and cluster membership and to calibrate out these effects. Serious problems remain, however, with very small corrections which depend upon the astrophysical evolution of massive galaxies in clusters. The problem of estimating the 'static' evolution of a massive galaxy in a cluster has already been mentioned. There is also the question of how the most massive galaxies in clusters form. If, as seems plausible, they are massive because matter in the cluster has nowhere else to fall, there could well be significant changes in their luminosities over cosmological time-scales. Recently Ostriker and Tremaine have discussed the problem of the central massive galaxy disrupting and 'swallowing' spiral galaxies in the cluster. Both of these effects tend to make the most massive galaxies in clusters brighter with cosmological epoch, a trend opposite to the normal evolutionary corrections

according to which galaxies become fainter with time. The astrophysics of these problems will undoubtedly be one of the most important research fields in the next few years.

There are now independent routes to  $\Omega$  and  $H_0$  which have made great advances recently. One approach is to investigate how smooth the expansion of the Universe is locally. If the mean density of the Universe is small, then the 'Hubble flow' for galaxies should be very uniform at the present time – i.e. the velocity distribution is not perturbed by large perturbations such as the Virgo cluster. There are conflicting views on this problem. On the one hand Sandage finds a very smooth velocity distribution for his local samples of galaxies; on the other hand Ford and Rubin find evidence for large perturbations in the Hubble flow for particular classes of galaxies. The origin of this discrepancy has yet to be resolved.

Another approach is purely astrophysical. The technique has been surveyed by Gott, Gunn, Schramm and Tinsley. The most important piece of evidence is the cosmic deuterium abundance. The observed abundance,  $N[D]/N[H] = 1.4 \times 10^{-5}$  is much greater than can have been produced by stars and it is inferred that it must be of primaeval origin. The amount of deuterium synthesized in the early stages of the hot model of the Universe is very sensitive to the mean baryon density and the observed abundance corresponds to values of  $\Omega \approx 0.1$ , i.e. an open Universe. Together with evidence on the age of the Galaxy and generous limits to the possible range of values of the Hubble constant, the permissible region of  $H_0$ ,  $\Omega$  parameter space is relatively small. The favoured values are

$$H_0 \approx 60 \text{ km s}^{-1} \text{ Mpc}^{-1}; \quad \Omega \approx 0.1$$

which are in agreement with the traditional approach to the problem.

### B. *The Large Scale Distribution of Matter in the Universe*

The evidence of the isotropy of the microwave background and surveys of discrete radio sources indicate that on a large scale, the Universe is impressively isotropic. On the other hand there is obviously a great deal of structure in the distribution of discrete objects such as galaxies and clusters of galaxies. The most significant recent work on this topic of central importance for cosmology is that of Peebles (for general survey, see *IAU Symp. 58* 'Dynamics and Formation of Galaxies'). Peebles has applied the powerful technique of power spectrum analysis to the distribution of galaxies and clusters of galaxies. Of most importance in these studies are the statistics derived directly from observation rather than the interpretation which is model dependent. The following facts emerge:

(i) Abell clusters are clustered on a scale of about 50 Mpc; the mean number of Abell clusters per association is about 2 – these associations of Abell clusters correspond to 'superclusters'.

(ii) Galaxies themselves are clustered on a scale of 50 Mpc. This includes regions free of Abell clusters and is derived from the counts of galaxies in the Shane-Wirtanen catalogue.

(iii) The covariance function which describes the clustering of faint galaxies is in excellent agreement with the covariance function for bright, nearby galaxies, once allowance is made for the different volumes of space sampled. This is the best evidence for the overall homogeneity of the Universe on a large scale – i.e. out to a redshift of about 0.15.

(iv) The covariance function is very smooth and can be adequately described by a power-law function of physical size,  $l$ ,

$$\xi(l) \propto l^{-1.77}$$

over the range 30 kpc to 50 Mpc. Scales larger than 50 Mpc are difficult to investigate because the effects of patchy obscuration in the Galaxy become important on the largest angular scales. This problem has been emphasized by the work of Holmberg on the patchiness of the distribution of galaxies in the Zwicky catalogues of galaxies. Holmberg argues that much of the apparent large scale clustering of galaxies may be due to patchy Galactic obscuration but it is difficult to see how this could explain all the observations. If 'superclusters' were only due to obscuration, the observed patchiness of the distribution of galaxies as described by Peebles' covariance functions would not scale properly with distance.

These analyses provide new data for theories of the formation of galaxies. The covariance

function for galaxies contains information about the spectrum of irregularities from which galaxies formed. The extension of these techniques to the very faintest magnitudes will eventually enable the homogeneity of the Universe to be tested to genuinely cosmological distances and provide information about the evolution of large scale structure in the Universe with cosmic epoch.

## 2. RADIO ASTRONOMY AND COSMOLOGY

(M. S. Longair)

The observational situation up to September 1973 is surveyed in the Proceedings of *IAU Symp. 63* 'Confrontations of Cosmological Theories and Observational Data' (Reidel, Dordrecht, ed. by M. S. Longair 1974). The field may be conveniently divided into those aspects concerning the microwave background radiation and those aspects involving discrete sources.

### A. *The Microwave Background Radiation*

#### I. *The Spectrum of the Microwave Background Radiation*

Of most interest has been the work of the groups at Queen Mary College, London (Robson *et al.*, 1974, *Nature* 251, 591) and the University of California at Berkeley (Woody *et al.*, 1975, *Phys. Rev. Letters* 34, 1036) who have performed balloon-flights with onboard detectors to measure the spectrum of the microwave background in the wavelength region 250  $\mu\text{m}$  to 3 mm where it is expected that the high frequency cut-off in a Planck spectrum of 2.7 K should be observed. As reported in *IAU Symp. 63* by Blair, previous balloon flights had established that there was no great excess of emission in this waveband and the results were consistent with a black body curve at 2.7 K but with very large error bars. Both the Berkeley and QMC experimenters now claim to have measured the curvature of the Planck spectrum near to its maximum and in the QMC case, they believe they have observed the decrease expected at wavelengths shorter than 1 mm. There are, however, problems of calibration and taken at face value there seems to be a discrepancy between the two sets of observations. The technical problems involved in these observations are very great: those which pose the greatest difficulties are the estimation of the residual contribution of the atmosphere even at the greatest balloon altitudes and providing absolute calibrations for the detectors. If the experiments have detected the turn-over to the Wien region of the spectrum, this proves that the microwave background has indeed a black-body spectrum. Limits can then be set to the amount of energy which could have been liberated during the evolution of the Universe according to the well-known arguments of Zeldovich and Sunyaev. Further balloon flights with improved detectors are planned to clear up the existing discrepancies.

#### II. *The Isotropy of the Microwave Background Radiation*

There have been no significant improvements on the upper limits to fluctuations in the angular distribution of the microwave background radiation over those quoted by Partridge and Boynton in *IAU Symp. 63*. The importance of these observations is emphasized by Zeldovich and Novikov in the accompanying report. The major problems in improving these limits are associated with fluctuations in the water vapour content of the atmosphere which, even at high altitude sites, impose larger fluctuations on the observed intensity of the background than those expected from theory. Unless the observer has particularly good luck and makes observations when the atmosphere is very stable, it would seem that these experiments must be performed from space vehicles. However, because of their great importance, they should form a major project for future space missions.

### III. *The Microwave Background in the Direction of Rich Clusters of Galaxies*

No observer has yet confirmed the result reported by Parijskij at the Cracow symposium, that there is a dip in the intensity of the microwave background radiation in the direction of the Coma cluster. Again, all other observers have been unable to approach the sensitivity limit claimed by Parijskij because of tropospheric fluctuations. However, a number of groups are now intensively working on this problem and the question should be resolved when observing conditions are good.

#### B. *Surveys of Discrete Radio Sources*

##### I. *Surveys of Sources*

The situation has developed considerably since the Cracow symposium, although few of the new results have yet been published. All of this new material will undoubtedly be reported at IAU Symposium No. 74 'Radio Astronomy and Cosmology' to be held in Cambridge, England in August 1976. A careful survey of the observations available up till the end of 1974 is given by Jauncey (*Ann. Rev. Astron. Astrophys.* 13, 1, 1975). At the time of the Cracow symposium, there were a number of reports of anisotropies and anomalies in the counts of sources and many of these points have now been resolved. The most important new results in order of increasing frequency are as follows.

At 408 MHz, the statistics are now greatly improved in the flux density range 0.1 to  $1 \times 10^{-26} \text{ w m}^{-2} \text{ Hz}^{-1}$  as a result of the Molonglo and Bologna surveys. They are in excellent agreement. At the high flux densities  $S_{408} \geq 10 \times 10^{-26} \text{ w m}^{-2} \text{ Hz}^{-1}$ , the statistics are derived from whole sky surveys and therefore the statistics will never improve (Robertson, 1973, *Australian J. Phys.* 26, 403). At low flux densities,  $S_{408} \geq 0.01 \times 10^{-26} \text{ w m}^{-2} \text{ Hz}$ , the statistics of 5 SC surveys have been compared and there is no evidence of anisotropies. SC 5 was performed close to the region of 5C1 and small discrepancies were detected in the position and flux densities of sources in 5C1 (Gillespie, 1975, *Monthly Notices Roy. Astron. Soc.* 170, 541 and Pearson, 1975, *Monthly Notices Roy. Astron. Soc.* 171, 475).

At 610 MHz, a deep survey made with the Westerbork telescope was supplemented by deep optical plates which enabled a surprisingly large number of these very faint radio sources to be associated with optical objects (Katjert-Merkelijn and Spinrad, 1974, *Astron. Astrophys.* 35, 393).

At 1400 MHz, a number of reported anisotropies in the source counts are no longer believed to exist in the light of further surveys. It is now known that the flux density scale of the Greenbank GA survey was slightly low at low flux densities and no discrepancy now exists between the Westerbork, Greenbank GA and GB surveys (Fomalont *et al.*, 1974, *Astron. Astrophys.* 36, 273). Spectral index anomalies between the 5C1 and 5C2 fields have been investigated by Gillespie (op. cit.) who shows that calibration errors in 5C1 were certainly responsible for part of the discrepancy. Additional surveys at 1400 MHz of the 5C3, 4 and 5 regions reveal no anisotropies in the spectral index distribution.

At 2700 MHz, the Parkes catalogue has now been completed and source counts derived for the whole sky at high flux densities. The results are in excellent agreement with the counts at lower frequencies, showing an initial steep rise in the counts at high flux densities  $N(\geq S) \propto S^{1.8}$  and then flattening. The distribution at very low flux densities has been examined using the  $P(D)$  technique and these observations show that the counts converge at the lowest flux densities (Wall and Cooke, 1975, *Monthly Notices Roy. Astron. Soc.* 171, 9). There appears to be some variation in the source count slope over the sky but there does not appear to be any systematic variation. It is likely that the variations are statistical in origin.

A powerful statistical technique, modifying the power-spectrum approach of Peebles, has been developed by Webster (1975, *Monthly Notices Roy. Astron. Soc.*, in press) to test the isotropy of the distribution of sources in complete surveys. The technique involves inspection of the components of the Fourier transform of the source distribution on the sky and looking for non-randomness. Application to the 4C, Greenbank GB and Molonglo surveys reveals no

evidence for real anisotropies in the source distribution. *Anti-clustering* is found but this is attributed to confusion effects at low flux densities.

## II. $V/V_{\max}$ Test for Quasars

Complete samples of quasars are now available from optical identification surveys of the sources catalogued in the 4C catalogue. The surveys of Schmidt and Wills (SRC/CERN/ESO conference on 'Programmes for the New Large Optical Telescopes 1974') independently find the same phenomenon as in the 3CR catalogue, i.e. the mean of  $V/V_{\max}$  for quasars is significantly greater than 0.5, implying strong evolution of the quasar population with cosmological epoch. An analysis of all data available in 1973 enabled Jackson (1974, *Monthly Notices Roy. Astron. Soc.* **166**, 281) to make best estimates of the luminosity functions, source evolution and 'colour' function for quasars.

## III. Angular Diameter-Redshift Tests

As more precise angular information becomes available on faint radio sources, it is possible to perform angular-diameter-redshift or angular diameter-flux density tests. The statistics of Miley's original work on the  $\theta$ - $Z$  test for quasars have not yet greatly improved although a number of surveys are in progress. His work showed that there is a well defined upper bound to the  $\theta - Z$  relation for radio galaxies and quasars which could be described roughly by  $\theta \propto Z^{-1}$ , a result inconsistent with all world models. There was no evidence for a minimum angular diameter as predicted by the isotropic homogeneous world models of general relativity. A similar result was found by Swarup and Kapahi (1975, *Monthly Notices Roy. Astron. Soc.* **172**, 501, 513) using lunar occultation data on faint radio sources – they found that faint radio sources had angular sizes smaller than those predicted by extrapolation from high flux densities. Both of these results refer to the overall sizes of radio sources. A comparison of the results from Ooty with those of Katgert-Merkelijn and Spinrad suggested that there may be a minimum in the angular size-flux density relation at sufficiently low flux densities.

A rather different result was found by Hewish, Readhead and Duffett-Smith (1974, *Nature* **252**, 657) who showed from interplanetary scintillation data that there was no evidence for a  $\theta \propto Z^{-1}$  relation for the compact radio features observed in the extended components of quasars. One possible interpretation of the data is that there is a minimum in the  $\theta$ - $Z$  relation which explains the scatter in their diagram. Alternatively, the observed  $\theta$ - $Z$  relation may only reflect some correlation between the physical size of the compact components of quasars and their intrinsic luminosities.

This is a general feature of such cosmological tests – one must attempt to eliminate all correlations which influence the  $\theta$ - $Z$  relation. The theory of radio sources is far from being able to account for these observations and hence no theoretical guide-lines exist.

The importance of determining whether or not there is a minimum in the angular size-redshift relation has been emphasized by Roeder and Dyer (see *Nature* **255**, 124). If a minimum is observed, one can set limits to the inhomogeneity of the matter distribution in the Universe. If there is no matter in the light cone subtended by the radio source, there is no minimum in the  $\theta$ - $Z$  relation.

## IV. Studies of Individual Sources

Radio astronomy continues to be a prime source for the discovery of objects of great importance for cosmology. The identification of quasars with redshifts up to 3.53 has been principally a result of the optical identification programmes for sources catalogued in high frequency surveys. These efforts continue.

Radio sources as probes of the intergalactic gas has always been an important possibility but it has never been certain whether the radio components actually penetrate into the space between clusters. Giant radio sources have recently been discovered with physical sizes up to 6 Mpc which is much greater than the scale of clusters of galaxies (Willis *et al.*, 1974, *Nature* **250**, 625). It may eventually be possible to use such objects as probes of the intercluster gas.

### 3. FLUCTUATIONS IN THE MICROWAVE BACKGROUND RADIATION CONNECTED WITH ANISOTROPIC COSMOLOGICAL MODELS AND THE FORMATION OF GALAXIES

(Ya. B. Zeldovich and I. D. Novikov)

#### A. *Fluctuations of the Microwave Background Connected with Galaxy Formation*

The problem of the formation of galaxies is undoubtedly one of the most fundamental in astrophysics. The most generally held view is that galaxies form from small initial inhomogeneities in the matter distribution in the Universe as a result of gravitational instability. Small initial fluctuations in an otherwise homogeneous distribution of matter and their subsequent growth must lead to fluctuations in the microwave background radiation. These fluctuations may be discovered, in principle, by observation and must give important information about the process of formation of galaxies.

From the point of view of theory, the most important results were developed some time ago. The following references may be cited: Sachs and Wolfe (*Astrophys. J.* **147**, 73, 1967); Silk (*Astrophys. J.* **151**, 459, 1968); Zeldovich and Sunyaev (*Astrophys. Space Sci.* **6**, 358, 1970). A review of these results and some new ones are given in the book by Zeldovich and Novikov *The Structure and Evolution of the Universe* (Russian edition, Nauka, Moscow, 1975, Chapter 15, pp. 450–481; English edition, Chicago University Press, 1976).

According to theory, the expected anisotropies in the microwave background can be split into two parts

$$\frac{\Delta T}{T} = \Delta_1 + \Delta_2$$

The first term  $\Delta_1$  is connected with perturbations of density on large scales ( $10^{13} - 10^{23} M_\odot$ ) and it can be shown that they are associated with the peculiar motion of the observer in the gravitational field of these perturbations on various scales. They must have a 24-hour angular distribution (i.e. a dipole distribution). The observations give an upper limit on the peculiar velocity of the Earth due to such perturbations of  $|u| < 300 \text{ km s}^{-1}$ . The existing measurements refer only to one great circle on the sky and therefore do not give complete information on all three components of the Earth's peculiar velocity. However, we may state that the root mean square peculiar velocity on a given scale is  $u < 4 \times 10^{-3} c$  with a probability of 93%. We note that the observed isotropy of the X-ray background gives an upper limit to the peculiar velocity of  $u < 800 \text{ km s}^{-1}$  in all three directions (the work of Schwartz (1970) from observations made with the OSO-3 satellite). In the cosmological model having density parameter  $\Omega_0 = 1$  ( $\Omega_0$  is the mean density of matter in the Universe relative the critical density  $\rho_{\text{crit}} = 3H_0^2/8\pi G$ ), this result means that at the present day the amplitude of fluctuations  $\Delta\rho/\rho$  on the scale of the horizon ( $r = ct$ ) are  $< 0.01 - 0.001$ .

If  $\Omega_0 \ll 1$ , the fluctuations  $\Delta\rho/\rho$  do not grow at redshifts  $z$  smaller than  $(1+z) \approx 0.4/\Omega_0$  and the peculiar velocity of motion of weakly interacting matter decreases proportional to  $(1+z)$ . Therefore for small values of  $\Omega_0$  one can compare the small observed amplitude  $\Delta T/T$  of the 24-hour anisotropy (and correspondingly the small value of  $\Delta\rho/\rho$  on the scale of the horizon) with the fact that clusters of galaxies exist with  $\Delta\rho/\rho \approx 1$  and masses  $M \approx 10^{15} M_\odot$ . The important conclusion of recent work appears to be that there is an absence of individual large scale structures in the Universe with  $M \gg 10^{15} M_\odot$ .

The second term  $\Delta_2$  is basically connected with the Doppler shifts resulting from scattering of the background radiation by the peculiar motions of inhomogeneities at the epoch of recombination of the primordial plasma during the expansion of the Universe at a redshift  $z \approx 1500$ ; galaxies and clusters of galaxies are eventually born out of these inhomogeneities. The angular scale of the inhomogeneities associated with the term  $\Delta_2$  as measured by an observer at the present epoch is

$$\theta = 10\Omega_0^{2/3}(M/10^{14} M_\odot)^{1/3} \text{ min arc}$$

according to theory. The amplitude of the fluctuations is about  $\Delta T/T \approx 10^{-4}$  on angular scales  $\theta \sim 10'$  according to the adiabatic theory. On smaller scales the amplitude of the fluctuations is expected to be much smaller because the temperature fluctuations are washed out because by the epoch of recombination the matter in the fluctuation is optically thin.

The following measurements of the upper limits to small scale structure on the microwave background radiation have been reported.

- (1) Carpenter, R. L., Gulkis, S., and Sato, H.: *Astrophys. J. Letters* **182**, L61, 1973;  $\lambda = 3.56$  cm,  $\Delta_2 < 7 \times 10^{-4}$  on angular scale  $2.3'$ .
- (2) Boynton, R. E. and Partridge, R. B.: *Astrophys. J.* **181**, 243, 1973;  $\lambda = 0.35$  cm,  $\Delta_2 < 3.7 \times 10^{-3}$  on an angular scale  $80''$ .
- (3) Parijskij, Yu. N.: *Astron. Zh.* **50**, 453, 1973.  $\lambda = 4$  cm,  $\Delta_2 < 4 \times 10^{-5}$  on scale  $12' \times 40'$ ; *Astrophys. J. Letters* **180**, L47,  $\Delta_2 \leq 3 \times 10^{-5}$ ,  $\lambda = 2.8$ ;  $\theta = 3' - 1^\circ$ . Later, Parijskij improved the precision of his result and it is not in contradiction with  $\Delta_2 \approx 2 \times 10^{-4}$  on a scale of  $3 - 5'$  (*Astrophys. J. Letters* **188**, 113, 1974).
- (4) Stankevich, K. S.: *Astron. Zh.* **51**, 216, 1974.  $\lambda = 11.1$  cm,  $\Delta_2 < 1.5 \times 10^{-4}$ ,  $\theta = 8' - 10'$ .

Thus small scale fluctuations have not yet been discovered. The sensitivities of the existing measurements are only slightly poorer than the amplitude of the perturbations predicted by theory. The search for small-scale fluctuations in the microwave background radiation continues to be one of the most important problems for observational radio astronomy.

*B. Fluctuations of the Microwave Background Associated with Possible Strongly Anisotropic Deformations in the Past*

The possibility has often been discussed of an anisotropic expansion of the whole Universe close to the singularity which eventually evolves into an isotropically expanding model. Residual anisotropies in the expansion lead to anisotropies in the microwave background radiation. From such anisotropies, if they are observed, it is possible to study parameters of the anisotropic model close to the singularity.

The theoretical basis for the study of possible strong anisotropies in the expansion of the whole Universe in the past has been homogeneous anisotropic models. A generalised approach to the theory of this problem is given by Doroshkevich, Lukash and Novikov (*Astron. Zh.* **51**, 940, 1974). A survey of the observations is given by R. B. Partridge (see *Confrontations of Cosmological Theories with Observational Data* (ed. by M. S. Longair), Reidel, Dordrecht, 1974). A survey of the whole problem is given by Zeldovich and Novikov in *The Structure and Evolution of the Universe*, Nauka, Moscow 1975.

Theory predicts the following:

If at the present day  $\rho \sim \rho_{\text{crit}}$  (i.e.  $\Omega_0 \approx 1$ ), then  $\Delta T/T$  should have a quadrupole distribution in angle on the celestial sphere, described by the following formula

$$\frac{\Delta T}{T} \approx \frac{8}{8 + \ln \left( \frac{t_c}{t_F} \right)} \left( \frac{t_c}{t_l} \right)^{2/3} \sin^2 \theta \tag{1}$$

where  $t_c$  is the moment in the evolution of the Universe when  $\rho_{\text{matter}} = \rho_{\text{radiation}}$ ;  $t_F$  is the moment when the expansion ceases to be anisotropic and becomes Friedmannian – i.e. describable by the equations for isotropic models,  $t_l$  (corresponding to redshift  $Z_l$ ) is the moment after which the Universe is transparent to radiation.

For  $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , formula (1) predicts the dependence of  $\Delta T/T$  upon  $t_F$  and  $t_l$  (or correspondingly  $Z_l$ ). The smallest value of  $t_F$  in Table 1 is the Planck time  $t_p = 10^{-43}$  s.

Table 1. The amplitude  $\Delta T/T$ 

$t_F$	$Z_l = 10^3$	$Z_l = 10$
$10^{-4.3}$ s	$5.5 \times 10^{-3}$	$5.5 \times 10^{-5}$
1 s	$2 \times 10^{-2}$	$2 \times 10^{-4}$
$10^{11}$ s	$7 \times 10^{-2}$	$7 \times 10^{-4}$

The observations give a limit to the quadrupole amplitude of the microwave background  $\Delta T/T < 3 \times 10^{-3}$  (see the survey by Partridge, 1974, *op cit.*). The conclusion from the comparison of theory and observation is that for the reasonable value  $Z_l = 10^3$ , corresponding to the epoch of recombination, we must have  $t_F \approx t_p \approx 10^{-4.3}$  s. Since  $t_p$  is the minimum quantum interval of time and smaller intervals than  $t_p$  have no meaning, then on this model of the Universe, the expansion must have been isotropic from the very beginning.

Let us now consider the case  $\Omega_0 \ll 1$ . Then it turns out that the distribution of  $\Delta T/T$  on the sky differs from a quadrupole variation and is shown in Figure 1.

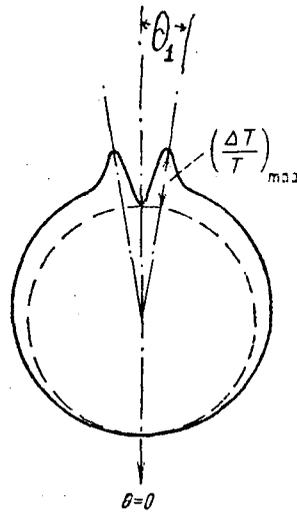


Fig. 1. The angular distribution of  $\Delta T/T$  on the sky in the case of models of the expansion of the Universe with highly anisotropic beginnings in the case  $\Omega_0 \ll 1$ . The distribution is symmetric with respect to the vertical axis.

$\Delta T/T$  differs significantly from zero only in a region of angular size  $\theta_1 \approx \Omega_0$ .

The maximum amplitude of  $\Delta T/T$  is given to order of magnitude by formula (1). Outside an angular size  $\theta_2 \approx Z_l^{1/2} Z_m^{-3/2}$  the distribution of  $\Delta T/T$  is dipolar on the sky. Here  $Z_m$  is the redshift corresponding to the epoch when gravitational forces acting on the matter in the Universe cease to influence the expansion, i.e. when the instantaneous value of  $\Omega$  is significantly less than 1.

For the case  $\Omega_0 \ll 1$ , we obtain the following table, similar to Table 1.

Table 2. Amplitude  $\Delta T/T$

$t_F$	$Z_I = 10^3$	$Z_I = 10$
$10^{-4.3}$ s	0.05	$5 \times 10^{-4}$
1 s	0.18	$2 \times 10^{-3}$
$10^{11}$ s	0.5	$5 \times 10^{-3}$

Thus in all cases [ $\Omega_0 = 1$  and  $\Omega_0 < 1$ ], if the moment at which the Universe becomes transparent corresponds to  $Z_I = 10$ ,  $\Delta T/T < 10^{-3}$  and the observed degree of isotropy agrees with any model in which the anisotropic expansion ends not later than the end of the radiation dominated stage i.e.  $t_F < t_c$ . However  $Z_I = 10$  is unlikely for many reasons and probably  $Z_I \approx 10^3$ . Then the smallness of  $(\Delta T/T)_{\max}$  can only occur if  $t_F$  is very small indeed because of the logarithmic dependence of  $(\Delta T/T)_{\max}$  on  $t_F$  (see Formula (1)). Therefore, if  $\Omega_0 \approx 1$ , the comparison of theory and observation shows that it is apparently essential that the expansion of the Universe be isotropic in practice from the very beginning – i.e. from the Planck time onwards. If  $\Omega_0 \ll 1$ , then it is impossible to make categorical conclusions in that case when  $\Delta T/T$  takes a large value only in some small region with angular size  $\theta_1 \approx \Omega_0$ . Outside this region  $\Delta T/T$  is considerably smaller. Because observations of  $\Delta T/T$  do not cover the whole sky, it is not excluded that there should be a small spot with  $\Delta T/T$ , say,  $10^{-1}$  or even 1.

To exclude such possibilities, it is essential to make special searches for such a spot and this is one of the most important observational problems.

#### 4. GALAXY FORMATION

(H. Nariai)

The most important publications on cosmology during the past three years are (1) *Cargèse Lectures in Physics*, Vol. 6 [(ed. by E. Schatzman), Gordon and Breach Sci. Publ., New York, 1973] and (2) Proc. of *IAU Symp. 63* [‘Confrontation of Cosmological Theories with Observational Data’ (ed. by M. S. Longair), Reidel, Dordrecht, 1974]. As regards galaxy formation, Harrison and Silk gave their lectures in (1), and the invited papers in (2) consist of Doroshkevich, Sunyaev and Zel’dovich’s, Ozernoy’s and Nariai’s.

Harrison dealt with a variety of problems: (1) the fragmentation versus clustering hypotheses on the origin of structure in a big-bang universe, (2) the initial conditions of galaxy formation with primordial or causal origin, (3) the observational estimation of angular momenta of galaxies, (4) the linear and non-linear theories of gravitational instability, (5) the rotation of galaxies due to either the tidal interaction between protogalaxies or the primordial turbulence, and (6) the spinning core theory (in which density and spin inhomogeneities of order unity are assumed to exist in the radiation era of a chaotic universe) as a rival to the theories of gravitational instability (cf. Peebles and Zel’dovich-Doroshkevich’s group) and primordial turbulence (cf. Ozernoy’s group, Dallaporta’s group and ours). Next, on the basis of Ozernoy and Chernin’s approach to the primordial turbulence, Silk dealt with the growth of density contrast generated from the subsonic isotropic turbulence before the decoupling epoch of matter and radiation (cf. Silk and Ames, *Astrophys. J.* 178, 77, 1973), together with the initial peculiar velocity spectrum of galaxies, the evidence and related theoretical development for intergalactic cosmic rays and the ones for the thermal component of intergalactic matter.

Doroshkevich, Sunyaev and Zel’dovich’s paper in (2) reviewed their work on the formation of galaxies and clusters of galaxies in terms of the non-linear theory of gravitational instability. The background universe adopted by them is an open Friedmann universe with the density parameter  $\Omega \equiv \rho/\rho_c = 0.1$  ( $\rho_c \equiv 3H^2/8\pi G$  and  $H$ (Hubble constant) = 50 km s<sup>-1</sup> Mpc<sup>-1</sup>), so that the redshifts of equality of radiation and matter density and of hydrogen decoupling coincide at  $z = z_D = 1300$  or  $t = t_D = 3 \times 10^{13}$  s. They assumed an initial ( $z > z_D$ ) density perturbation corresponding to the dimensionless metric perturbation  $h = 10^{-4}$  with flat spectrum. Then the

density contrast in a region with mass  $M$  larger than the Jeans mass  $M_J \propto t^{3/2}$  can grow in harmony with Lifshitz's classical result,  $\delta\rho/\rho = 10^{-4} (M_J/M)^{2/3} \propto t$ , up to  $z \sim 10$ , and thereafter ( $10 \gtrsim z \gtrsim 4 \sim 5$ ) dense gas clouds (called pancakes by them) with  $M = 10^{13} - 10^{15} M_\odot$  (clusters of galaxies) are formed entering into the realm of non-linear gravitational and thermal instability. The formed pancakes are accompanied by their fragmentation into galaxies in such a way that fragments in the central cool region and those in the outer hot region evolve to giant elliptical galaxies and spiral galaxies, respectively. Evolution of fragments in the respective regions was dealt with by Doroshkevich and Shandarin (*Soviet Astron. - AJ* 18, 24, 1974). As regards the origin of rotation of galaxies, Doroshkevich (*Astrophys. Lett.* 14, 11, 1973; *Soviet Astron. - AJ* 16, 986, 1973) proposed that the galactic rotation originates as vorticity which is generated by shock waves arising mainly in the outer region of a pancake, while the tidal interaction between fragments (cf. Peebles, *Astrophys. J.* 155, 393, 1969) could not generate the required vorticity (cf. Tomita, *Publ. Astron. Soc. Japan* 25, 287, 1973). Moreover, the reason why they started from the Friedmann universe with a small initial adiabatic perturbation is that it is easy to reconcile with observational data about the relic radiation and the cosmic helium abundance.

Ozernoy reviewed in [II] the present state of the theory of primordial turbulence which may be responsible for the origin of galaxies and galaxy systems (groups and clusters, etc.) in the light of Kurskov and his recent works (*Soviet Astron. - AJ* 18, 157, 300, 1974; 18, 700, 1975) on the evolution of cosmological turbulence, i.e., (a) the inertial redistribution of vortex spectrum in the subsonic turbulence when  $z < z_D \sim 10^3$ , (b) the dissipation of the subsonic motions and alternative (rough and calm) modes in the evolution when  $z > z_D$ , and (c) the spectrum of density inhomogeneities produced by the turbulence. According to them, whatever the initial eddy spectrum, the resulting turbulent spectrum at  $t = t_D$  has the universal Kolmogorov form and its amplitude essentially does not depend in detail on the initial spectrum, provided that its scale  $R$  is smaller than the maximum scale  $R_{\max} \equiv R_0 \{1 + \pi v_0 (\tau - \tau_0)/R_0\}^{2/(m+3)}$ , where  $R_0$  and  $v_0$  stand for the initial values of the main energy containing scale and the vortex velocity,  $(\tau - \tau_0)$  is a certain time scale at  $t = t_D$  and  $m$  the power in the initial wavenumber spectrum. If  $v_0/c \lesssim 0.3$  and  $\Omega h^2$  is not too small, the post-decoupling fate of turbulence on scales such that  $M \gg 10^{12} M_\odot$  (protoclusters of galaxies) is rather simple. In such a case, by a procedure similar to Silk and Ames's, there arises at  $t \gtrsim 3 t_D$  the density contrast  $\delta\rho/\rho = (9/20)(z_D t_D)^2 (\nabla(v \cdot \nabla)v)_D (t/t_D)^{2/3}$  which grows with time up to unity at which epoch the system is roughly isolated from the cosmic expansion. This leads to the mean-density versus effective-radius relation  $\langle \rho \rangle \approx 10^{-26} (\Omega h^2)^{3/7} (R/1 \text{ Mpc})^{-12/7} \text{ g cm}^{-3}$  consistent with observational data for galaxy systems. In contrast to the large scales, the pictures for a small scale eddy ( $M \lesssim 10^{12} M_\odot$ ) is much more complicated, because the efficiency of the viscous dissipation is different according as the post-decoupling turbulence is rough or calm. They also examined how the dissipation of cosmological turbulence distorts the relic radiation spectrum (cf. Sunyaev and Zel'dovich, *Astrophys. Space Sci.* 7, 20, 1970; 9, 368, 1970; Tomita, Nariai, Sato, Matsuda, and Takeda, *Prog. Theor. Phys.* 43, 1511, 1970) with an affirmative conclusion. Moreover, Ozernoy (Preprint No. 102, Lebedev Phys. Inst., 1973) pointed out that there is some observational evidence for the rotation of clusters of galaxies and superclusters supporting the primordial turbulence hypothesis.

Nariai's paper in (2) is a short review of his work (*Prog. Theor. Phys.* 50, 472, 1973; 51, 1368, 1974) on the generation of sound from the primordial cosmic turbulence before the decoupling epoch. The motivation is to make the vortex cosmogony free from Jones' criticism (*Astrophys. J.* 181, 269, 1973) that cosmic turbulence cannot be supported against decay because of the generation of sound after the epoch  $t_*$  at which matter density is equal to radiation density. Matsuda, Takeda and Sato (*Prog. Theor. Phys.* 49, 1770, 1973) pointed out that the criticism is erroneous because of his mis-application of the Lighthill-Crighton theory (*Proc. Roy. Soc. A* 211, 564, 1952; *Proc. Camb. Phil. Soc.* 65, 557, 1969) for the sound generation from a turbulent static medium to the turbulent expanding medium. By making use of an appropriate retarded Green's function for sound waves in the hot big-bang universe and taking account of the situation that the reception point when  $t \approx t_D$  is not far from the source region, he derived a new formula for the intensity of sound at the center of the expanding

turbulent fluid sphere within the acoustic event horizon. (The underlying expression for  $\delta\rho/\rho$  is different from Silk and Ames's which is due to the local source.) On the basis of that formula, it was numerically shown by Tanabe, Nariai, Matsuda, and Takeda (*Prog. Theor. Phys.* **54**, 719, 1975) that the acoustic decay of primordial cosmic turbulence is negligible irrespective of a concrete form of the spectral function.

As regards the galaxy formation from primordial turbulence, there are works by Dallaporta's group (Dallaporta and Lucchin, *Astron. Astrophys.* **19**, 123, 1972; **26**, 325, 1973; Bonometto, Danese, and Lucchin, *Astron. Astrophys.* **41**, 55, 1975), but Ozernoy pointed out in (2) that their treatment of turbulent dissipation is rough. The problem was also dealt with by Stein (*Astron. Astrophys.* **35**, 17, 1974) to show that density fluctuations on the scale of clusters of galaxies ( $10^{13} - 10^{15} M_{\odot}$ ) could be gravitationally bound but galactic mass fluctuations would not be the case, so that galaxies would be formed from the latter by their mutual collisions as the former started to recollapse. His estimation for density fluctuations is based on the Lighthill-mechanism, but it seems to be rough compared with ours.

As one of the clustering theory, Press and Schechter (*Astrophys. J.* **187**, 425, 1974) proposed that, without any ad hoc assumption about the spectrum of long-wavelength initial perturbation, the non-linear  $N$ -body interactions of mass points in the Friedmann universe leads to the picture that galaxies and their clusters are formed by self-similar gravitational condensation from isothermal seed masses ( $\sim 3 \times 10^7 M_{\odot}$ ) and galaxies, respectively (cf. Zel'dovich's comment in (2)). On the other hand, Peebles (*Astrophys. J.* **185**, 413, 1973; *Astrophys. J. Suppl.* **252**, 1974; *Astrophys. J. Letters* **189**, 51, 1974; *Astron. Astrophys.* **32**, 197, 1974) and Peebles and Hauser (*Astrophys. J.* **185**, 757, 1973; *Astrophys. J. Suppl.* **252**, 1974) made extensive studies on the distribution of galaxies. According to them, its covariance function varies as a simple power law over a large range of the argument, i.e., there is no evidence of the anti-correlation of galaxy positions (required for the clustering theory) and of a natural division between groups and clusters of galaxies or between clusters and superclusters, which may fit well with the gravitational instability picture for the evolution of irregularities in the expanding universe. A similar result was also obtained by Miyoshi and Kihara (*Publ. Astron. Soc. Japan* **27**, 333, 1975). Such a kind of study is to be performed from the standpoint of the vortex cosmogony too.

Moreover, in contrast with the above dynamical (gravitational instability) and hydrodynamical (primordial turbulence) approaches, there are two other approaches. One is an astrophysical approach asking what properties must have had in their formative period in order to produce the observed present properties of galaxies. This is important owing to its possible fulfilment of the requirement that cosmological results should after all be connected with astrophysical ones in the usual sense. As works belonging to this category, see various papers in Rep. of IAU Symp. No. 58, 'Formation and Dynamics of Galaxies' [(ed. by J. R. Shakeshaft), Reidel, Dordrecht, 1973], and those of Partridge (*Astrophys. J.* **192**, 241, 1974) and Davis and Wilkinson (*Astrophys. J.* **192**, 251, 1974). Another is an attempt to seek for a possible unification of elementary particle (or high energy) physics with cosmology in an early stage of the universe. As regards Omnes' charge symmetric cosmology and his idea for the galaxy formation, Zel'dovich reviewed in Rep. of IAU Commission No. 47 (1973) (cf. Omnes, *Phys. Rep.* **3C**, 1972). Next, on the basis of the bootstrap and dual resonance theories of hadrons (cf. Hagedorn's lecture in (1)) whose equivalence to the quantum theory of relativistic strings (cf. Rebbi, *Phys. Rep.* **12C**, 1974) was shown several years ago, Carlitz, Frautschi and Nahm (*Astron. Astrophys.* **26**, 171, 1973) put forward the idea that galaxies are causally formed from an initially homogeneous and isotropic state after the grainy structure characteristic in the hadron era disappeared at about  $t \sim 10^5$  yr, while Peebles (*Astron. Astrophys.* **32**, 391, 1974) doubted its possibility. In spite of this, such a program of study is attractive because, so far as the universe is of the big-bang origin, the hadron era ( $t \sim 10^{-23}$  s and  $\rho \sim 10^{56}$  g cm $^{-3}$ ) is far nearer to the epoch of helium production ( $t \sim 1$  s and  $\rho \sim 10^6$  g cm $^{-3}$ ) than the Planck era ( $t \sim 10^{-43}$  s and  $\rho \sim 10^{93}$  g cm $^{-3}$ ) in which even gravity must be quantized. This means that the physical state in the hadron era may control those in the later stages at which the nuclear synthesis and the initial perturbations required for the galaxy formation become the subjects of study, and vice versa.

## 5. COSMOLOGY AND CURVED SPACE QUANTUM THEORY

(M. A. H. MacCallum)

There are several questions in cosmology which may be affected when the proper treatment of curved space quantum theory is known. Among these are the problems of the existence and nature of the initial 'big-bang' singularity, the isotropisation and homogenisation of 'chaotic' models, and the existence and effects of primordial black holes. One might also hope, for example, for an explanation of the observed entropy per baryon.

A number of approaches to a synthesis of gravity and quantum theory have been attempted (Isham *et al.*, 1975). From dimensional arguments one can deduce that vacuum fluctuations of a quantised gravity field should become important at densities around  $10^{94} \text{ g cm}^{-3}$ , which in a conventional Robertson–Walker cosmology would occur when the 'radius' of the universe is about  $10^{-33} \text{ cm}$ . Thus to attack the singularity question one needs a theory of quantised gravity usable in strong fields. This motivated the cosmological application of the canonical quantisation of gravity, 'quantum cosmology'. It turns out that this can also have consequences for the behaviour of the universe at other phases of expansion.

The second relevant approach is that of generalising quantum field theory to a (classical) curved space background. The aim is to discuss particle creation, either in the case of cosmological curvature, when the back-reaction on the metric may explain observed homogeneity and isotropy, or in the case of primordial black holes.

Quantum cosmology has been recently reviewed elsewhere (Misner, 1972; Ryan, 1972; MacCallum, in Isham *et al.*, 1975). The idea is first to cast the gravitational equations into the form of classical Hamiltonian mechanics, and then quantise the resultant system just as in the usual first-quantisation. There are unfortunately a series of difficulties in completing this procedure. I shall try to describe these briefly. More adequate accounts are in the reviews listed above and in Arnowitt, Deser and Misner (ADM, 1962), and Kuchar (1973).

One starts from the usual Lagrangian for the Einstein field equations, and expresses it in terms of the metric of three dimensional spatial sections and the components of the time axis relative to the normal direction of these surfaces. The latter act like Lagrange multipliers of the constraints (which are just those 4 Einstein equations which involve only the initial values on a space section). The momentum conjugate to the three-space metric is a tensor density formed from the extrinsic curvature of the three-spaces. Of the  $16 \infty^3$  degrees of freedom (16 functions of 3 spatial variables),  $4 \infty^3$  are accounted for by transformations of the time coordinate,  $3 \infty^3$  by transformations of the space coordinates, and  $4 \infty^3$  by the constraints. The remaining variables are position and momentum variables of  $2 \infty^3$  degrees of freedom, together with an intrinsically defined time (Baierlein *et al.*, 1962).

The difficulty at this stage lies in eliminating the unwanted variables in practice (ADM, Wheeler, 1970; York, 1973). A considerable effort has therefore been put into studying the constraints (Kuchar, 1973). It is known that three of them show that the state of the Universe depends only on the three-geometry, that the remaining one occupies a position analogous to that of the Schrodinger equation in parametrised versions of standard theories, and that the four together show the evolution is path-independent. With weak additional assumptions, the path independence leads back to the form of the constraints.

On quantising, the deepest difficulty is in the meaning of the wavefunction for the universe. The usual 'Copenhagen' interpretation of quantum mechanics breaks down. The most plausible alternative is that of Everett and Wheeler recently re-examined by DeWitt (1973), whose work has in turn been criticised by Clarke (1974). Misner has suggested that a mixture of states representing universes with and without particle horizons might circumvent the classical implausibility of his 'mixmaster universe' suggestion for enhancing dissipative mechanisms for isotropisation by having very early causal communication between distant parts of the Universe. The relevance of this depends on the interpretation of the wavefunction.

The next difficulty in quantising is in choice of variables. Since there is no complete prescription for reduction to the true degrees of freedom, this problem leads on to the problem of the treatment of the constraints in quantum theory. Available answers range from Dirac's (1959) method where the physical states are picked out as the eigenstates of constraint

operators with eigenvalue zero, through 'semi-Dirac' and 'semi-ADM' methods where some but not all constraints are eliminated before quantisation, to the original ADM idea. The various prescriptions have only been implemented fully in cases where most of the degrees of freedom have been suppressed. These 'quantum models' of the full theory have doubtful status because the suppression of a degree of freedom violates the Uncertainty Principle. Nevertheless they have considerable interest. A further associated problem is that of factor-ordering.

In those quantum models where the ADM procedure can be completed, the 'true' Hamiltonian is typically the square-root of a time-dependent function. Quantising this is awkward. It has been done only for the simplest cases (Blyth and Isham, 1975). A re-interpretation, analogous to the Dirac equation has been used for Bianchi I models (Ryan, 1972). The commonest response has been to use the superspace method ('semi-ADM') in which wavefunctions are defined on the space of all three-geometries, and one constraint remains. This is the one analogous to the Schrodinger equation of parametrised theories and on quantisation it gives a Klein-Gordon-like equation. Classically it is the square of the 'true' Hamiltonian in most cases, but this is not true quantum-mechanically because of the time dependence (Blyth and Isham, 1975).

Classical superspace, introduced by Wheeler, has been studied in detail by Fischer and DeWitt among others (Carmeli *et al.*, 1970). The spatially-homogeneous cosmological models give the simplest quantum models in the quantised theory. One hope was that the wavefunction would vanish on the singular three-geometries, and that this would represent avoidance of the classical 'big-bangs'. Blyth and Isham pointed out that the latter would really require the vanishing of the wavefunction in some region around the singular geometries. In any event most of the cases calculated so far do not lead naturally to avoidance of the singularity. In the Bianchi I case, for example, the volume evolution rate is essentially a conserved quantity, but the result is not so clear in Bianchi IX. The Robertson-Walker case, which, unless the matter is quantised, has no dynamical degrees of freedom, gives varying results depending on the choice of variables (Blyth and Isham, 1975; Isham *et al.*, 1975). In the Bianchi IX case the semi-Dirac and ADM methods seem to be inequivalent (Ryan, 1972). In the Kantowski-Sachs case, the wave packet seems to spread most near the maximum of expansion, rather than the dense phase being the important one. Misner has pointed out that in the 'minisuperspaces' of the quantum models, symmetries may lead to natural choices of variable, so resolving that difficulty.

The most recent work of this type is the discussion, by a semi-Dirac method, of spin- $\frac{1}{2}$  fields in Robertson-Walker spaces by Isham and Nelson (1974).

I have left until last the work of Berger (1975a) on the Gowdy 3-torus inhomogeneous cosmology, because it forms a bridge to discussion of quantum field theory in classical curved space backgrounds. Berger shows that one can cast the Gowdy model, via the ADM method, into the form of a field of  $1 \infty^1$  degrees of freedom on a curved background. Particle states in the initial and final stages of evolution are defined and graviton creation thus discussed. Misner (1973) treated the same model as having 2 quantised degrees of freedom and found that treating the initial stage by considering the asymptotic equations near the big-bang (rather than introducing a switching-on of the Einstein equations as Berger did) led to infinite graviton production. Raine and Winlove (1975) have introduced an alternative definition of vacuum leading to finite (but unbounded) particle production rates. They point out that the governing equations of the field are the same as for a scalar field in a Bianchi I spacetime. This latter situation has been considered by Berger (1975b), and by Zel'dovich *et al.* (1974).

The background to these calculations has recently been reviewed by DeWitt (1975). Most calculations so far refer only to scalar fields. Suppose one has a self-adjoint differential operator  $F$  with the field  $\phi$  obeying  $F\phi = 0$ . By taking a suitable basis of solutions one can define creation and annihilation operators, and then define the vacuum by the condition that it is annihilated by the annihilation operators. A stress tensor can be defined as the functional derivative of the action with respect to the metric, and then we can compute the stress. A change of the basis of solutions which mixes the original basis and its conjugate gives rise to a Bogoliubov transformation of the creation and annihilation operators with the result that the 'old vacuum' contains 'new' particles. This may not be unitarily implementable (Castagnino *et al.*, 1975). This is avoided if there are globally defined positive (and negative) frequency

solutions to act as a basis, and this is true if there is a globally timelike Killing vector, i.e. if spacetime is stationary. In cosmology this is not so, and one therefore expects the disparity in the vacuum state at different times to lead to particle creation. The problem is to find a satisfactory scheme for defining the vacuum at various times and to regularise the stress tensor so that its expectation value can be used as the source in the otherwise unquantised Einstein equations.

There are several suggestions for solving this problem. They are reviewed by DeWitt (1975). The main groups are those of Parker (see e.g. Parker and Fulling, 1974) and of Zeldovich *et al.* (1974). DeWitt's scheme, which follows Schwinger, makes use of an effective Lagrangian calculated from the curved space Feynman propagator and incorporates the effect of gravitational fluctuations away from a curved space background. DeWitt shows that the Zeldovich method, which, as originally described appeared to depend on the special properties of the Bianchi I spaces, can be generalized so as to agree with his proposal. Parker and Fulling have shown that the Zeldovich formula is equivalent to their 'adiabatic regularisation' scheme. This itself also seems to be linked to the symmetries of the spaces to which they have applied it.

One difficulty is that the Schwinger method does not respect conformal invariance. One could correct this failing if one could perform dimensional regularisation but this may not be generally possible and is not unambiguous. The method has been used for the De Sitter cosmology (Candelas and Raine, 1975). Further calculations on this example have been performed by J. S. Dowker and R. Critchley of Manchester (unpublished).

It should be remarked that Schwinger's method leads to renormalisation of the cosmological and gravitational constants. More general, but less securely based, corrections of the same kind can lead to singularity avoidance (Nariai *et al.*, 1971).

Zeldovich *et al.* find that the rate of particle creation, and the resulting anisotropic stresses, become infinite as the singularity is approached in their model, and thus that there is a very strong tendency to isotropisation! Zeldovich had previously made heuristic and dimensional arguments for the same conclusion. Raine and Winlove argue for similar results. Further calculations (for Bianchi I) have been done by Fulling, Parker and Hu (1974) and for Bianchi IX by Hu *et al.*, 1973. In the latter case mode-mixing between different frequencies can occur.

Hawking (Isham *et al.*, 1975) has shown that curved space quantum field theory leads to radiation by and evaporation of black holes. Carr (1975a) has shown that observational limits put strong constraints on early universe black hole formation. He has subsequently (1975b) discussed the effect of the Hawking process, which significantly affects black holes of mass  $10^{15}$  g or less. Constraints are placed on their existence by the  $\gamma$ -ray background, but very early evaporation of very small holes could produce the energy of the microwave background.

Hawking's path integral method for creation due to black hole event horizons has recently been adapted to de Sitter space by Gibbons and Hawking and extended to deal with thermal Green's functions in de Sitter space by Gibbons and Perry (unpublished).

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## 6. UNCONVENTIONAL AND PATHOLOGICAL WORLD MODELS

(Y. Ne'eman)

In this section, we survey recent work in some of the more speculative approaches to Cosmology. Such attempts derive their motivation from the following premises:

(1) trying to account for observational data in cases where there appear to arise difficulties in a straightforward utilization of the laws of General Relativity (GR) as applied to an assumed homogeneous and isotropic cosmology (i.e. the familiar Friedmann universe, FU). We list the main difficulties involved (although several of these may just be due to observational error):

- (a) apparent anomalies in red shift ( $z$ )/magnitude and similar counts, especially for quasars.
- (b) anomalous values of  $z$  in apparent quasar-galaxy associations, and within apparent groups of galaxies.
- (c) the large power output of quasars, in a cosmological interpretation of their  $z$  values; also, some radio-interferometry reports of faster-than-light velocities within some multiple systems.
- (d) the apparent high degree of isotropy of the 2.7 K Background Radiation (BR).
- (e) theoretical difficulties in accounting for the creation of the galaxies.
- (f) unexplained numerical coincidences, many of which were pointed out by Dirac and Eddington in the thirties. A new one is the equality (within 1–2 orders of magnitude) between the energy density produced by starlight, and the value for BR.
- (g) the emergence of an infinite-density singularity in the past, a situation in which we have no understanding of the laws of physics.

(2) continuing the exploration of the wealth of cosmological solutions allowed by GR, and attempting at the same time to define the additional topological constraints which have to be imposed so as to exclude unphysical situations. This may in turn generate the need for new types of measurements.

(3) in the absence to date of a theory of Quantized Gravitation, evaluating the possible effects of quantization, in an ad-hoc approach.

(4) attempting to fit in some of the symmetries (including their 'spontaneous' breakdown) of the Physics of Particles and Fields: T, CP, scale-invariance, the charge-space symmetries of the Strong Interactions, etc.

The data mentioned in (1) cannot be considered at present to be in strong contradiction with GR; moreover, there exists, to date, no alternative theory accounting for all the difficulties listed in (1). It is plausible to assume that future observations, together with further work on (2) (3) and (4) will improve the fit between theory (incorporating GR) and observations. Some of the developments we review here may contribute to this improved application of Covariant Cosmology. However, it seems both useful and healthy to explore in addition other possibilities, and we shall touch upon two non-Einsteinian attempts which present new versions of a Steady State model (thus answering in particular point (1g)): Segal's stationary universe ('Chronogeometry') and the Hoyle–Narlikar most recent reformulation of the original Steady State model.

#### A. *Doing away with our Singular Past by going beyond General Relativity*

Both non-Einsteinian models we mentioned refute the Big Bang and thus attribute the 2.7 K BR to starlight (the effective Olbers sky). Both replace GR covariance by the assumption of conformal symmetry (i.e. the Poincaré group, scale invariance and 'pure' conformal transformations), with GR becoming a local approximation.

In *Segal's chronogeometry* (Segal, 1972, 1975),  $z$  is not due to a Doppler shift, and there is no expansion of the universe. We are back at Einstein's original static closed model as far as cosmology is concerned (there is no corresponding dynamical theory appended to date). The universe has a radius  $\sim H_0^{-1}$  and finite volume, (and thus no Olbers paradox) and the red-shifts are due to an 'aging' of light as first suggested by Shamir and Fox (1967). Aging involves a distinction between two time-axes,  $\tau$  and  $t$ ,  $t = \tan \tau$ . 'Local' time, as used in our special-relativistic physics is  $t$ , and coincides with  $\tau$  (global time) for small values. The scale of  $\tau$  is derived from  $(H_0)^{-1}$ . The theory claims to provide a better fit to  $z$ -magnitude counts, with a red-shift distance-squared law (the density distribution function is  $z^{1/2}(1+z)^{-2} dz$ ).

Some of the difficulties we listed in (1) are alleviated, but not resolved: e.g. a very different  $z$  within a 'quintet' of galaxies (such as the W 172 chain) cannot be interpreted as representing very high relative velocities, but it now uniquely implies different distances and an illusory grouping. One important test of this model requires the existence of very old galaxies, since there has been no evolution from a BB. The order of magnitude fit between the values of the age of the Universe as given by nucleosynthesis, stellar evolution and  $H_0$  is here an approximation due to the  $\tan \tau$  distribution. The 2.7 K BR is assumed to represent starlight after very many scatterings and circumnavigations of the Universe. There is no direct explanation for the (perhaps incorrect) order of magnitude insufficiency of starlight density.

Since there is to date no dynamical theory, a most important test would be the theoretical one of proving the dynamical stability of this static system. In the laboratory, experiments of the type suggested by Shamir and Fox (1967) should provide clearcut answers, when the necessary precision is reached.

In the *Hoyle–Narlikar* (Hoyle 1974, 1975; Hoyle–Narlikar 1974) ('conformal gravitation') Steady-State model, masses are just local values of a massless scalar field coupled to massless matter fields, although it is not clear whether or not this should involve observable effects. The possibility of a change in the muon's mass from  $m_\mu = m_e$  to  $m_\mu \sim 200 m_e$  has been suggested (Hoyle, 1974). Such changes would imply very different processes in the early universe! On the other hand, the observed expanding universe is assumed to represent just one 'region' where the mass field has positive values. The preceding era had negative values for that field; however, it is assumed that the gravitational interaction was still the same, i.e. the coupling to the mass field also changes sign under time-reversal. We are reminded of a model with a CP inverted universe in the preceding phase, suggested in 1967 by E. Schücking. The usual singularity of the Friedmann model is replaced by the zero-mass interface of the two regions. The difference of a factor 10–50 between the observed density of BR and that which could be due to starlight is explained by the contribution of hydrogen burning in the previous era.

Additional regions with negative values of the mass-field coexist with our region, in non-communicating space-sectors. One can speculate about the nature of the static (Newtonian) contribution of gravitation between sectors. As to horizons, the vanishing of masses (and of Schwarzschild radii) ensures that they should not appear in the early stages of our era.

This present version of the Steady State model is not yet a complete and consistent theory, and much remains to be done in order to fix on definite exact predictions which could allow a proper Popper-test.

There are in addition several theories of gravitation which differ from GR (see the analysis of C. Will, 1972; Thorne, Lee and Lightman, 1973) but their cosmological implications have not been explored. It is possible that some of these theories might also yield non-singular cosmologies. One such possibility has been pointed out by Trautman (1973) for an Einstein–Cartan theory of gravitation. Here the singularity may be averted by all particle spins being correlated in the first  $10^{-10}$  s of the Universe (see also Raychaudhuri, 1975).

In the oscillating FU model suggested by Gott (Gott, 1974), we also have a T (and CP) inverted past era (or region). However, because of the thermodynamical arrow of time, the 'inhabitants' of that era also think of themselves as residing in an expanding universe, rather than a contracting one. If CPT is conserved, this is consistent, provided they redefine matter as anti-matter (Ne'eman, 1968, 1970; Aharony and Ne'eman, 1970, 1973). In addition, there are 'space-like' solutions representing a tachyon (Bilaniuk, Deshpande, and Sudarshan, 1962) universe, where one is doomed to experience the 'inexorable flow of space' instead of our own feeling of the inexorability of the passage of time (Ne'eman, 1975). This is because our own always-allowed rest-frames (leaving only  $\Delta t \neq 0$ ) are replaced by the availability of time-arrested frames (with only  $\Delta x \neq 0$ ). All of these regions are joined together analytically by getting rid of the singularity through the introduction of an ad hoc negative pressure term in the earliest moments ( $10^{-40}$  s) of our own universe. Such a negative pressure would just stand (Parker and Fulling, 1973; see also Beckenstein, 1975) for as yet little explored effects (or an effective viscosity, as in some of the other models we shall mention), in the same spirit as the suggested emergence of an effective  $\Lambda$  cosmological constant from quantum effects (Sakharov, 1967). A positive non-zero  $\Lambda$  acts as a negative energy-density and can also cancel the singularity.

Having noted the attempts at cosmological completeness under CPT, we should add that an important insight into a necessary connection with the Physics of Particles and Fields has come from attempts to check the cosmological implications of those models of Particle Quantum Symmetry-Breaking in which the theory is made invariant, whereas it is the vacuum which is assumed to break the symmetry ('spontaneous' symmetry-breaking like in Heisenberg's ferromagnet). In particular, Kobsarev, Grun and Zeldovich (1974) have shown that the isotropy of BR (up to  $10^{-3}$ ) makes it rather implausible to believe that the CP violation (or T violation) discovered in  $K^0$  meson decay could be 'blamed' on the vacuum. If that were true, we would have observed a domain structure in the universe, with 'walls' (i.e. anisotropies). However, this argument is proved as yet only in the simplest models.

#### B. *Phenomenological Rehabilitation within the Framework of Einsteinian Law*

The task of fitting the observational material (optical, radio,  $\gamma$ , X-ray and microwave) to a conventional Friedmann universe (homogeneous and isotropic) is covered by the other reviewers. We shall only mention that although the various items in the data have still not reached the required confidence level, the impression one receives is that the normal range of Robertson–Walker metrics will contain a model that will fit the general structure of the post BR universe. As to the earlier phase, this reviewer tends to adopt some of the more recent versions of the approach suggested by Misner (1968) of a homogeneous but anisotropic model later evolving into the present FU. The original model was received with scepticism, since it appeared the 'mixing' might be insufficient or too slow (Doroshkevich *et al.*, 1971). The analysis by Belinskii and Khalatnikov (1975) of an improved version in which the singularity is cancelled through an assumption of viscosity (effectively, a negative pressure, violating only the stronger of Hawking's criteria) following Matzner and Misner (1972) and Murphy (1973) indicates that such an evolution is highly plausible.

As a philosophical digression, we note the failure of the ‘Copernican Principle’ as it was applied to the universe, namely that ‘we do not occupy a privileged position in space-time’ (Bondi, 1960) or that ‘we may assume that our view of the universe is not a preferred picture’ (Ellis, 1971). This was in fact the second, and more speculative principle, with the first just implying ‘that local physical laws are the same everywhere in the Universe’ (Ellis, 1971a). In all recent treatments (including the newest version of the Steady State) except for Segal’s, and especially in treatments respecting GR, we are led to at least two different phases in the history of the last 10–20 billion years, and we may be observing in our distant past just that border region between the two eras. Our present conditions are only typical of the second phase; indeed it is only roughly typical of it, since the density of matter has changed tremendously within that phase itself. In the method of physical science, such symmetry assumptions as the second Copernican Principle, make excellent starting points for the unifying synthesis stage in the construction of a theory. They tend to be premature in the exploratory stage, where it seems more appropriate to adopt Hamlet’s statement that ‘there are more things in Heaven and Earth, Horatio than are dreamt of in your philosophy’.

Returning to the pre-BR universe, present studies have exploited the homogeneous (but anisotropic) models of GR, as classified by Bianchi, Schüking and Behr (Bianchi, 1897; Schüking, 1962; Behr, 1962, 1965a, 1965b; Estabrook *et al.*, 1968; Jantzen, 1975). Generally speaking, the anisotropy (such as having three different functions of  $t$  multiplying  $dx^2$ ,  $dy^2$  and  $dz^2$  in the metric of type I) presumably generates elongated light cones that manage to sweep away the horizons expected in the isotropic case. Treatments introducing macroscopic concepts such as the viscosity parameters still require a check of relativistic microscopic consistency, as a guarantee against non-local (and non-relativistic) effects.

We leave the detailed discussion of the Bianchi types and of quantum effects to other reviewers, and treat in the broadest lines recent studies of pathologies. These apply topological and other methods in an attempt at improving our understanding of the structure and role of the primordial singularity, and the implications of time-ordering or of causality.

### C. Pathologies

The study of topological pathologies appearing in relativistic world model has gone through three main phases to date. In the first phase, investigations dealt with the question of the singularities’ inevitability for the physical universe. After some optimism generated by the work of Lifshitz and Khalatnikov (1963), the more pessimistic (and fatalistic . . .) theorems of Hawking and Penrose (see Hawking and Ellis, 1973, ch. 8) settled the issue. Singularities are unavoidable, under some relatively mild conditions, e.g. the existence of a non-negative energy density (‘the energy condition’), as measured by any observer, the inexistence of closed time-loops allowing a long-lived observer to return to his own past, as in the Gödel (1949) universe (i.e. an expression of causality), universal expansion, and a geometric generic condition.

In the second stage, it was noticed that singularities did not always imply an infinite density of matter (Shepley, 1969). Some singularities appeared to represent catastrophic world-line features: rather than an infinite squeeze of the observer, they might just cause him to cease to exist after a given moment. Such vanishing (when a certain parameter intrinsic to the trajectory, the ‘generalized affine parameter’, would reach some given value) is called ‘incompleteness’ (or ‘confined’ incompleteness for that particular type, where the world-line is confined to one part of the model). This ‘topological death’ of the subject may take the form of an infinite spiral which never crosses the fatal moment [Taub-NUT space: (Taub, 1951; Newman *et al.*, 1963)]. Another characteristic of this particular type of incompleteness reflects itself in non-Hausdorff properties of the ‘Schmidt-completion’ (Schmidt, 1971, 1973) of that World-Model, a particular type of extension of the physical universe in which one has been able to adjoin to space-time some unphysical ‘missing pieces’. ‘Hausdorff’ means that any two distinct points can respectively be embedded in two disjoint open subspaces (this is not true of the fork-point on the two branches of a half-opened zipper, for instance i.e. a zipper is non-Hausdorff). For an observer’s world-line this ‘zipper behaviour’ is not a healthy state and

can be regarded as another type of split-personality (Hajicek, 1971). However, there are less inconvenient non-Hausdorff models, and one may end up allowing some such classes in a physical model (Miller and Kruskal, 1973). On some space-time completion, this behaviour can be just a way of identifying 'confined incompleteness' singularities of the embedded physical model.

The study of singularities and understanding of the connections between material and world-line types requires the invention of new and ever more powerful topological tools (Schmidt, 1971; Eardley, *et al.*, 1972; Sachs, 1973; Duncan and Shepley, 1974, 1975), and we are still very far from achieving an understanding based on sound deductive methods. Meanwhile, the third and more recent phase has seen several inductive programs. With the availability of a complete classification of all homogeneous but anisotropic world-models (Bianchi-types) and a certain familiarity with these cosmologies in the context of the Misner program of 'mixmaster' isotropization, it became possible to search for characteristic singularities in these models. This was achieved in one way: singularity splits in two (with different evolutions). However, Eardley (1974) has looked at the 'lagging core' model astrophysically, and claims that such white holes can only be very short-lived (it would require  $M \approx 10^{17} M_{\odot}$  to survive for  $10^7$  yr, until the end of the period of recombination). They would attract photons and matter and would become black holes. Since no such black holes are known to lie around, there were probably no large white holes, according to Eardley, and the pre-BR universe cannot have been too inhomogeneous (Carr, 1975) [quantum tunneling effects will have evaporated away lumps of less than  $10^{15}$  g only (Hawking, 1974; Page, 1975)]. Note that according to a recent study by Hawking (1975), black and white holes are indistinguishable to an outside observer, due to quantum statistical effects. See also Narlikar–Apparao (1975) for new white hole prediction.

This problem is also related to the conjecture that GR forbids the existence of 'naked' singularities, i.e. singularities which are not hidden by matter. Yodzis *et al.* (1973) produce a counter-example, so that the conjecture cannot be true in its crudest form.

One general conclusion from the above works is the importance of the equation of state. Actually, the final answer seems to depend only in part on the topological properties, and the physical conditions are an essential input. A further illustration of this state of affairs can be derived from a comparison between the opposite results found by Collins and Hawking (1973) on the one hand, and Belinskii and Khalatnikov (1975) on the other hand. Both studies were searching for an answer to the same problem: can the present models in which the matter content of space-time is a perfect fluid, but with the fluid flow vector not normal ('tilted') to the 3-surfaces of homogeneity. The matter may thus move with non-zero expansion, rotation and shear (King and Ellis, 1973; Ellis and King, 1974; Collins, 1974). The Bianchi classification does not segregate world-line singularities from the others, and examples can apparently be found in all models. All non-tilted models (perhaps with the exception of type IX) have a matter singularity. At least some type V models have an 'intermediate' singularity, where the matter density does not diverge (though it may do it elsewhere in the model). There are as yet no clear-cut gains in understanding, as a result of this inductive method, beyond the first example (Shepley, 1969), although much effort has gone into the construction of 'tilted' models.

Another inductive program has dealt with velocity-dominated singularities in irrotational dust cosmologies (Eardley *et al.*, 1972). In these ELS models [including the Tolman–Bondi (Bondi, 1947) spherically symmetric types] an iterative approach yields some information about 3-space at the time of the singularity (including 'dating', i.e. if there is more than one singularity, we learn about their relative timing). This represents an advance in the treatment of inhomogeneous models (see also Szekeres, 1975 for a generalization) in which there is more than one BB (Novikov, 1964; Ne'eman, 1965), i.e. a universe with large white-holes. ELS do indeed find one explicit model in which the singularity is 'regional' in the beginning, i.e. most of the universe is Friedmann-like, while there is 'a lump' in one region for a while. In another example, the original Friedmann universe has evolved naturally out of a 'chaotic' anisotropic model. The Cambridge team proved that the set of self-isotropization models is of measure zero, i.e. a Friedmann universe is a most improbable outcome of dynamical evolution, and we

exist by yet another Darwinian evolutionary chance-mutation, at the cosmological level. The Landau Institute team have just found just the opposite result: all possible anisotropic beginnings lead to the same inevitable Friedmann universe.

This contradiction is due to the different physical assumptions: the Landau Institute team introduced viscosity and thereby removed the singularity altogether. This approach seems justifiable for the early universe; however, when the isotropic stage is reached there is probably very little viscosity left. Thus an improved model should lie in between the two calculations, and a more reliable answer should thus await the next iteration. It should also be interesting to check on the actual class of Friedmann models reached (since the Cambridge calculation points out the low probability of a model with escape velocity).

In an entirely different line of work, some progress has been made in the study of the intrinsic global topology of space-time in view of explaining its causal structure. Hawking, King and McCarthy (1975) have suggested a topology determining the causal, differential and conformal structure of space-time, replacing a suggestion of Zeeman (1964). Zeeman had noted that the group preserving time-ordering along paths of free particles is the Poincaré group and scale invariance. The new topology preserves such partial ordering (time-ordering, or along a null line) along arbitrary Feynman paths, i.e. interacting particles. This selects the full conformal group, perhaps at the price of replacing continuity by countability in space-like directions, a point to be further studied.

Yet one more line of topological investigation deals with the global geometry of the universe. For Friedmann world models,  $k = +1$  (closed, spherical) fits only  $S^3$  (the 3-dimensional sphere) or  $P^3$  (real projective 3-space). For  $k = 0$  (flat) only  $R^3$  (real 3-dimensional) is allowed; for  $k = -1$  only  $H^3$  (open). For anisotropic models, the possibilities are much more numerous (see review in Criss *et al.*, 1975). It is dangerous to exclude altogether inhomogeneous and anisotropic spaces, because they could look like Friedmann models locally (Ellis, 1971).

Our last remark relates to a social context. In recent years we have been exposed to strong criticism from both outside and inside the scientific establishment, for not caring enough about the 'relevance' of our work. Both Astronomy and Relativity Physics are considered as relatively parasitic in that utilitarian approach, and Cosmology is in an even worse position. However, a recent study (Tipler, 1974) discusses the gravitational field of a rapidly rotating infinite cylinder (van Stockum, 1937). This has closed time-like lines, as in the Gödel universe (Gödel, 1949). It seems possible to preserve these lines even for a very long finite cylinder, and we may have here a formula for a time-machine. This should certainly prove the usefulness of cosmology.

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