Non-linear effects in laminar flow
over wavy surfaces

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We examine linear and non-linear effects in laminar flows over several
types of wavy surfaces for various régimes of the wave amplitude ε and
the Reynolds number R . In particular, we are interested in the surface
pressure and shear stress, which are important in considering wave growth.

In Section 1 we extend linear theory for parallel flows to include
both third-order terms and finite Reynolds numbers. The Fourier-type
expansion in ε produces forced Orr-Sommerfeld problems which are solved
numerically for various values of ε and R . With increasing ε , the
surface stress profiles become distorted from sinusoids and the location of
the surface extrema migrate from their linear positions. Effects of
'fluid' boundary conditions, as opposed to 'solid' and 'flexible sheet'
conditions are examined. The stress contours reveal elevated extrema out
out of phase with the surface values. It is shown in two ways that an
expansion of the above type requires $\varepsilon^2 R^{4/3} \ll 1$, a much stricter
condition than previously recognized.

In Section 2 we examine perturbations to the Blasius flow over a flat
plate caused by two types of disturbances on the plate. Calculations are
performed in parabolic co-ordinates (σ, τ) and expansions made in powers
of ε and then in powers of $R^{-\frac{1}{2}}$. The inner and outer solutions of the

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resulting matched asymptotic expansion scheme are solved to $O(\varepsilon^2)$. The surface stresses exhibit different behaviour to those of Section 1 and we obtain the restriction $\varepsilon \ll x^{\frac{1}{2}}\varepsilon < 0.08$ for the expansion to be valid.

    To try and overcome this severe restriction, triple-deck scalings for the wave amplitude and wavelength are assumed in Section 3, eventually reducing the problem to solving the lower deck equations. Analytical results are obtained for the linearized case, and then the full non-linear equations are solved numerically via a specially modified spectral method. Some of the linear results of Section 1 are derived as a limiting case of the linear theory. We study effects of increasing non-linearity and distance downstream and examine the resulting asymmetries, the behaviour near the leading edge of the disturbance and the onset of separation. Finally, it is shown that the first term in the lower deck expansion is enough to provide the first two terms in the main deck.

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